Transition radiation on semi-infinite plate and Smith-Purcell effect

N F Shul'ga[1] and V V Syshchenko[2]

[1] Akhiezer Institute for Theoretical Physics of the NSC "KIPT", Akademicheskaya Street, 1, Kharkov 61108, Ukraine
[2] Belgorod State University, Pobedy Street, 85, Belgorod 308015, Russian Federation

E-mail: shulga@kipt.kharkov.ua, syshch@bsu.edu.ru, syshch@yandex.ru

Abstract. The Smith-Purcell radiation is usually measured when an electron passes over the grating of metallic stripes. However, for high frequencies (exceeding the plasma frequency of the grating material) none material could be treated as a conductor, but ought to be considered as a dielectric with plasma-like permittivity. So for describing Smith-Purcell radiation in the range of high frequencies new theoretical approaches are needed. In the present paper we apply the simple variant of eikonal approximation developed earlier to the case of radiation on the set of parallel semi-infinite dielectric plates. The formulae obtained describe the radiation generated by the particles both passing through the plates (traditionally referred as "transition radiation") and moving in vacuum over the grating formed by the edges of the plates (traditionally referred as "diffraction radiation", and, taking into account the periodicity of the plates arrangement, as Smith-Purcell radiation).

1. Introduction

An interest to Smith-Purcell effect [1] both as a prospective way for generation of electromagnetic radiation from TeraHertz frequencies [2] to soft X-ray range [3] and as a novel method of beam monitoring (see, e.g., [4]) continuously grows during last years. Usually the Smith-Purcell radiation is registered when an electron passes over the grating of metallic stripes. However, for high frequencies (exceeding the plasma frequency ω_p of the grating material) none material could be treated as a perfect conductor, but ought to be considered as a dielectric with permittivity $\varepsilon_\omega = 1 - \omega_p^2/\omega^2$. So for describing Smith-Purcell radiation in the range of high frequencies new theoretical approaches are needed.

A simple variant of eikonal approximation suitable for description of transition radiation on the targets with complex geometry was proposed in [5, 6]. In the present paper we apply that method to the case of radiation on the set of parallel semi-infinite dielectric plates. The formulae obtained describe the radiation generated by the particles both passing through the plates (traditionally referred as "transition radiation") and moving in vacuum over the grating formed by the edges of the plates (traditionally referred as "diffraction radiation", and, taking into account the periodicity of the plates arrangement, as Smith-Purcell radiation).

In our article we use the system of units where the speed of light $c = 1$.

2. Radiation on the semi-infinite plate

The spectral-angular density of the transition radiation (TR) could be expressed in the form [5, 6, 7]

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{\omega^2}{(8\pi^2)^2} \left| \mathbf{k} \times \mathbf{I} \right|^2,$$ (1)

where $\mathbf{k}$ is the wave vector of the radiated wave,

$$\mathbf{I} = \int (1 - \varepsilon_\omega(\mathbf{r})) \mathbf{E}_\omega(\mathbf{r}) e^{-i\mathbf{kr}} d^3 r,$$ (2)

$\mathbf{E}_\omega$ is the Fourier component by time,

$$\mathbf{E}_\omega(\mathbf{r}) = \int_{-\infty}^\infty \mathbf{E}(\mathbf{r}, t) e^{i\omega t} dt,$$

of the electric field produced by the moving particle in the substance of the target with the dielectric permittivity $\varepsilon_\omega(\mathbf{r})$.

If $|1 - \varepsilon_\omega(\mathbf{r})| \ll 1$, the precise value of the field in the target in (2) could be replaced in the first approximation by non-disturbed Coulomb field of the uniformly moving particle in vacuum. Although this approximation permits to calculate the TR characteristics for the targets with complex geometry [7], its applicability is restricted by the range of extremely high frequencies,

$$\omega \gg \gamma \omega_p,$$ (3)

where $\gamma = \sqrt{1 - v^2}$ is the particle's Lorentz factor. For investigation of TR in the range of more soft photons, some different approximate method is needed.

A simple variant of eikonal approximation in TR theory was developed in [5, 6]. In that approach the component of $\mathbf{I}$ perpendicular to the particle's velocity $\mathbf{v}$ could be written in the form

$$\mathbf{I}_\perp^{(eik)} = i\frac{4e}{v^2\gamma} \int d^2\rho\, e^{-i\mathbf{k}_\perp \boldsymbol{\rho}} \frac{\boldsymbol{\rho}}{\rho} K_1\left(\frac{\omega\rho}{v\gamma}\right) \left\{ \exp\left[-i\frac{\omega}{2} \int_{-\infty}^\infty (1 - \varepsilon_\omega(\mathbf{r}))\, dz \right] - 1 \right\},$$ (4)

where $K_n(x)$ is the modified Bessel function of the third kind, $\boldsymbol{\rho}$ is the component of $\mathbf{r}$ perpendicular to $\mathbf{v}$. The eikonal approximation in TR theory is valid for

$$\omega \gg \omega_p$$ (5)

(compare with the condition(3)), but only for small radiation angles,

$$\theta \ll 1/\sqrt{a\omega},$$ (6)

where a is the thickness of the target (the last constraint leads to the possibility of taking into account only the transverse component of $\mathbf{I}$).

Consider the radiation arising under the interaction of the moving particle with the semi-infinite dielectric plate of the thickness a (figure 1). In this case the application of (4) and (1) leads to the following result for the spectral-angular density of the radiation:

$$\frac{d\mathcal{E}}{d\omega d\Omega} = \frac{e^2\gamma^2}{2\pi^2} \left[1 - \cos\left(\frac{a\omega}{2} \frac{\omega_p^2}{\omega^2} \right) \right] F(\theta_x, \theta_y),$$ (7)

where θ_x and θ_y are the components of the two-dimensional radiation angle $\boldsymbol{\theta}$,

$$F(\theta_x, \theta_y) = \frac{1 + 2\gamma^2\theta_y^2}{(1 + \gamma^2\theta_y^2)(1 + \gamma^2\theta^2)} \exp\left(-2\left|x_0\right| (\omega/\gamma)\sqrt{1 + \gamma^2\theta_y^2} \right),$$ (8a)
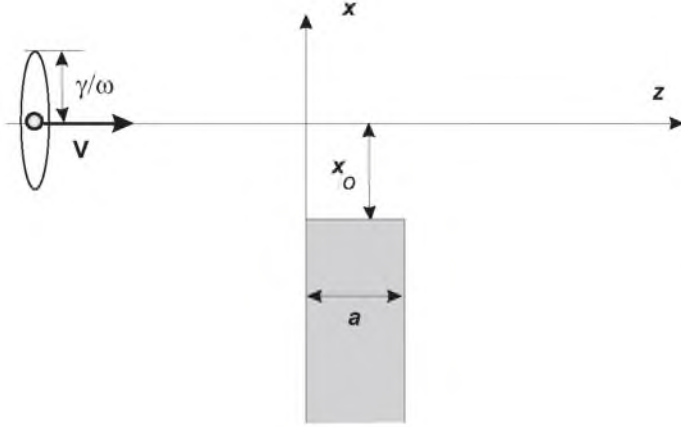
Figure 1. Particle's motion near the edge of the plate. The value γ/ω is the characteristic transverse dimension of the Fourier component of the particle's Coulomb field corresponding to the frequency ω. Positive values of the impact parameter x_0 correspond to the case of penetration of the particle through the plate.

under $x_0 \leq 0$, and

$$F(\theta_x, \theta_y) = \frac{4\gamma^2\theta^2}{(1+\gamma^2\theta^2)^2} + \tag{8b}$$

$$+\frac{1+2\gamma^2\theta_y^2}{\left(1+\gamma^2\theta_y^2\right)\left(1+\gamma^2\theta^2\right)} \exp\left(-2\left|x_0\right|(\omega/\gamma)\sqrt{1+\gamma^2\theta_y^2}\right) -$$

$$-\frac{4\exp\left(-\left|x_0\right|(\omega/\gamma)\sqrt{1+\gamma^2\theta_y^2}\right)}{(+\gamma^2\theta^2)^2}\left(\frac{\gamma\theta_x\sin(x_0\omega\theta_x)}{\sqrt{1+\gamma^2\theta_y^2}} + \gamma^2\theta^2\cos(x_0\omega\theta_x)\right),$$

under $x_0 > 0$. This formula is in agreement with the result of the paper [8] obtained using the method [9].

Surface plots of the function $F(\theta_x, \theta_y)$ for some values of the impact parameter x_0 are presented on the figure 2.

3. Radiation on the grating

Consider now the radiation arising under incidence on the periodic set of such plates (figure 3).

Direct account of the periodic structure of the target is impossible because of the integration along the z axis in the exponent in (4). Nevertheless, if the formation length of the radiation on the single plate do not exceed the distance between two plates,

$$l_{coh} = \frac{2\gamma^2/\omega}{1+\gamma^2\theta^2+\gamma^2\omega_p^2/\omega^2} < b - a, \tag{9}$$

the problem is easy for computation: the effect of the target's complexity will consist in the interference of the electromagnetic waves emitted under the interaction of the moving particle with separated plates.

The radiation in this case would be described by equation (7) multiplied by the factor

$$2\pi N \sum_{m=-\infty}^{\infty} \delta\left\{\omega b\left(\frac{1}{v} - \cos\theta\right) - 2\pi m\right\}, \tag{10}$$

where N is the total number of plates, $N \gg 1$ (like in the most part of the papers devoted to TR, we neglect the refraction of the emitted radiation in the substance of the target). These
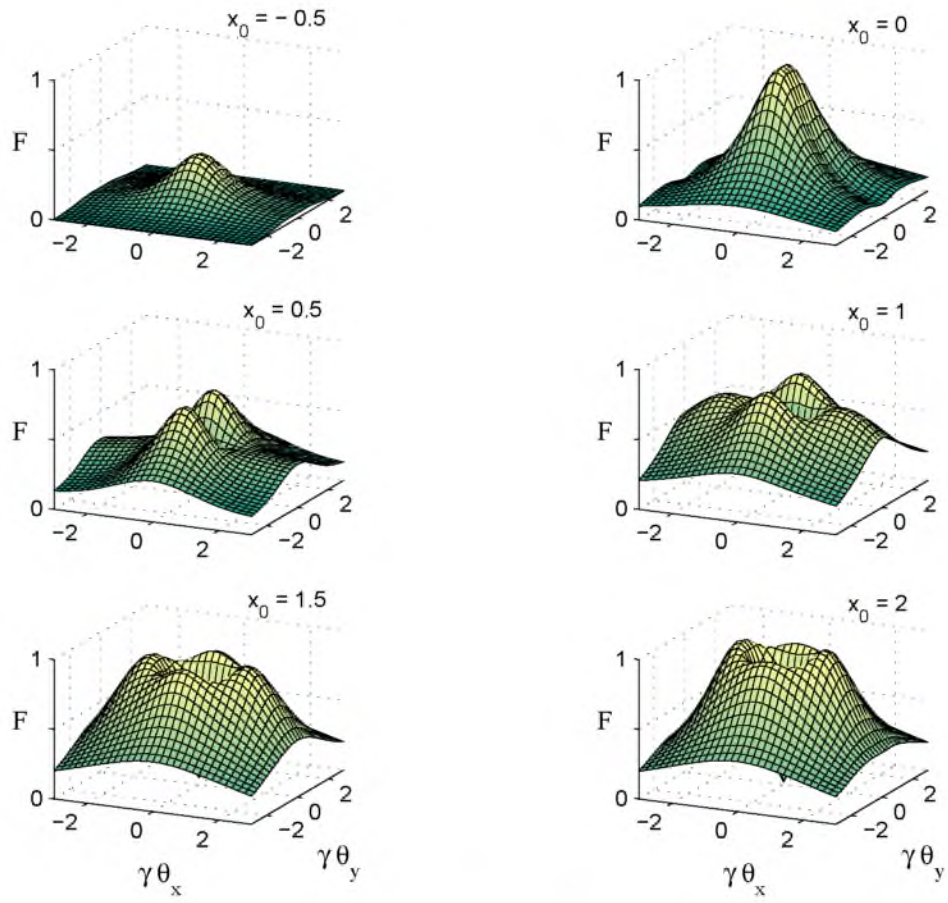
Figure 2. Angular distribution of radiation on a single semi-infinite plate according to (8). Impact parameter x_0 is given in the units of γ/ω.
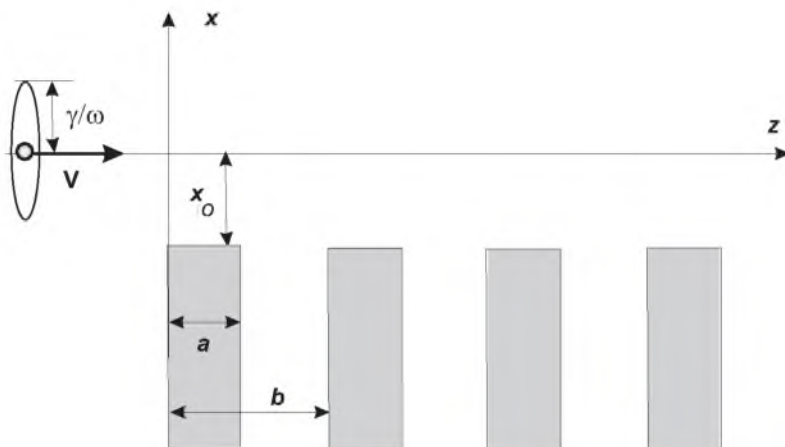

Figure 3. Grazing incidence of the particle onto the grating

δ-functions mean that the radiation under angle θ would take the place only for the frequencies satisfying the condition

$$\omega = \frac{2\pi m}{b\left(\dfrac{1}{v} - \cos\theta\right)} \qquad (11a)$$

(m is the positive integer) or, for small angles,

$$\omega_m = \frac{2\gamma^2}{1 + \gamma^2\theta_m^2}\,\frac{2\pi}{b}\,m. \qquad (11b)$$

This is well known Smith-Purcell condition [1].

4. Conclusion

Equations (7), (8a), (10) describe the Smith-Purcell radiation in the high-frequency limit,

$$\omega \gg \omega_p.$$

The exponent in (8a) describes the radiation intensity dependence on the impact parameter. The characteristic values of the last one, on which the radiation is substantial, are

$$\left. x_0\right|_{eff} = \gamma/2\omega.$$

Note that such the exponent is present in the formulae describing Smith-Purcell effect independently on the range of frequencies and the computation method (see, e.g., [10]; the results of different models are distinct from each other only by pre-exponential factors).

In the region $x_0 > 0$ when the particle's trajectory penetrates the plates, the radiation arising is described by equations (7), (8b), (10). The first term in (8b) describes the transition radiation on the infinite plate in the eikonal approximation [5, 6], the second term accounts the edge of the plate (its contribution evidently coincides with Smith-Purcell radiation), the third term describes the interference of that two mechanisms.

In the frames of our approach, for both negative and positive x_0 the angular distribution of the radiation is symmetric in relation to the (y, z) plane.

Acknowledgments

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