Acoustic and filtration properties of a thermoelastic porous medium: Biot's equations of thermo-poroelasticity

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Abstract. A linear system of differential equations describing the joint motion of a thermoelastic porous body and an incompressible thermofluid occupying a porous space is considered. Although the problem is linear, it is very hard to tackle due to the fact that its main differential equations involve non-smooth rapidly oscillating coefficients, inside the differentiatial operators. A rigorous substantiation based on Nguetseng's two-scale convergence method is carried out for the procedure of the derivation of homogenized equations (not containing rapidly oscillating coefficients), which for different combinations of the physical parameters can represent Biot's system of equations of thermo-poroelasticity, the system consisting of Lamé's non-isotropic equations of thermoelasticity for the solid component and the acoustic equations for the fluid component of a two-temperature two-velocity continuum, or Lamé's non-isotropic thermoelastic system for a two-temperature one-velocity continuum.

Bibliography: 16 titles.

Introduction

In this paper we consider the problem of a joint motion of a thermoelastic deformable solid (the thermoelastic *skeleton*) perforated by a system of channels and pores and an incompressible thermofluid occupying the *porous space*. We refer to this as the (**NA**) model. In dimensionless variables (without primes)

$$\mathbf{x}' = L\mathbf{x}, \qquad t' = \tau t, \qquad \mathbf{w}' = L\mathbf{w}, \qquad \theta' = \vartheta_* \frac{L}{\tau v_*}\,\theta$$

the differential equations of the model for small values of the dimensionless displacement vector $\mathbf{w}$ and small deviations of the dimensionless temperature θ in a domain $\Omega \subset \mathbb{R}^3$ have the following form:

$$\alpha_\tau \overline{\rho}\,\frac{\partial^2 \mathbf{w}}{\partial t^2} = \operatorname{div}\mathbb{P} + \overline{\rho}\mathbf{F}, \tag{0.1}$$

$$\alpha_\tau \overline{c}_p \frac{\partial \theta}{\partial t} = \operatorname{div}(\overline{\alpha}_\varkappa \nabla_x \theta) - \overline{\alpha}_\theta \frac{\partial}{\partial t}\operatorname{div}\mathbf{w} + \Psi, \tag{0.2}$$

where the stress tensor of the continuous medium

$$\mathbb{P} = \overline{\chi}\mathbb{P}^f + (1-\overline{\chi})\mathbb{P}^s \tag{0.3}$$

AMS 2000 *Mathematics Subject Classification.* Primary 35Q72, 74Q10, 74B05, 76M50, 76S05.

coincides with the viscous stress tensor

$$\mathbb{P}^f = \alpha_\mu \mathbb{D}\Big(x, \frac{\partial \mathbf{w}}{\partial t}\Big) - (p_f + \alpha_{\theta f}\theta)\mathbb{I} \tag{0.4}$$

in the fluid and the elastic stress tensor

$$\mathbb{P}^s = \alpha_\lambda \mathbb{D}(x, \mathbf{w}) - (-\alpha_\eta \operatorname{div} \mathbf{w} + \alpha_{\theta s}\theta)\mathbb{I} \tag{0.5}$$

in the skeleton. The pressure p_f in the fluid can be found from the continuity equation

$$p_f + \overline{\chi}\alpha_p \operatorname{div} \mathbf{w} = 0. \tag{0.6}$$

Here and throughout we use the notation

$$\mathbb{D}(x, \mathbf{u}) = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T),$$

$$\overline{\rho} = \overline{\chi}\rho_f + (1 - \overline{\chi})\rho_s, \qquad \overline{c}_p = \overline{\chi}c_{pf} + (1 - \overline{\chi})c_{ps},$$

$$\overline{\alpha}_\varkappa = \overline{\chi}\alpha_{\varkappa f} + (1 - \overline{\chi})\alpha_{\varkappa s}, \qquad \overline{\alpha}_\theta = \overline{\chi}\alpha_{\theta f} + (1 - \overline{\chi})\alpha_{\theta s}.$$

The characteristic function $\overline{\chi}(\mathbf{x})$ of the porous space $\Omega_f \subset \Omega$ is assumed to be known.

For the derivation of (0.1)–(0.6) and the description of dimensionless constants (which are all strictly positive) see [1].

We endow the model (**NA**) with homogeneous initial and boundary conditions

$$\mathbf{w}\big|_{t=0} = 0, \quad \frac{\partial \mathbf{w}}{\partial t}\Big|_{t=0} = 0, \quad \theta\big|_{t=0} = 0, \qquad \mathbf{x} \in \Omega, \tag{0.7}$$

$$\mathbf{w} = 0, \quad \theta = 0, \qquad \mathbf{x} \in S = \partial\Omega, \quad t \geqslant 0. \tag{0.8}$$

From the mathematical point of view the corresponding initial-boundary value problem is well-posed in the following sense: it is uniquely soluble in a suitable function space on any finite time interval (see [1]). However, this model is inefficient from the standpoint of possible applications, such as numerical calculations. Therefore, the question of finding approximate models is of importance. If the model involves a small parameter ε, then the optimal approximations must have limiting regimes that tend to the exact model as ε approaches zero. A natural small parameter in the model under consideration is the characteristic size l of the pores divided by the characteristic size L of the domain:

$$\varepsilon = \frac{l}{L}.$$

Such an approximation significantly simplifies the original problem and at the same time preserves all its main features. However, even with a small parameter the model (**NA**) remains rather complicated and some additional simplifying assumptions are required. In terms of the geometric properties of the medium the most appropriate such simplification is to postulate that the porous structure is periodic. In what follows we will call this ‘submodel’ of the model (**NA**) the model $(\mathbf{NB})^\varepsilon$.

Our main aim is to derivate limit regimes (homogenized equations) for the model $(\mathbf{NB})^\varepsilon$.

We make the following assumptions.

Assumption 1. The domain $\Omega = (0,1)^3$ is a periodic repetition of an elementary cell $Y^\varepsilon = \varepsilon Y$, where $Y = (0,1)^3$. The quantity $1/\varepsilon$ is integer, so that Ω contains an integer number of elementary cells. Let Y_s be the 'solid part' of Y and assume that the 'fluid part' Y_f is its open complement. We also set $\gamma = \partial Y_f \cap \partial Y_s$. The boundary γ must be a C^1-surface, the porous space Ω_f^ε is the periodic repetition of the elementary cell εY_f, the solid skeleton Ω_s^ε is the periodic repetition of the elementary cell εY_s, and the C^1-boundary $\Gamma^\varepsilon = \partial\Omega_s^\varepsilon \cap \partial\Omega_f^\varepsilon$ is the periodic repetition in Ω of the boundary $\varepsilon\gamma$. The skeleton $\mathbf{\Omega}_s$ is a connected domain.

In these assumptions

$$\overline{\chi}(\mathbf{x}) = \chi^\varepsilon(\mathbf{x}) = \chi\left(\frac{\mathbf{x}}{\varepsilon}\right),$$

$$\begin{aligned}
\overline{c}_p &= c_p^\varepsilon(\mathbf{x}) = \chi^\varepsilon(\mathbf{x})c_{pf} + \big(1-\chi^\varepsilon(\mathbf{x})\big)c_{ps},\\
\overline{\rho} &= \rho^\varepsilon(\mathbf{x}) = \chi^\varepsilon(\mathbf{x})\rho_f + \big(1-\chi^\varepsilon(\mathbf{x})\big)\rho_s,\\
\overline{\alpha}_\varkappa &= \alpha_\varkappa^\varepsilon(\mathbf{x}) = \chi^\varepsilon(\mathbf{x})\alpha_{\varkappa f} + \big(1-\chi^\varepsilon(\mathbf{x})\big)\alpha_{\varkappa s},\\
\overline{\alpha}_\theta &= \alpha_\theta^\varepsilon(\mathbf{x}) = \chi^\varepsilon(\mathbf{x})\alpha_{\theta f} + \big(1-\chi^\varepsilon(\mathbf{x})\big)\alpha_{\theta s};
\end{aligned}$$

where $\chi(\mathbf{y})$ is the characteristic function of Y_f in Y.

We say that the *porous space is disconnected* (isolated pores) if $\gamma \cap \partial Y = \varnothing$.

In this paper we suppose that all the dimensionless parameters below depend on the small parameter ε and there exist (finite or infinite) limits

$$\lim_{\varepsilon\searrow 0}\alpha_\mu(\varepsilon) = \mu_0, \qquad \lim_{\varepsilon\searrow 0}\alpha_\lambda(\varepsilon) = \lambda_0, \qquad \lim_{\varepsilon\searrow 0}\alpha_\tau(\varepsilon) = \tau_0, \qquad \lim_{\varepsilon\searrow 0}\alpha_p(\varepsilon) = p_*.$$

Moreover, we only consider the case when $\tau_0 < \infty$ and

$$\mu_0 = 0, \qquad p_* = \infty, \qquad 0 < \lambda_0 < \infty.$$

If $\tau_0 = \infty$, then we renormalize the displacement vector and the temperature by setting

$$\mathbf{w} \to \alpha_\tau \mathbf{w}, \qquad \theta \to \alpha_\tau\theta$$

and reduce the problem to the previous case. The condition $p_* = \infty$ means that the fluid under consideration is incompressible.

Using Nguetseng's two-scale convergence method (see [2]) we shall show that, depending on the relations between the dimensionless parameters of the model and the geometry of the elementary cells Y_s and Y_f, the limit regimes can be: Biot's system of equations of thermo-poroelasticity, the system consisting of Lamé's non-isotropic equations of thermoelasticity for the solid component and the acoustic equations for the fluid component of a two-temperature two-velocity continuum, or Lamé's non-isotropic system for a two-temperature one-velocity continuum.

We now discuss the subject of this paper in greater detail. Since the differential equations under consideration contain discontinuous coefficients inside differentiatial operators, the original system of equations reduces in a natural fashion to a system of integral identities with well-defined terms. The homogenization of this system for a family of solution $(\mathbf{w}^\varepsilon, \theta^\varepsilon)$ depending on a small parameter ε reduces to the following steps:

– picking a subsequence of solutions convergent as $\varepsilon \searrow 0$ (and finding the limit solution);
– finding a system of equations (the homogenized system) solved by the limit solution.

For the first step we require bounds for solutions that are uniform in ε. Finding estimates for the solutions in the cases $\tau_0 = 0$ and $p_* = \infty$ is non-trivial.

At the second step we must pass to the limit as $\varepsilon \searrow 0$ in integrals in the situation when some terms are products of several factors each of which converges only weakly in $L^2(\Omega_T)$. It is at this point that we use Nguetseng's method, which is popular in homogenization theory (see, for instance, the survey [3] and Zhikov's papers [4]–[6]). This method is fairly simple in concept, but the solution of the corresponding microscopic equations on the elementary cell is technically complicated and requires many calculations, which we shall normally leave out, presenting only the final result.

We also point out an interesting fact: if the entire medium is incompressible from the outset (the case when $\alpha_p = \alpha_\eta = \infty$ or $\operatorname{div} \mathbf{w}^\varepsilon = 0$), then the system of equation splits: the heat equation can be solved independently of the dynamical equations. But if this property holds asymptotically ($p_* = \eta_0 = \infty$, where $\eta_0 = \lim_{\varepsilon\searrow 0} \alpha_\eta(\varepsilon)$), then the system of equations remain coupled in the general case. Moreover, in this case, as in the case of compressible media the homogenized equations contain non-local expressions (functionals), which are not taken into account in standard phenomenological models.

We point out that in this paper we only consider a small number of the possible limit cases (homogenized equations). Obviously, finding all possible consistent mathematical models that give asymptotic approximations to the original commonly accepted model is an important and interesting problem, both from a mathematical and a practical standpoint. It is equally obvious that in the solution of actual physical problems one does not resort to a limiting procedure. The researcher has at his disposal only concrete physical constants (the density of the medium, the viscosity of the fluid, the elastic constants of the solid skeleton, and so on) and two variables: the characteristic size L of the domain under consideration and the characteristic time τ of the physical process. Changing these variables within the application range of the mathematical model he can discover laws for the behaviour of dimensionless complexes α_μ, α_τ, α_λ, ..., which will suggest the choice of one or other limit regime in the exact model. It is at this point that as complete a list of homogenized equations as possible is required, because different limit regimes correspond to different physical situations and it is virtually impossible to guess in advance whether one situation or another is more likely.

Simpler models for isothermic media were considered in [7]–[13].

§ 1. Formulation of the main results

As usual, equations (0.1), (0.2) are understood in the sense of distributions. They involve equations (0.1), (0.2) in the proper sense, in the domains Ω_f^ε and Ω_s^ε, and also the boundary conditions

$$[\theta] = 0, \qquad [\mathbf{w}] = 0, \qquad \mathbf{x}_0 \in \Gamma^\varepsilon, \quad t \geqslant 0, \tag{1.1}$$

$$[\mathbb{P} \cdot \mathbf{n}] = 0, \quad [\alpha_\varkappa^\varepsilon \nabla\theta \cdot \mathbf{n}] = 0, \qquad \mathbf{x}_0 \in \Gamma^\varepsilon, \quad t \geqslant 0, \tag{1.2}$$

at the boundary Γ^ε, where $\mathbf{n}$ is the unit normal to the boundary, and

$$[\varphi](\mathbf{x}_0) = \varphi_{(s)}(\mathbf{x}_0) - \varphi_{(f)}(\mathbf{x}_0),$$
$$\varphi_{(s)}(\mathbf{x}_0) = \lim_{\substack{\mathbf{x}\to\mathbf{x}_0 \\ \mathbf{x}\in\Omega_s^\varepsilon}} \varphi(\mathbf{x}), \qquad \varphi_{(f)}(\mathbf{x}_0) = \lim_{\substack{\mathbf{x}\to\mathbf{x}_0 \\ \mathbf{x}\in\Omega_f^\varepsilon}} \varphi(\mathbf{x}).$$

Condition (1.1) is a natural consequence of the definition of the solution class: we seek solutions (the temperature θ and the displacement $\mathbf{w}$) with minimal continuity properties. The first condition in (1.2) is a consequence of the momentum balance at strong (contact) cracks and the second condition in (1.2) is a consequence of energy conservation.

There exist various ways to represent equations (0.1), (0.2) and boundary conditions (1.1), (1.2), which are equivalent in the sense of distributions. In what follows it will be convenient to write them in the form of integral equalities.

Definition 1. A system of functions $(\mathbf{w}^\varepsilon, \theta^\varepsilon, p_f^\varepsilon, q_f^\varepsilon, p_s^\varepsilon, q_s^\varepsilon)$ is called a *generalized solution in the model* $(\mathbf{NB})^\varepsilon$ if they satisfy the regularity conditions

$$\mathbf{w}^\varepsilon,\ \nabla\mathbf{w}^\varepsilon,\ \operatorname{div}\mathbf{w}^\varepsilon,\ p_f^\varepsilon,\ p_s^\varepsilon,\ q_f^\varepsilon,\ q_f^\varepsilon,\ \theta^\varepsilon,\ \nabla\theta^\varepsilon \in L^2(\Omega_T)$$

in the domain $\Omega_T = \Omega \times (0,T)$, boundary conditions (0.8), the equations

$$q_f^\varepsilon = p_f^\varepsilon + \chi^\varepsilon \alpha_{\theta f}\theta^\varepsilon, \tag{1.3}$$

$$\frac{1}{\alpha_p} p_f^\varepsilon = -\chi^\varepsilon \operatorname{div}\mathbf{w}^\varepsilon - \frac{\chi^\varepsilon}{m}\beta^\varepsilon, \tag{1.4}$$

$$q_s^\varepsilon = p_s^\varepsilon + (1-\chi^\varepsilon)\alpha_{\theta s}\theta^\varepsilon, \tag{1.5}$$

$$\frac{1}{\alpha_\eta} p_s^\varepsilon = -(1-\chi^\varepsilon)\operatorname{div}\mathbf{w}^\varepsilon + \beta^\varepsilon \frac{1-\chi^\varepsilon}{1-m} \tag{1.6}$$

a.e. in Ω_T, the integral identity

$$\int_{\Omega_T} \Big(\alpha_\tau \rho^\varepsilon \mathbf{w}^\varepsilon \cdot \frac{\partial^2 \boldsymbol{\varphi}}{\partial t^2} - \chi^\varepsilon \alpha_\mu \mathbb{D}(x,\mathbf{w}^\varepsilon) : \mathbb{D}\Big(x, \frac{\partial\boldsymbol{\varphi}}{\partial t}\Big) - \rho^\varepsilon \mathbf{F}\cdot\boldsymbol{\varphi}$$
$$+ \big\{(1-\chi^\varepsilon)\alpha_\lambda \mathbb{D}(x,\mathbf{w}^\varepsilon) - (q_f^\varepsilon + q_s^\varepsilon)\mathbb{I}\big\} : \mathbb{D}(x,\boldsymbol{\varphi})\Big)\, dx\, dt = 0 \tag{1.7}$$

for all smooth vector-valued functions $\boldsymbol{\varphi} = \boldsymbol{\varphi}(\mathbf{x},t)$ such that $\boldsymbol{\varphi}\big|_{\partial\Omega} = \boldsymbol{\varphi}\big|_{t=T} = \partial\boldsymbol{\varphi}/\partial t\big|_{t=T} = 0$, and the integral identity

$$\int_{\Omega_T} \Big((\alpha_\tau c_p^\varepsilon \theta^\varepsilon + \alpha_\theta^\varepsilon \operatorname{div}\mathbf{w}^\varepsilon)\frac{\partial\xi}{\partial t} - \alpha_\varkappa^\varepsilon \nabla\theta^\varepsilon \cdot \nabla\xi + \Psi\xi \Big)\, dx\, dt = 0 \tag{1.8}$$

for all smooth functions $\xi = \xi(\mathbf{x},t)$ such that $\xi\big|_{\partial\Omega} = \xi\big|_{t=T} = 0$.

In the definition of a generalized solution we introduced new unknown functions p_s^ε, q_f^ε and q_s^ε, which by analogy with p_f^ε we shall call *pressures*. In addition, we shall call (1.4) and (1.6) the *continuity equations* and (1.3) and (1.5) the *state equations* in the solid and fluid components, respectively. We introduced the normalization term

$$\beta^\varepsilon = \int_\Omega (1-\chi^\varepsilon)\operatorname{div}\mathbf{w}^\varepsilon\,dx$$

so that

$$\int_\Omega p_f^\varepsilon\,dx = \int_\Omega p_s^\varepsilon\,dx = 0.$$

These conditions are necessary to ensure that the family of solutions p_f^ε, p_s^ε, q_f^ε and q_s^ε is bounded uniformly in ε in the $L^2(\Omega_T)$ space.

In (1.7) we denote by $A:B$ the convolution of two second-rank tensors with respect to both indices, that is, $A:B = \operatorname{tr}(B^*\circ A) = \sum_{i,j=1}^3 A_{ij}B_{ji}$.

In addition to the assumptions we made in the introduction suppose that there exist (finite or infinite) limits

$$\lim_{\varepsilon\searrow 0}\alpha_\eta(\varepsilon) = \eta_0; \qquad \lim_{\varepsilon\searrow 0}\alpha_{\varkappa s}(\varepsilon) = \varkappa_{0s}; \qquad \lim_{\varepsilon\searrow 0}\alpha_{\theta f}(\varepsilon) = \beta_{0f};$$
$$\lim_{\varepsilon\searrow 0}\alpha_{\theta s}(\varepsilon) = \beta_{0s}, \qquad \lim_{\varepsilon\searrow 0}\frac{\alpha_\mu}{\varepsilon^2} = \mu_1, \qquad \lim_{\varepsilon\searrow 0}\frac{\alpha_{\varkappa f}}{\alpha_\mu} = \varkappa_f.$$

In what follows we also make the following assumption.

Assumption 2. 1) The dimensionless parameters in the model $(\mathbf{NB})^\varepsilon$ satisfy the following restrictions:

$$\mu_0 = 0; \qquad \tau_0,\ \varkappa_f,\ \varkappa_{0s},\ \beta_{0f},\ \beta_{0s},\ \lambda_0 < \infty;$$
$$\tau_0+\mu_1,\ \varkappa_{0s},\ \varkappa_f,\ \lambda_0,\ \eta_0 > 0.$$

2) The functions $|\mathbf{F}|$, $|\partial\mathbf{F}/\partial t|$, Ψ, $\partial\Psi/\partial t$ belong to the space $L^2(\Omega_T)$.

Throughout what follows the parameters can take all the values permitted by the assumptions made in the theorems. For example, if $\tau_0=0$ or $\eta_0^{-1}=0$, then the terms containing these parameters disappear from the equations.

The main results of this paper are Theorems 1 and 2.

Theorem 1. *Under the above assumptions, for all $\varepsilon>0$, on an arbitrary time interval $[0,\max\limits_{0\leqslant t\leqslant T}\|$re exists a unique generalized solution of the model $(\mathbf{NB})^\varepsilon$ and*

$$\max_{0\leqslant t\leqslant T}\big\||\mathbf{w}^\varepsilon(t)| + \sqrt{\alpha_\mu}\,\chi^\varepsilon|\nabla\mathbf{w}^\varepsilon(t)| + (1-\chi^\varepsilon)|\nabla\mathbf{w}^\varepsilon(t)|\big\|_{2,\Omega} \leqslant C_0, \tag{1.9}$$

$$\|\theta^\varepsilon\|_{2,\Omega_T} + \sqrt{\alpha_{\varkappa f}}\,\|\chi^\varepsilon\nabla_x\theta^\varepsilon\|_{2,\Omega_T} + \|(1-\chi^\varepsilon)\nabla\theta^\varepsilon\|_{2,\Omega_T} \leqslant C_0; \tag{1.10}$$

$$\big\||q_f^\varepsilon| + |p_f^\varepsilon| + |q_s^\varepsilon| + |p_s^\varepsilon|\big\|_{2,\Omega_T} \leqslant C_0, \tag{1.11}$$

where the constant C_0 is independent of the small parameter ε.

Theorem 2. *The functions $\mathbf{w}^\varepsilon$ and θ^ε admit extensions $\mathbf{u}^\varepsilon$ and ϑ^ε, respectively, from $\Omega^\varepsilon_{s,T}=\Omega^\varepsilon_s\times(0,T)$ into the domain Ω_T such that the sequences $\{\mathbf{u}^\varepsilon\}$ and $\{\vartheta^\varepsilon\}$ converge strongly in $L^2(\Omega_T)$ and weakly in $L^2((0,T);W^1_2(\Omega))$ to the functions $\mathbf{u}$ and ϑ, respectively. At the same time the sequences $\{\mathbf{w}^\varepsilon\}$, $\{\theta^\varepsilon\}$, $\{p^\varepsilon_f\}$, $\{q^\varepsilon_f\}$, $\{p^\varepsilon_s\}$ and $\{q^\varepsilon_s\}$ converge weakly in $L^2(\Omega_T)$ to $\mathbf{w}$, θ, p_f, q_f, p_s and q_s, respectively.*

(I) *If $\mu_1=\infty$, then $\mathbf{w}=\mathbf{u}$, $\theta=\vartheta$ and the functions $\mathbf{u}$, ϑ, p_f, q_f, p_s and q_s satisfy the following initial-boundary value problem in Ω_T:*

$$\tau_0\widehat{\rho}\frac{\partial^2\mathbf{u}}{\partial t^2}+\nabla(q_f+q_s)-\widehat{\rho}\,\mathbf{F} = \operatorname{div}\Big\{\lambda_0\mathbb{A}^s_0:\mathbb{D}(x,\mathbf{u})+B^s_0\Big(\operatorname{div}\mathbf{u}-\frac{\beta_{0s}}{\eta_0}\vartheta\Big)+B^s_1 q_f\Big\}, \tag{1.12}$$

$$\tau_0\widehat{c}_p\frac{\partial\vartheta}{\partial t}-\operatorname{div}(B^\theta\cdot\nabla\vartheta)-\Psi-\frac{\beta_{0s}}{\eta_0}\frac{\partial p_s}{\partial t}=(\beta_{0f}-\beta_{0s})\frac{d\beta}{dt}, \tag{1.13}$$

$$\frac{1}{\eta_0}p_s+C^s_0:\mathbb{D}(x,\mathbf{u})+(1-m+a^s_0)\operatorname{div}\mathbf{u} = \frac{\beta_{0s}}{\eta_0}a^s_0(\vartheta-\langle\vartheta\rangle_\Omega)-a^s_1(q_f-\langle q_f\rangle_\Omega), \tag{1.14}$$

$$q_s=p_s+(1-m)\beta_{0s}\vartheta,\qquad q_f=p_f+m\beta_{0f}\vartheta, \tag{1.15}$$

$$\frac{1}{\eta_0}p_s+\operatorname{div}\mathbf{u}=0, \tag{1.16}$$

where

$$\widehat{\rho}=m\rho_f+(1-m)\rho_s,\qquad \widehat{c}_p=mc_{pf}+(1-m)c_{ps},$$
$$\beta=\frac{\beta_{0s}}{\eta_0}a^s_2\langle\vartheta\rangle_\Omega+a^s_3\langle q_f\rangle_\Omega,\qquad \langle\varphi\rangle_\Omega=\int_\Omega\varphi\,dx.$$

The quantity m, which is called the porosity of the solid skeleton, is defined by

$$m=\int_Y\chi(\mathbf{y})\,d\mathbf{y}=\langle m\rangle_Y.$$

The symmetric strictly positive constant rank-4 tensor $\mathbb{A}^s_0$, the constant matrices C^s_0, B^s_0 and B^s_1, the strictly positive definite constant matrix B^ϑ and the constants a^s_k, $k=0,1,2,3$, are defined below by formulae (4.38), (4.39) and (4.42).

The differential equations (1.12)–(1.16) are endowed with the homogeneous initial conditions

$$\tau_0\mathbf{u}=\tau_0\frac{\partial\mathbf{u}}{\partial t}=0, \tag{1.17}$$

$$\tau_0\widehat{c}_p\vartheta-\frac{\beta_{0s}}{\eta_0}p_s-(\beta_{0f}-\beta_{0s})\beta=0 \tag{1.18}$$

for $t=0$ and $\mathbf{x}\in\Omega$ and with the homogeneous boundary conditions

$$\vartheta(\mathbf{x},t)=0,\quad \mathbf{u}(\mathbf{x},t)=0,\qquad \mathbf{x}\in S,\quad t>0. \tag{1.19}$$

(II) *If the porous space is disconnected, then* $\mathbf{w}=\mathbf{u}$ *and in* Ω_T *the strong/weak limits* $\mathbf{u}$, ϑ, p_f, q_f, p_s *and* q_s *together with the weak limit* θ^f *of the sequence* $\{\chi^\varepsilon\theta^\varepsilon\}$ *satisfy equations* (1.12), (1.14), (1.16), *the state equations*

$$q_s = p_s + (1-m)\beta_{0s}\vartheta, \qquad q_f = p_f + \beta_{0f}\theta^f, \tag{1.20}$$

and the heat equation

$$\begin{aligned} &\tau_0 c_{pf}\frac{\partial \theta^f}{\partial t} + \tau_0 c_{ps}(1-m)\frac{\partial \vartheta}{\partial t} - \operatorname{div}(B^\theta\cdot\nabla\vartheta) \\ &\quad = \Psi + \frac{\beta_{0s}}{\eta_0}\frac{\partial p_s}{\partial t} + (\beta_{0f}-\beta_{0s})\frac{d\beta}{dt}\,. \end{aligned}\tag{1.21}$$

The temperature θ^f *of the fluid is defined below by formulae* (4.45)–(4.47), *where the particular choice of the function* θ^f *depends on the parameters* μ_1 *and* τ_0.

The problem is endowed with boundary conditions (1.18), *initial conditions* (1.17) *for the displacement, and the initial condition*

$$\tau_0 c_{ps}\vartheta - \frac{\beta_{0s}}{\eta_0}p_s - (\beta_{0f}-\beta_{0s})\beta = 0 \tag{1.22}$$

for the temperature ϑ *in the solid skeleton for* $t=0$ *and* $\mathbf{x}\in\Omega$.

(III) *If* $\mu_1<\infty$, *then in* Ω_T *the strong/weak limits* $\mathbf{u}$, ϑ, $\mathbf{w}^f$, θ^f, p_f, q_f, p_s *and* q_s *of the sequences* $\{\mathbf{u}^\varepsilon\}$, $\{\vartheta^\varepsilon\}$, $\{\chi^\varepsilon\mathbf{w}^\varepsilon\}$, $\{\chi^\varepsilon\theta^\varepsilon\}$, $\{p_f^\varepsilon\}$, $\{q_f^\varepsilon\}$, $\{p_s^\varepsilon\}$ *and* $\{q_s^\varepsilon\}$ *satisfy the initial-boundary value problem consisting of the momentum balance equation*

$$\begin{aligned} &\tau_0\left(\rho_f\frac{\partial^2\mathbf{w}^f}{\partial t^2} + \rho_s(1-m)\frac{\partial^2\mathbf{u}}{\partial t^2}\right) + \nabla(q_f+q_s) - \hat{\rho}\,\mathbf{F} \\ &\quad = \operatorname{div}\left\{\lambda_0 A_0^s : \mathbb{D}(x,\mathbf{u}) + B_0^s\left(\operatorname{div}\mathbf{u} - \frac{\beta_{0s}}{\eta_0}\vartheta\right) + B_1^s q_f\right\} \end{aligned}\tag{1.23}$$

and the continuity equation (1.14) *for the solid component, where* $\mathbb{A}_0^s$, B_0^s, *and* B_1^s *are as in* (1.12), *the continuity equation*

$$\frac{1}{\eta_0}p_s + \operatorname{div}\mathbf{w}^f = (m-1)\operatorname{div}\mathbf{u}, \tag{1.24}$$

the state equation (1.20), *the heat equation* (1.21), *and the relation*

$$\begin{aligned} &\frac{\partial \mathbf{w}^f}{\partial t}(\mathbf{x},t) = m\frac{\partial \mathbf{u}}{\partial t}(\mathbf{x},t) + \int_0^t B_1(\mu_1, t-\tau)\cdot\mathbf{z}(\mathbf{x},\tau)\,d\tau, \\ &\mathbf{z}(\mathbf{x},t) = -\frac{1}{m}\nabla q_f(\mathbf{x},t) + \rho_f\mathbf{F}(\mathbf{x},t) - \tau_0\rho_f\frac{\partial^2\mathbf{u}}{\partial t^2}(\mathbf{x},t), \end{aligned}\tag{1.25}$$

for $\tau_0>0$ *and* $\mu_1>0$, *or Darcy's law in the form*

$$\frac{\partial \mathbf{w}^f}{\partial t} = m\frac{\partial \mathbf{u}}{\partial t} + B_2(\mu_1)\cdot\left(-\frac{1}{m}\nabla q_f + \rho_f\mathbf{F}\right), \tag{1.26}$$

for $\tau_0 = 0$, *or finally, the momentum balance equation for the fluid component, which has the following form:*

$$\tau_0\rho_f\frac{\partial^2\mathbf{w}^f}{\partial t^2} = \tau_0\rho_f B_3\cdot\frac{\partial^2\mathbf{u}}{\partial t^2} + (m\mathbb{I} - B_3)\cdot\Big(-\frac{1}{m}\nabla q_f + \rho_f\mathbf{F}\Big), \tag{1.27}$$

for $\mu_1 = 0$.

The problem is completed by the initial and boundary conditions (1.17), (1.18) *and* (1.22) *for the displacement* $\mathbf{u}$ *and the temperature* θ *of the solid component, the boundary condition*

$$\mathbf{w}^f(\mathbf{x},t)\cdot\mathbf{n}(\mathbf{x}) = 0, \qquad (\mathbf{x},t)\in S, \quad t>0, \tag{1.28}$$

and the initial condition

$$\tau_0\mathbf{w}^f = \tau_0\frac{\partial\mathbf{w}^f}{\partial t} = 0 \tag{1.29}$$

for the displacement $\mathbf{w}^f$ *of the fluid component.*

In equations (1.25)–(1.28) $\mathbf{n}(\mathbf{x})$ *is the unit normal to* S *at the point* $\mathbf{x}\in S$, *while the matrix* $B_1(\mu_1,t)$ *and the symmetric positive definite matrices* $B_2(\mu_1)$ *and* $(m\mathbb{I} - B_3)$ *are defined below by formulae* (4.60)–(4.65).

§ 2. Preliminaries

2.1. Two-scale convergence. The proof of Theorem 2 relies on the systematic use of the method of two-scale convergence put forward by Nguetseng [2].

Definition 2. A sequence $\{\varphi^\varepsilon\}\subset L^2(\Omega_T)$ is said to be *two-scale convergent* to a limit $\varphi\in L^2(\Omega_T\times Y)$ if and only if for each smooth function $\sigma=\sigma(\mathbf{x},t,\mathbf{y})$, 1-periodic in $\mathbf{y}$

$$\lim_{\varepsilon\searrow 0}\int_{\Omega_T}\varphi^\varepsilon(\mathbf{x},t)\sigma\Big(\mathbf{x},t,\frac{\mathbf{x}}{\varepsilon}\Big)\,dx\,dt = \int_{\Omega_T}\int_Y\varphi(\mathbf{x},t,\mathbf{y})\sigma(\mathbf{x},t,\mathbf{y})\,d\mathbf{y}\,dx\,dt. \tag{2.1}$$

The existence and the main properties of two-scale convergent sequences are established by the following result (see [2], [3]).

Theorem 3 (Nguetseng's theorem). 1. *Each bounded sequence in* $L^2(\Omega_T)$ *contains a subsequence two-scale convergent to a limit* $\varphi\in L^2(\Omega_T\times Y)$.

2. *Let* $\{\varphi^\varepsilon\}$ *and* $\{\varepsilon\nabla\varphi^\varepsilon\}$ *be uniformly bounded sequences in* $L^2(\Omega_T)$. *Then there exist a function* $\varphi=\varphi(\mathbf{x},t,\mathbf{y})$ *1-periodic in* $\mathbf{y}$ *and a subsequence of* $\{\varphi^\varepsilon\}$ *such that* $\varphi,\nabla_y\varphi\in L^2(\Omega_T\times Y)$ *and* φ^ε, $\varepsilon\nabla\varphi^\varepsilon$ *are two-scale convergent to* φ *and* $\nabla_y\varphi$, *respectively.*

3. *Let* $\{\varphi^\varepsilon\}$ *and* $\{\nabla\varphi^\varepsilon\}$ *be uniformly bounded sequences in* $L^2(\Omega_T)$. *Then there exist functions* $\varphi\in L^2(\Omega_T)$ *and* $\psi\in L^2(\Omega_T\times Y)$ *and a subsequence of* $\{\varphi^\varepsilon\}$ *such that* $\nabla\varphi\in L^2(\Omega_T)$, *the function* ψ *is 1-periodic in* $\mathbf{y}$, $\nabla_y\psi\in L^2(\Omega_T\times Y)$, *and* $\nabla\varphi^\varepsilon$ *is two-scale convergent to* $\nabla\varphi(\mathbf{x},t)+\nabla_y\psi(\mathbf{x},t,\mathbf{y})$.

Corollary. *Let* $\sigma\in L^2(Y)$, $\sigma^\varepsilon(\mathbf{x})=\sigma(\mathbf{x}/\varepsilon)$, *and let* $\{\varphi^\varepsilon\}\subset L^2(\Omega_T)$ *be a sequence two-scale convergent to* $\varphi\in L^2(\Omega_T\times Y)$. *Then the sequence* $\{\sigma^\varepsilon\varphi^\varepsilon\}$ *is two-scale convergent to* $\sigma\varphi$.

2.2. An extension lemma. The following feature is typical in problems similar to the $(\mathbf{NB})^{\varepsilon}$ model: bounds for the displacement gradient $\nabla\mathbf{w}^{\varepsilon}$ are distinct in Ω_s and Ω_f (in the liquid and the solid phases), which does not allow one an immediate use of stronger estimates. This difficulty can be overcome by constructing an extension to the whole of Ω of the displacement field defined in Ω_s, while preserving the bound for the norm of the gradient in Ω_s. We have the following result (see [14], [15]), which we state in a form appropriate for us.

Lemma 1. *Suppose that the assumptions on the geometry of Ω^{ε}_s hold and let $\psi^{\varepsilon}\in W^1_2(\Omega^{\varepsilon}_s)$, with $\psi^{\varepsilon}=0$ at the boundary $S^{\varepsilon}_s=\partial\Omega^{\varepsilon}_s\cap\partial\Omega$. Then there exists a function $\sigma^{\varepsilon}\in W^1_2(\Omega)$ such that its restriction to the subdomain Ω^{ε}_s coincides with ψ^{ε}, that is,*

$$\big(1-\chi^{\varepsilon}(\mathbf{x})\big)\big(\sigma^{\varepsilon}(\mathbf{x})-\psi^{\varepsilon}(\mathbf{x})\big)=0,\qquad \mathbf{x}\in\Omega. \tag{2.2}$$

Moreover,

$$\|\sigma^{\varepsilon}\|_{2,\Omega}\leqslant C\|\psi^{\varepsilon}\|_{2,\Omega^{\varepsilon}_s},\qquad \|\nabla\sigma^{\varepsilon}\|_{2,\Omega}\leqslant C\|\nabla_x\psi^{\varepsilon}\|_{2,\Omega^{\varepsilon}_s}, \tag{2.3}$$

where the constant C depends only on the geometry of the cell Y and does not depend on ε.

2.3. The Friedrichs–Poincaré inequality in a periodic structure. The following result is well known. It refines the value of the constant in the case of an ε-periodic geometric structure.

Lemma 2. *Suppose that the assumptions about the geometry of the domain Ω^{ε}_f hold. Then for each function $\varphi\in W^1_2(\Omega^{\varepsilon}_f)$ the inequality*

$$\int_{\Omega^{\varepsilon}_f}|\varphi|^2\,dx\leqslant C\varepsilon^2\int_{\Omega^{\varepsilon}_f}|\nabla\varphi|^2\,dx \tag{2.4}$$

holds with constant C independent of ε.

In what follows we use the following notation:

1) $\langle\Phi\rangle_Y=\displaystyle\int_Y\Phi\,dy,\quad \langle\Phi\rangle_{Y_f}=\int_Y\chi\Phi\,dy,\quad \langle\Phi\rangle_{Y_s}=\int_Y(1-\chi)\Phi\,dy,$

$$\langle\varphi\rangle_{\Omega}=\int_{\Omega}\varphi\,dx,\qquad \langle\varphi\rangle_{\Omega_T}=\int_{\Omega_T}\varphi\,dx\,dt;$$

2) if $\mathbf{a}$ and $\mathbf{b}$ are two vectors, then the matrix $\mathbf{a}\otimes\mathbf{b}$ is defined by the formula

$$(\mathbf{a}\otimes\mathbf{b})\cdot\mathbf{c}=\mathbf{a}(\mathbf{b}\cdot\mathbf{c})$$

for each vector $\mathbf{c}$;

3) if B and C are two matrices, then $B\otimes C$ is a rank-4 tensor such that its convolution with an arbitrary matrix A is defined by the formula

$$(B\otimes C):A=B(C:A);$$

4) we denote by $\mathbb{I}^{ij}$ the matrix with exactly one non-zero entry: it is equal to one and is placed at the intersection of the ith row and the jth column;

5) $J^{ij}=\frac{1}{2}(\mathbb{I}^{ij}+\mathbb{I}^{ji})=\frac{1}{2}(\mathbf{e}_i\otimes\mathbf{e}_j+\mathbf{e}_j\otimes\mathbf{e}_i)$, where $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ are the standard Cartesian basis vectors.

§ 3. Proof of Theorem 1

For $\tau_0 > 0$ the estimates (1.9), (1.10) follow from the energy identity

$$\begin{aligned}
&\frac{d}{dt}\bigg\{\int_\Omega \alpha_\tau\bigg(\rho^\varepsilon\Big(\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\Big)^2 + c_p^\varepsilon\Big(\frac{\partial\theta^\varepsilon}{\partial t}\Big)^2\bigg)\,dx \\
&\qquad + \alpha_\lambda\int_\Omega (1-\chi^\varepsilon)\mathbb{D}\Big(x,\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big) : \mathbb{D}\Big(x,\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big)\,dx \\
&\qquad + \alpha_p\int_\Omega \chi^\varepsilon\Big(\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big)^2\,dx + \alpha_\eta\int_\Omega (1-\chi^\varepsilon)\Big(\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big)^2\,dx\bigg\} \\
&\quad + \int_\Omega \alpha_\varkappa^\varepsilon\Big|\nabla\frac{\partial\theta^\varepsilon}{\partial t}\Big|^2\,dx + \alpha_\mu\int_\Omega \chi^\varepsilon\mathbb{D}\Big(x,\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\Big) : \mathbb{D}\Big(x,\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\Big)\,dx \\
&= \int_\Omega \rho^\varepsilon\frac{\partial\mathbf{F}}{\partial t}\cdot\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\,dx \\
&\quad + \beta^\varepsilon\bigg(\frac{\alpha_p}{m}\int_\Omega \chi^\varepsilon\operatorname{div}\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\,dx + \frac{\alpha_\eta}{1-m}\int_\Omega (1-\chi^\varepsilon)\operatorname{div}\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\,dx\bigg),
\end{aligned}$$

obtained by differentiating the equations for $\mathbf{w}^\varepsilon$ and θ^ε with respect to time, multiplying the first equation by $\partial^2\mathbf{w}^\varepsilon/\partial t^2$, the second by $\partial\theta^\varepsilon/\partial t$, integrating by parts, and summing. In the process we have expressed the pressures in terms of the displacements with the use of the continuity equations and the state equations.

Since

$$\frac{1}{m}\Big(\int_\Omega \chi^\varepsilon\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\,dx\Big)^2 \leqslant \int_\Omega \chi^\varepsilon\Big(\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big)^2\,dx,$$

$$\frac{1}{1-m}\Big(\int_\Omega (1-\chi^\varepsilon)\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\,dx\Big)^2 \leqslant \int_\Omega (1-\chi^\varepsilon)\Big(\operatorname{div}\frac{\partial\mathbf{w}^\varepsilon}{\partial t}\Big)^2\,dx,$$

the estimates (1.9) and (1.10) follow from the inequality

$$\begin{aligned}
&\max_{0<t<T}\bigg(\sqrt{\alpha_\tau}\,\Big\|\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}(t)\Big\|_{2,\Omega} + \sqrt{\alpha_\lambda}\,\Big\|\nabla\frac{\partial\mathbf{w}^\varepsilon}{\partial t}(t)\Big\|_{2,\Omega_s^\varepsilon} + \sqrt{\alpha_\tau}\,\Big\|\frac{\partial\theta^\varepsilon}{\partial t}(t)\Big\|_{2,\Omega}\bigg) \\
&\qquad + \sqrt{\alpha_{\varkappa s}}\,\Big\|(1-\chi^\varepsilon)\nabla\frac{\partial\theta^\varepsilon}{\partial t}\Big\|_{2,\Omega_T} + \sqrt{\alpha_{\varkappa f}}\,\Big\|\chi^\varepsilon\nabla_x\frac{\partial\theta^\varepsilon}{\partial t}\Big\|_{2,\Omega_T} \\
&\qquad + \sqrt{\alpha_\mu}\,\Big\|\chi^\varepsilon\nabla\frac{\partial^2\mathbf{w}^\varepsilon}{\partial t^2}\Big\|_{2,\Omega_T} \leqslant \frac{C_0}{\sqrt{\alpha_\tau}}, \qquad (3.1)
\end{aligned}$$

where C_0 is independent of ε.

The same estimate (3.1) guarantees the existence and uniqueness of the generalized solution for the model $(\mathbf{NB})^\varepsilon$ because here we do not require estimates uniform in ε, while the boundedness of the pressures non-uniform in ε is also an easy consequence of (3.1).

Estimate (1.11) for the pressures, which is uniform in ε, follows from the integral identity (1.7) and the bounds (3.1), as an estimate for the corresponding functional, once we recall that

$$\int_\Omega \big(p_f^\varepsilon(\mathbf{x},t) + p_s^\varepsilon(\mathbf{x},t)\big)\,dx = 0.$$

Indeed, expressing the pressures q_f^ε and q_s^ε in (1.7) using the state equations (1.3) and (1.5), in view of (3.1), we obtain

$$\left|\int_\Omega (p_f^\varepsilon + p_s^\varepsilon)\,\mathrm{div}\,\boldsymbol{\psi}\,dx\right| \leqslant C\|\nabla\boldsymbol{\psi}\|_{2,\Omega}.$$

Now choosing $\boldsymbol{\psi}$ such that $p_f^\varepsilon + p_s^\varepsilon = \mathrm{div}\,\boldsymbol{\psi}$ we arrive at the desired estimate for the sum of the pressures $p_f^\varepsilon + p_s^\varepsilon$. Such a choice is always possible (see [16]) if we set

$$\boldsymbol{\psi} = \nabla\varphi + \boldsymbol{\psi}_0, \qquad \mathrm{div}\,\boldsymbol{\psi}_0 = 0, \quad \Delta\varphi = p_f^\varepsilon + p_s^\varepsilon,$$
$$\varphi\big|_{\partial\Omega} = 0, \quad (\nabla\varphi + \boldsymbol{\psi}_0)\big|_{\partial\Omega} = 0.$$

Note that identity (1.7) allows only an estimate for the sum $p_f^\varepsilon + p_s^\varepsilon$. However, the product of these functions vanishes, so this is sufficient for the derivation of bounds for each of them. We estimate the pressures q_f^ε and q_s^ε on the basis of state equations (1.3) and (1.5) using (3.1).

The greatest difficulty lies in finding an estimate for $\mathbf{w}^\varepsilon$ in the case $\tau_0 = 0$.

Assume that $\mu_1 > 0$ and $\tau_0 = 0$. As usual, for the basic estimate we must multiply the equation for $\mathbf{w}^\varepsilon$ (with pressures expressed in terms of the divergence of the displacement and its time derivative, with the help of the state equation and the continuity equations) by $\partial\mathbf{w}^\varepsilon/\partial t$ and the equations for θ^ε by θ^ε, add the results and integrate by parts. Only two terms in the result, $\rho^\varepsilon\mathbf{F}\cdot\partial\mathbf{w}^\varepsilon/\partial t$ and $\Psi\cdot\theta^\varepsilon$, require additional treatment. First of all, using Lemma 1 we construct an extension $\mathbf{u}^\varepsilon$ of the function $\mathbf{w}^\varepsilon$ from the domain Ω_s^ε into Ω_f^ε such that $\mathbf{u}^\varepsilon = \mathbf{w}^\varepsilon$ in Ω_s^ε, $\mathbf{u}^\varepsilon \in W_2^1(\Omega)$, and

$$\|\mathbf{u}^\varepsilon\|_{2,\Omega} \leqslant C\|\nabla\mathbf{u}^\varepsilon\|_{2,\Omega} \leqslant \frac{C}{\sqrt{\alpha_\lambda}}\|(1-\chi^\varepsilon)\sqrt{\alpha_\lambda}\,\nabla\mathbf{w}^\varepsilon\|_{2,\Omega}.$$

After that we find an estimate for $\|\mathbf{w}^\varepsilon\|_{2,\Omega}$ with the help of Poincaré's inequality (Lemma 2 for the difference $\mathbf{u}^\varepsilon - \mathbf{w}^\varepsilon$):

$$\begin{aligned}
\|\mathbf{w}^\varepsilon\|_{2,\Omega} &\leqslant \|\mathbf{u}^\varepsilon\|_{2,\Omega} + \|\mathbf{u}^\varepsilon - \mathbf{w}^\varepsilon\|_{2,\Omega} \\
&\leqslant \|\mathbf{u}^\varepsilon\|_{2,\Omega} + C\varepsilon\|\chi^\varepsilon\nabla(\mathbf{u}^\varepsilon - \mathbf{w}^\varepsilon)\|_{2,\Omega} \\
&\leqslant \|\mathbf{u}^\varepsilon\|_{2,\Omega} + C\varepsilon\|\nabla_x\mathbf{u}^\varepsilon\|_{2,\Omega} + C(\varepsilon\alpha_\mu^{-1/2})\|\chi^\varepsilon\sqrt{\alpha_\mu}\,\nabla_x\mathbf{w}^\varepsilon\|_{2,\Omega} \\
&\leqslant \frac{C}{\sqrt{\alpha_\lambda}}\|(1-\chi^\varepsilon)\sqrt{\alpha_\lambda}\,\nabla_x\mathbf{w}^\varepsilon\|_{2,\Omega} + C(\varepsilon\alpha_\mu^{-1/2})\|\chi^\varepsilon\sqrt{\alpha_\mu}\,\nabla_x\mathbf{w}^\varepsilon\|_{2,\Omega}.
\end{aligned}$$

We apply the same method to θ^ε; hence there exists an extension ϑ^ε of the function θ^ε from Ω_s^ε into Ω_f^ε such that $\vartheta^\varepsilon = \theta^\varepsilon$ in Ω_s^ε, $\vartheta^\varepsilon \in W_2^1(\Omega)$ and

$$\|\vartheta^\varepsilon\|_{2,\Omega} \leqslant C\|\nabla\vartheta^\varepsilon\|_{2,\Omega} \leqslant \frac{C}{\sqrt{\alpha_{\varkappa s}}}\|(1-\chi^\varepsilon)\sqrt{\alpha_{\varkappa s}}\,\nabla\theta^\varepsilon\|_{2,\Omega},$$
$$\|\theta^\varepsilon\|_{2,\Omega} \leqslant \frac{C}{\sqrt{\alpha_{\varkappa s}}}\|(1-\chi^\varepsilon)\sqrt{\alpha_{\varkappa s}}\,\nabla\theta^\varepsilon\|_{2,\Omega} + C(\varepsilon\alpha_{\varkappa s}^{-1/2})\|\chi^\varepsilon\sqrt{\alpha_{\varkappa s}}\,\nabla\theta^\varepsilon\|_{2,\Omega}.$$

Next, we carry over the time derivative from $\partial\mathbf{w}^\varepsilon/\partial t$ to $\rho^\varepsilon\mathbf{F}$ and find bounds for all positive terms (including $\alpha_\nu\chi^\varepsilon\,\mathrm{div}(\partial\mathbf{w}^\varepsilon/\partial t)^2$) in the usual way, with the help of

Hölder's and Gronwall's inequalities. The rest of the proof is as for $\tau_0 > 0$, provided that we use the following consequence of (3.1):

$$\max_{0<t<T} \alpha_\tau \left\| \frac{\partial^2 \mathbf{w}^\varepsilon}{\partial t^2}(t) \right\|_{2,\Omega} \leqslant C_0.$$

§ 4. Proof of Theorem 2

4.1. Weak and two-scale limits of sequences of displacements, temperatures and pressures. In view of Theorem 1, the sequences $\{\theta^\varepsilon\}$, $\{p_f^\varepsilon\}$, $\{q_f^\varepsilon\}$, $\{p_s^\varepsilon\}$, $\{q_s^\varepsilon\}$ and $\{\mathbf{w}^\varepsilon\}$ are bounded uniformly in ε in the space $L^2(\Omega_T)$. Hence there exists a sequence of $\{\varepsilon > 0\}$ and functions θ, p_f, q_f, p_s, q_s and $\mathbf{w}$ such that

$$\theta^\varepsilon \to \theta,\ p_f^\varepsilon \to p_f,\ q_f^\varepsilon \to q_f,\ p_s^\varepsilon \to p_s,\ q_s^\varepsilon \to q_s,\ \mathbf{w}^\varepsilon \to \mathbf{w} \quad \text{weakly in } L^2(\Omega_T)$$

as $\varepsilon \searrow 0$.

By Lemma 1 there exists a function $\mathbf{u}^\varepsilon \in L^\infty((0,T); W_2^1(\Omega))$ such that $\mathbf{u}^\varepsilon = \mathbf{w}^\varepsilon$ in $\Omega_s \times (0,T)$ and the family $\{\mathbf{u}^\varepsilon\}$ is bounded in the space $L^\infty((0,T); W_2^1(\Omega))$ uniformly with respect to ε. Hence it is possible to extract a subsequence of $\{\varepsilon > 0\}$ such that

$$\mathbf{u}^\varepsilon \to \mathbf{u} \quad \text{weakly in } L^2((0,T); W_2^1(\Omega))$$

as $\varepsilon \searrow 0$.

Using Lemma 1 again we conclude that there exists a function

$$\vartheta^\varepsilon \in L^2((0,T); W_2^1(\Omega))$$

such that $\vartheta^\varepsilon = \theta^\varepsilon$ in $\Omega_s \times (0,T)$ and the family $\{\vartheta^\varepsilon\}$ is bounded in $L^2((0,T); W_2^1(\Omega))$ uniformly with respect to ε. Hence there exists a subsequence of $\{\varepsilon > 0\}$ such that

$$\vartheta^\varepsilon \to \vartheta \quad \text{weakly in } L^2((0,T); W_2^1(\Omega))$$

as $\varepsilon \searrow 0$. Moreover,

$$\chi^\varepsilon \alpha_\mu \mathbb{D}(x, \mathbf{w}^\varepsilon) \to 0, \quad \chi^\varepsilon \alpha_{\varkappa f} \nabla \theta^\varepsilon \to 0 \quad \text{strongly in } L^2(\Omega_T) \tag{4.1}$$

as $\varepsilon \searrow 0$.

Relabelling if necessary we assume that the sequences themselves are convergent.

We now use Nguetseng's theorem: there exist functions $\Theta(\mathbf{x},t,\mathbf{y})$, $P_f(\mathbf{x},t,\mathbf{y})$, $P_s(\mathbf{x},t,\mathbf{y})$, $Q_f(\mathbf{x},t,\mathbf{y})$, $Q_s(\mathbf{x},t,\mathbf{y})$, $\mathbf{W}(\mathbf{x},t,\mathbf{y})$, $\Theta^s(\mathbf{x},t,\mathbf{y})$ and $\mathbf{U}(\mathbf{x},t,\mathbf{y})$ that are 1-periodic in $\mathbf{y}$, such that the sequences $\{\theta^\varepsilon\}$, $\{p_f^\varepsilon\}$, $\{p_s^\varepsilon\}$, $\{q_s^\varepsilon\}$, $\{q_f^\varepsilon\}$, $\{\mathbf{w}^\varepsilon\}$, $\{\nabla\vartheta^\varepsilon\}$ and $\{\nabla\mathbf{u}^\varepsilon\}$ converge two-scale to $\Theta(\mathbf{x},t,\mathbf{y})$, $P_f(\mathbf{x},t,\mathbf{y})$, $P_s(\mathbf{x},t,\mathbf{y})$, $Q_s(\mathbf{x},t,\mathbf{y})$, $Q_f(\mathbf{x},t,\mathbf{y})$, $\mathbf{W}(\mathbf{x},t,\mathbf{y})$, $\nabla\vartheta + \nabla_y\Theta^s(\mathbf{x},t,\mathbf{y})$ and $\nabla\mathbf{u} + \nabla_y\mathbf{U}(\mathbf{x},t,\mathbf{y})$, respectively.

Note that the sequence $\{\operatorname{div}\mathbf{w}^\varepsilon\}$ converges weakly to $\operatorname{div}\mathbf{w}$ and ϑ, $|\mathbf{u}| \in L^2((0,T); W_2^1(\Omega))$. For a disconnected porous space the last assertion follows from the inclusion $\vartheta^\varepsilon, |\mathbf{u}^\varepsilon| \in L^2((0,T); W_2^1(\Omega))$. For a connected porous space this follows from the Friedrichs–Poincaré inequality for $\mathbf{u}^\varepsilon$ and ϑ^ε in the ε-layer at the boundary S and from the convergence of the sequences $\{\mathbf{u}^\varepsilon\}$ and $\{\vartheta^\varepsilon\}$ to $\mathbf{u}$ and ϑ, respectively, strongly in $L^2(\Omega_T)$ and weakly in $L^2((0,T); W_2^1(\Omega))$.

4.2. Micro- and macroscopic equations. I.

Lemma 3. *For all* $\mathbf{x} \in \Omega$ *and* $\mathbf{y} \in Y$ *weak and two-scale limits of the sequences* $\{p_f^\varepsilon\}$, $\{q_f^\varepsilon\}$, $\{p_s^\varepsilon\}$, $\{q_s^\varepsilon\}$, $\{\theta^\varepsilon\}$, $\{\mathbf{w}^\varepsilon\}$, $\{\vartheta^\varepsilon\}$ *and* $\{\mathbf{u}^\varepsilon\}$ *satisfy the following relations:*

$$Q_f = \frac{1}{m}\chi q_f, \qquad Q_f = P_f + \chi\beta_{0f}\Theta, \tag{4.2}$$

$$Q_s = P_s + (1-\chi)\beta_{0s}\vartheta, \tag{4.3}$$

$$\frac{1}{\eta_0} P_s = -(1-\chi)\Big(\operatorname{div}\mathbf{u} + \operatorname{div}_y \mathbf{U} - \frac{\beta}{1-m}\Big), \tag{4.4}$$

$$\operatorname{div}_y \mathbf{W} = 0, \tag{4.5}$$

$$\mathbf{W} = \chi\mathbf{W} + (1-\chi)\mathbf{u}, \qquad \Theta = \chi\Theta + (1-\chi)\vartheta, \tag{4.6}$$

$$q_f = p_f + \beta_{0f}\theta^f, \tag{4.7}$$

$$q_s = p_s + (1-m)\beta_{0s}\vartheta, \tag{4.8}$$

$$\frac{1}{\eta_0} p_s = -(1-m)\operatorname{div}\mathbf{u} - \langle \operatorname{div}_y \mathbf{U}\rangle_{Y_s} + \beta, \tag{4.9}$$

$$\frac{1}{\eta_0} p_s + \operatorname{div}\mathbf{w} = 0, \tag{4.10}$$

where $\theta^f = \langle\Theta\rangle_{Y_f}$ *and* $\beta = \langle\langle \operatorname{div}_y \mathbf{U}\rangle_{Y_s}\rangle_\Omega$.

Proof. To prove (4.2) we substitute a test function $\boldsymbol{\psi}^\varepsilon = \varepsilon\boldsymbol{\psi}(\mathbf{x}, t, \mathbf{x}/\varepsilon)$ in integral identity (1.7), where $\boldsymbol{\psi}(\mathbf{x}, t, \mathbf{y})$ is an arbitrary function 1-periodic and compactly supported in Y_f with respect to $\mathbf{y}$. Passing to the limit as $\varepsilon \searrow 0$ we obtain

$$\nabla_y Q_f(\mathbf{x}, t, \mathbf{y}) = 0, \qquad \mathbf{y} \in Y_f. \tag{4.11}$$

The weak and the two-scale limiting procedures in (1.3) yield (4.7) and the second equation in (4.2).

Performing now the two-scale limiting procedure in the equalities

$$(1-\chi^\varepsilon)p_f^\varepsilon = 0, \qquad (1-\chi^\varepsilon)q_f^\varepsilon = 0$$

we obtain

$$(1-\chi)P_f = 0, \qquad (1-\chi)Q_f = 0,$$

which proves (4.2).

Equations (4.3)–(4.5) and (4.7)–(4.10) are the result of passing to the two-scale limit in equations (1.3)–(1.6) with appropriate test functions. For example, equation (4.8) is a consequence of (1.6), while (4.5) and (4.10) result from the two-scale limiting procedure for the sum of equations (1.4) and (1.6) with test functions of the form $\psi^\varepsilon = \varepsilon\psi(\mathbf{x}, t, \mathbf{x}/\varepsilon)$ and with test functions independent of the 'fast' variable $\mathbf{y} = \mathbf{x}/\varepsilon$.

To prove (4.6) we must consider the two-scale limit in the relations

$$(1-\chi^\varepsilon)(\mathbf{w}^\varepsilon - \mathbf{u}^\varepsilon) = 0, \qquad (1-\chi^\varepsilon)(\theta^\varepsilon - \vartheta^\varepsilon) = 0.$$

Lemma 4. *For all* $(\mathbf{x},t)\in\Omega_T$ *and* $y\in Y$ *the relation*

$$\operatorname{div}_y\Big\{\lambda_0(1-\chi)\big(\mathbb{D}(y,\mathbf{U})+\mathbb{D}(x,\mathbf{u})\big)-\Big(Q_s+\frac{1}{m}q\chi\Big)\cdot\mathbb{I}\Big\}=0 \tag{4.12}$$

holds.

Proof. Substituting a test function of the form $\boldsymbol{\psi}^\varepsilon=\varepsilon\boldsymbol{\psi}(\mathbf{x},t,\mathbf{x}/\varepsilon)$ in (1.7), where $\boldsymbol{\psi}(\mathbf{x},t,\mathbf{y})$ is an arbitrary function 1-periodic in $\mathbf{y}$, vanishing at the boundary S, then passing to the limit as $\varepsilon\searrow 0$ we obtain the desired microscopic equation on the cell Y.

In the same way, using the continuity equations (1.4) and (1.6) to eliminate the terms $\chi^\varepsilon\operatorname{div}\mathbf{w}^\varepsilon$ and $(1-\chi^\varepsilon)\operatorname{div}\mathbf{w}^\varepsilon$, from the integral identity (1.8) we obtain the following result for the temperature.

Lemma 5. *For all* $(\mathbf{x},t)\in\Omega_T$,

$$\begin{aligned}\Delta_y\Theta^s&=0, && \mathbf{y}\in Y_s,\\ \frac{\partial\Theta^s}{\partial n}&=-\nabla\vartheta\cdot\mathbf{n}, && \mathbf{y}\in\gamma.\end{aligned} \tag{4.13}$$

We now proceed to the macroscopic equations for the solid displacement.

Lemma 6. *Let* $\widehat{\rho}=m\rho_f+(1-m)\rho_s$, $\mathbf{w}^f=\langle\mathbf{W}\rangle_{Y_f}$. *Then in* Ω_T *the functions* $\mathbf{u}$, $\mathbf{w}^f$, p_s, q_f, q_s, θ^f *and* ϑ *satisfy the system of macroscopic equations*

$$\begin{aligned}&\tau_0\rho_f\frac{\partial^2\mathbf{w}^f}{\partial t^2}+\tau_0\rho_s(1-m)\frac{\partial^2\mathbf{u}}{\partial t^2}-\widehat{\rho}\mathbf{F}\\ &\quad=\operatorname{div}\big\{\lambda_0\big((1-m)\mathbb{D}(x,\mathbf{u})+\langle\mathbb{D}(y,\mathbf{U})\rangle_{Y_s}\big)-(q_f+q_s)\mathbb{I}\big\},\end{aligned} \tag{4.14}$$

$$\begin{aligned}&\tau_0\Big(c_{pf}\frac{\partial\theta^f}{\partial t}+c_{ps}(1-m)\frac{\partial\vartheta}{\partial t}\Big)-\frac{\beta_{0s}}{\eta_0}\frac{\partial p_s}{\partial t}-\Psi\\ &\quad=\varkappa_{0s}\operatorname{div}\big\{(1-m)\nabla\vartheta+\langle\nabla_y\Theta^s\rangle_{Y_s}\big\}+(\beta_{0f}-\beta_{0s})\frac{\partial\beta}{\partial t}\,.\end{aligned} \tag{4.15}$$

Proof. Equations (4.14) and (4.15) arise as the limits of (1.7) and (1.8) with test functions that have compact support in Ω_T and are independent of ε. As in the proof of the previous result, we have used the continuity equations (1.4) and (1.6) in (1.8).

Remark. Using the same procedure we arrive at the initial conditions

$$\tau_0\big(\rho_f\mathbf{w}^f+\rho_s(1-m)\mathbf{u}\big)\big|_{t=0}=\tau_0\Big(\rho_f\frac{\partial\mathbf{w}^f}{\partial t}+\rho_s(1-m)\frac{\partial\mathbf{u}}{\partial t}\Big)\Big|_{t=0}=0, \tag{4.16}$$

$$\Big(\tau_0\big(c_{pf}\theta^f+c_{ps}(1-m)\vartheta\big)-\frac{\beta_{0s}}{\eta_0}(p_s)-(\beta_{0f}-\beta_{0s})(\beta)\Big)\Big|_{t=0}=0. \tag{4.17}$$

4.3. Micro- and macroscopic equations. II.

Lemma 7. *If* $\mu_1=\infty$, *then* $\mathbf{u}=\mathbf{w}$ *and* $\theta=\vartheta$.

Proof. To verify this result it is sufficient to consider the differences $\mathbf{u}^\varepsilon - \mathbf{w}^\varepsilon$ and $\theta^\varepsilon - \vartheta^\varepsilon$ and use the Friedrichs–Poincaré inequality just as in the proof of Theorem 1.

Lemma 8. *Assume that $\mu_1 < \infty$ and let $\mathbf{V} = \chi\,\partial\mathbf{W}/\partial t$. Then*

$$\tau_0\rho_f\frac{\partial\mathbf{V}}{\partial t} - \rho_f\mathbf{F} = \mu_1\Delta_y\mathbf{V} - \nabla_y R - \frac{1}{m}\nabla q_f, \qquad \mathbf{y}\in Y_f, \tag{4.18}$$

$$\tau_0 c_{pf}\frac{\partial\Theta}{\partial t} = \varkappa_1\mu_1\Delta_y\Theta + \frac{\beta_{0f}}{m}\frac{d\beta}{dt} + \Psi, \qquad \mathbf{y}\in Y_f, \tag{4.19}$$

$$\mathbf{V} = \frac{\partial\mathbf{u}}{\partial t}, \quad \Theta = \vartheta, \qquad \mathbf{y}\in\gamma, \tag{4.20}$$

for $\mu_1 > 0$ and

$$\tau_0\rho_f\frac{\partial\mathbf{V}}{\partial t} = -\nabla_y R - \frac{1}{m}\nabla q_f + \rho_f\mathbf{F}, \qquad \mathbf{y}\in Y_f, \tag{4.21}$$

$$\tau_0 c_{pf}\frac{\partial\Theta}{\partial t} = \frac{\beta_{0f}}{m}\frac{d\beta}{dt} + \Psi, \qquad \mathbf{y}\in Y_f, \tag{4.22}$$

$$(\chi\mathbf{W} - \mathbf{u})\cdot\mathbf{n} = 0, \qquad \mathbf{y}\in\gamma, \tag{4.23}$$

for $\mu_1 = 0$.

In either case ($\mu_1 > 0$ or $\mu_1 = 0$) the functions $\mathbf{V}$ and Θ satisfy the homogeneous initial conditions

$$\mathbf{V}(\mathbf{y},0) = 0, \quad \Theta(\mathbf{y},0) = 0, \qquad \mathbf{y}\in Y_f. \tag{4.24}$$

In the boundary condition (4.21) $\mathbf{n}$ is the outward normal to the boundary γ.

Proof. The differential equations (4.18) and (4.21) follow as $\varepsilon \searrow 0$ from the integral identity (1.7) with test functions $\boldsymbol{\psi} = \boldsymbol{\varphi}(\mathbf{x}/\varepsilon)h(\mathbf{x},t)$, where $\boldsymbol{\varphi}$ is a solenoidal vector-valued function with compact support in Y_f.

The same arguments apply to equations (4.19) and (4.22), provided that we use continuity equation (1.4) to eliminate the term $\chi^\varepsilon \operatorname{div}(\partial\mathbf{w}^\varepsilon/\partial t)$ in integral identity (1.8)

The first boundary condition in (4.20) is a consequence of the first equation in (4.6) and the two-scale convergence of the sequence $\{\alpha_\mu^{1/2}\nabla\mathbf{w}^\varepsilon\}$ to the function $\mu_1^{1/2}\nabla_y\mathbf{W}(\mathbf{x},t,\mathbf{y})$. In view of this convergence, the function $\nabla_y\mathbf{W}(\mathbf{x},t,\mathbf{y})$ is bounded in $L^2(Y)$ uniformly in $(\mathbf{x},t)$ for $\mu_1 > 0$. Similar arguments hold for the second boundary condition in (4.20). The boundary condition (4.23) follows from equations (4.5) and (4.6).

Lemma 9. *If the porous space is disconnected (the case of isolated pores), then* $\mathbf{w} = \mathbf{u}$.

Proof. Indeed, for $0 \leqslant \mu_1 < \infty$ the system of equations (4.5), (4.18)–(4.20) or (4.5), (4.21)–(4.23) has the unique solution $\mathbf{V} = \partial\mathbf{u}/\partial t$.

4.4. Homogenized equations. I.

Lemma 10. *If $\mu_1=\infty$, then $\mathbf{w}=\mathbf{u}$, $\theta=\vartheta$, and the strong/weak limits $\mathbf{u}$, ϑ, p_f, q_f, p_s and q_s solve the following initial-boundary value problem in Ω_T:*

$$\begin{aligned}&\tau_0\widehat{\rho}\,\frac{\partial^2\mathbf{u}}{\partial t^2}+\nabla(q_f+q_s)-\widehat{\rho}\,\mathbf{F}\\&\qquad=\operatorname{div}\Big\{\lambda_0\mathbb{A}^s_0:\mathbb{D}(x,\mathbf{u})+B^s_0\Big(\operatorname{div}\mathbf{u}-\frac{\beta_{0s}}{\eta_0}\,\vartheta\Big)+B^s_1q_f\Big\},\end{aligned}\tag{4.25}$$

$$\begin{aligned}&\tau_0\widehat{c}_p\frac{\partial\vartheta}{\partial t}-\operatorname{div}(B^\vartheta\cdot\nabla\vartheta)-\Psi-\frac{\beta_{0s}}{\eta_0}\frac{\partial p_s}{\partial t}\\&\qquad=(\beta_{0f}-\beta_{0s})\Big(\frac{\beta_{0s}}{\eta_0}\,a^s_2\Big\langle\frac{\partial\vartheta}{\partial t}\Big\rangle_\Omega+a^s_3\Big\langle\frac{\partial q_f}{\partial t}\Big\rangle_\Omega\Big),\end{aligned}\tag{4.26}$$

$$\begin{aligned}&\frac{1}{\eta_0}\,p_s+C^s_0:\mathbb{D}(x,\mathbf{u})+(1-m+a^s_0)\operatorname{div}\mathbf{u}\\&\qquad=\frac{\beta_{0s}}{\eta_0}\,a^s_0(\vartheta-\langle\vartheta\rangle_\Omega)-a^s_1(q_f-\langle q_f\rangle_\Omega),\end{aligned}\tag{4.27}$$

$$q_s=p_s+(1-m)\beta_{0s}\vartheta,\qquad q_f=p_f+m\beta_{0f}\vartheta,\tag{4.28}$$

$$\frac{1}{\eta_0}\,p_s+\operatorname{div}\mathbf{u}=0,\tag{4.29}$$

where the symmetric strictly positive definite constant rank-4 tensor $\mathbb{A}^s_0$, the constant matrices C^s_0, B^s_0, B^s_1, the strictly positive definite symmetric matrix B^ϑ, and the constants a^s_k, $k=0,1,2,3$, are defined below by formulae (4.38), (4.39) and (4.42).

The differential equations (4.25) and (4.26) are endowed with the homogeneous initial conditions

$$\tau_0\mathbf{u}=\tau_0\frac{\partial\mathbf{u}}{\partial t}=0,\tag{4.30}$$

$$\tau_0\widehat{c}_p\vartheta-\frac{\beta_{0s}}{\eta_0}\,p_s-(\beta_{0f}-\beta_{0s})\Big(\frac{\beta_{0s}}{\eta_0}\,a^s_2\langle\vartheta\rangle_\Omega+a^s_3\langle q_f\rangle_\Omega\Big)=0\tag{4.31}$$

for $t=0$ and $\mathbf{x}\in\Omega$ and with the homogeneous boundary conditions

$$\vartheta(\mathbf{x},t)=0,\quad \mathbf{u}(\mathbf{x},t)=0,\qquad \mathbf{x}\in S,\quad t>0.\tag{4.32}$$

Proof. We observe in the first place that $\mathbf{u}=\mathbf{w}$ and $\theta=\vartheta$ by Lemma 7.

The differential equations (4.25) follows from the macroscopic equations (4.14) after substituting the expression

$$\lambda_0\langle\mathbb{D}(y,\mathbf{U})\rangle_{Y_s}=\lambda_0\mathbb{A}^s_1:\mathbb{D}(x,\mathbf{u})+B^s_0\Big(\operatorname{div}\mathbf{u}-\frac{\beta_{0s}}{\eta_0}\,\vartheta\Big)+B^s_1q_f+A(t).$$

In its turn, this expression results from the solution of equations (4.3), (4.5) and (4.12) on the pattern cell Y_s. Indeed, setting

$$\mathbf{U} = \sum_{i,j=1}^{3} \mathbf{U}^{ij}(\mathbf{y}) D_{ij} + \mathbf{U}_0(\mathbf{y})\Big(\operatorname{div}\mathbf{u} - \frac{\beta_{0s}}{\eta_0}(\vartheta - \langle\vartheta\rangle_\Omega)\Big) + \frac{1}{m}\mathbf{U}_1(\mathbf{y})(q_f - \langle q_f\rangle_\Omega) + \frac{\beta_{0s}}{\eta_0}\mathbf{U}_2(\mathbf{y})\langle\vartheta\rangle_\Omega + \mathbf{U}_3(\mathbf{y})\langle q_f\rangle_\Omega.$$

$$Q_s = \lambda_0 \sum_{i,j=1}^{3} Q_s^{ij}(\mathbf{y}) D_{ij} + Q_s^0(\mathbf{y})\Big(\operatorname{div}\mathbf{u} - \frac{\beta_{0s}}{\eta_0}(\vartheta - \langle\vartheta\rangle_\Omega)\Big) + \frac{1}{m}Q_s^1(\mathbf{y})(q_f - \langle q_f\rangle_\Omega) + \frac{\beta_{0s}}{\eta_0}Q_s^2(\mathbf{y})\langle\vartheta\rangle_\Omega + Q_s^3(\mathbf{y})\langle q_f\rangle_\Omega,$$

where

$$D_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big),$$

we arrive at the following boundary-value problems in Y:

$$\begin{gathered} \operatorname{div}_y\{(1-\chi)(\mathbb{D}(y,\mathbf{U}^{ij}) + J^{ij}) - Q_s^{ij}\cdot\mathbb{I}\} = 0, \\ \frac{\lambda_0}{\eta_0} Q_s^{ij} + (1-\chi)\operatorname{div}_y \mathbf{U}^{ij} = 0; \end{gathered} \tag{4.33}$$

$$\begin{gathered} \operatorname{div}_y\{\lambda_0(1-\chi)\mathbb{D}(y,\mathbf{U}_0) - Q_s^0\cdot\mathbb{I}\} = 0, \\ \frac{1}{\eta_0} Q_s^0 + (1-\chi)(\operatorname{div}_y \mathbf{U}_0 + 1) = 0; \end{gathered} \tag{4.34}$$

$$\begin{gathered} \operatorname{div}_y\{\lambda_0(1-\chi)\mathbb{D}(y,\mathbf{U}_1) - (Q_s^1 + \chi)\cdot\mathbb{I}\} = 0, \\ \frac{1}{\eta_0} Q_s^1 + (1-\chi)\operatorname{div}_y \mathbf{U}_1 = 0; \end{gathered} \tag{4.35}$$

$$\begin{gathered} \operatorname{div}_y\{\lambda_0(1-\chi)\mathbb{D}(y,\mathbf{U}_2) - Q_s^2\cdot\mathbb{I}\} = 0, \\ -\frac{1}{\eta_0} Q_s^2 = (1-\chi)\Big(\operatorname{div}_y \mathbf{U}_2 - 1 - \frac{1}{1-m}\langle\operatorname{div}_y \mathbf{U}_2\rangle_{Y_s}\Big); \end{gathered} \tag{4.36}$$

$$\begin{gathered} \operatorname{div}_y\Big\{\lambda_0(1-\chi)\mathbb{D}(y,\mathbf{U}_3) - \Big(Q_s^3 + \frac{1}{m}\chi\Big)\mathbb{I}\Big\} = 0, \\ -\frac{1}{\eta_0} Q_s^3 = (1-\chi)\Big(\operatorname{div}_y \mathbf{U}_3 - \frac{1}{1-m}\langle\operatorname{div}_y \mathbf{U}_3\rangle_{Y_s}\Big). \end{gathered} \tag{4.37}$$

Note that

$$\beta = \frac{\beta_{0s}}{\eta_0}\langle\operatorname{div}_y \mathbf{U}_2\rangle_{Y_s}\langle\vartheta\rangle_\Omega + \langle\operatorname{div}_y \mathbf{U}_3\rangle_{Y_s}\langle q_f\rangle_\Omega$$

in view of the homogeneous boundary conditions for $\mathbf{u}(\mathbf{x},t)$.

Under our assumptions about the geometry of the pattern cell Y_s the problems (4.33)–(4.37) have a unique solution (up to an arbitrary constant vector). To get rid of these arbitrary constant vectors we set

$$\langle\mathbf{U}^{ij}\rangle_{Y_s} = \langle\mathbf{U}_k\rangle_{Y_s} = 0, \qquad k = 0,1,2,3, \quad i,j = 1,2,3.$$

Thus,

$$\mathbb{A}_0^s = (1-m)\sum_{i,j=1}^{3} J^{ij}\otimes J^{ij} + \mathbb{A}_1^s, \qquad \mathbb{A}_1^s = \sum_{i,j=1}^{3}\langle D(y,\mathbf{U}^{ij})\rangle_{Y_s}\otimes J^{ij}. \tag{4.38}$$

In [10] we proved that the tensor $\mathbb{A}_0^s$ is symmetric and strictly positive definite.

Finally, the continuity and state equations (4.27)–(4.29) for the pressures follow from equations (4.7)–(4.10) after substituting the expression

$$\begin{aligned}\langle \operatorname{div}_y \mathbf{U}\rangle_{Y_s} = C_0^s : \mathbb{D}(x,\mathbf{u}) + a_0^s\Big(\operatorname{div}\mathbf{u} - \frac{\beta_{0s}}{\eta_0}(\vartheta - \langle\vartheta\rangle_\Omega)\Big)\\ + \frac{1}{m}a_1^s(\mathbf{y})(q_f - \langle q_f\rangle_\Omega) + \frac{\beta_{0s}}{\eta_0}a_2^s\langle\vartheta\rangle_\Omega + a_3^s\langle q_f\rangle_\Omega,\end{aligned}$$

where

$$\begin{aligned} B_k^s &= \lambda_0\langle \mathbb{D}(y,\mathbf{U}_k)\rangle_{Y_s}, \qquad a_k^s = \langle \operatorname{div}_y \mathbf{U}_k\rangle_{Y_s},\\ C_0^s &= \sum_{i,j=1}^{3}\langle \operatorname{div}_y \mathbf{U}^{ij}\rangle_{Y_s} J^{ij}, \qquad k=0,1,2,3.\end{aligned} \tag{4.39}$$

Now, for $i=1,2,3$ we consider the model boundary-value problems

$$\begin{aligned}\Delta_y\Theta_i^s &= 0, \qquad \mathbf{y}\in Y_s,\\ \frac{\partial\Theta_i^s}{\partial n} &= -\mathbf{e}_i\cdot\mathbf{n}, \qquad \mathbf{y}\in\gamma;\end{aligned} \tag{4.40}$$

and set

$$\Theta^s = \sum_{i=1}^{3}(\Theta_i^s\otimes\mathbf{e}_i)\cdot\nabla_x\vartheta. \tag{4.41}$$

Then Θ^s solves problem (4.13) and after substituting $\langle\nabla_y\Theta^s\rangle_{Y_s}$ in (4.15) we obtain

$$B^\theta = \varkappa_{0s}\Big((1-m)\mathbb{I} + \sum_{i=1}^{3}\langle\nabla_y\Theta_i^s\rangle_{Y_s}\otimes\mathbf{e}_i\Big). \tag{4.42}$$

The properties of the matrix B^θ are well known (see [5], [15]).

Lemma 11. *If the porous space is disconnected, then* $\mathbf{w}=\mathbf{u}$ *and in* Ω_T *the weak limits* θ^f, $\mathbf{u}$, ϑ, p_f, q_f, p_s *and* q_s *satisfy equations (4.24), (4.26) and (4.28), where* $\mathbb{A}_0^s$, C_0^s, B_0^s, B_1^s, B^ϑ, a_k^s, $k=0,1,2,3$, *are as in Lemma 10, the state equation*

$$q_s = p_s + (1-m)\beta_{0s}\vartheta, \qquad q_f = p_f + \beta_{0f}\theta^f \tag{4.43}$$

and the heat equation

$$\begin{aligned}\tau_0 c_{pf}\frac{\partial\theta^f}{\partial t} + \tau_0 c_{ps}(1-m)\frac{\partial\vartheta}{\partial t} - \operatorname{div}(B^\theta\cdot\nabla\vartheta) - \frac{\beta_{0s}}{\eta_0}\frac{\partial p_s}{\partial t}\\ = \Psi + (\beta_{0f}-\beta_{0s})\Big(\frac{\beta_{0s}}{\eta_0}a_2^s\Big\langle\frac{\partial\vartheta}{\partial t}\Big\rangle_\Omega + a_3^s\Big\langle\frac{\partial q_f}{\partial t}\Big\rangle_\Omega\Big),\end{aligned} \tag{4.44}$$

where

$$\theta^f(\mathbf{x},t)=\int_0^t\Big\{b^\theta_f(t-\tau)\Big(\frac{\beta_{0f}}{m}\frac{d\beta}{d\tau}(\tau)+\Psi(\mathbf{x},\tau)\Big)-\tau_0c_{pf}\frac{\partial\vartheta}{\partial\tau}(\mathbf{x},\tau)+m\frac{\partial\vartheta}{\partial\tau}(\mathbf{x},\tau)\Big\}\,d\tau \tag{4.45}$$

for $\mu_1>0$ *and* $\tau_0>0$. *If* $\mu_1>0$ *and* $\tau_0=0$, *then*

$$\theta^f(\mathbf{x},t)=m\vartheta(\mathbf{x},t)-c^\theta_f\Big(\frac{\beta_{0f}}{m}\frac{d\beta}{dt}(t)+\Psi(\mathbf{x},t)\Big) \tag{4.46}$$

and finally, if $\mu_1=0$, *then*

$$\theta^f(\mathbf{x},t)=\frac{m}{\tau_0c_{pf}}\int_0^t\Big(\frac{\beta_{0f}}{m}\frac{d\beta}{dt}(\tau)+\Psi(\mathbf{x},\tau)\Big)\,d\tau. \tag{4.47}$$

Here $b^\theta_f(t)$ *and* c^θ_f *are defined below by formulae* (4.50)–(4.52) *and*

$$\beta=\frac{\beta_{0s}}{\eta_0}\,a^s_2\langle\vartheta\rangle_\Omega+a^s_3\langle q_f\rangle_\Omega. \tag{4.48}$$

The problem has initial and boundary conditions given by (4.30) *and* (4.32) *and the initial condition*

$$\tau_0c_{ps}\vartheta-\frac{\beta_{0s}}{\eta_0}p_s-(\beta_{0f}-\beta_{0s})\beta=0 \tag{4.49}$$

for $t=0$ *and* $\mathbf{x}\in\Omega$.

Proof. The only difference from the previous Lemma 10 is the heat equation for ϑ and the second state equation in (4.28) because $\theta\neq\vartheta$. The fluid temperature $\theta^f=\langle\Theta\rangle_{Y_f}$ is now determined from the microscopic equation (4.19) with boundary and initial conditions (4.20) and (4.24) for $\mu_1>0$ and from the microscopic equation (4.22) with initial condition (4.24) for $\mu_1=0$.

Indeed, the solutions of these problems are given by the formula

$$\Theta=\vartheta(\mathbf{x},t)+\int_0^t\Theta^f_1(\mathbf{y},t-\tau)h(\mathbf{x},\tau)\,d\tau$$

for $\mu_1>0$ and $\tau>0$ and by

$$\Theta=\vartheta(\mathbf{x},t)-\Theta^f_0(\mathbf{y})\Big(\frac{\beta_{0f}}{m}\frac{\partial\beta}{\partial t}(t)+\Psi(\mathbf{x},t)\Big),$$

for $\mu_1>0$ and $\tau=0$, where

$$h=\frac{\beta_{0f}}{m}\frac{\partial\beta}{\partial t}+\Psi-\tau_0c_{pf}\frac{\partial\vartheta}{\partial t}$$

and the functions Θ^f_1 and Θ^f_0 are solutions that are periodic in $\mathbf{y}$ of the following problems:

$$\begin{gathered}\tau_0c_{pf}\frac{\partial\Theta^f_1}{\partial t}=\varkappa_1\mu_1\Delta_y\Theta^f_1,\quad \mathbf{y}\in Y_f,\\ \Theta^f_1(\mathbf{y},0)=\frac{1}{\tau_0c_{pf}},\quad \mathbf{y}\in Y_f;\qquad \Theta^f_1=0,\quad \mathbf{y}\in\gamma,\end{gathered} \tag{4.50}$$

and

$$\varkappa_1\mu_1\Delta_y\Theta_0^f = 1, \quad \mathbf{y}\in Y_f; \qquad \Theta_0^f = 0, \quad \mathbf{y}\in\gamma. \tag{4.51}$$

Then (in accordance with the definition) θ^f is given by formula (4.45) or (4.46). where

$$b_f^\theta(t) = \langle\Theta_1^f\rangle_{Y_f}, \qquad c_f^\theta = \langle\Theta_0^f\rangle_{Y_f}. \tag{4.52}$$

If $\mu_1 = 0$, then Θ is found simply by integrating with respect to time.

4.5. Homogenized equations. II. Assume that $\mu_1 < \infty$. In the same manner as in § 4.4 we verify that the strong limit $\mathbf{u}$ of the sequence $\{\mathbf{u}^\varepsilon\}$ satisfies an initial-boundary value problem similar to (4.25)–(4.29), but different from the latter in general because the weak limit $\mathbf{w}$ of the sequence $\{\mathbf{w}^\varepsilon\}$ is distinct from $\mathbf{u}$ in the general case. More precisely, the following result holds.

Lemma 12. *If $\mu_1 < \infty$, then the strong/weak limits $\mathbf{u}$, $\mathbf{w}^f$, θ^f, ϑ, p_f, q_f, p_s and q_s of the sequences $\{\mathbf{u}^\varepsilon\}$, $\{\chi^\varepsilon\mathbf{w}^\varepsilon\}$, $\{\chi^\varepsilon\theta^\varepsilon\}$, $\{\vartheta^\varepsilon\}$, $\{p_f^\varepsilon\}$, $\{q_f^\varepsilon\}$, $\{p_s^\varepsilon\}$ and $\{q_s^\varepsilon\}$ satisfy the initial-boundary value problem in Ω_T consisting of the momentum balance equation*

$$\begin{aligned}\tau_0\Big(\rho_f\frac{\partial^2\mathbf{w}^f}{\partial t^2} + \rho_s(1-m)\frac{\partial^2\mathbf{u}}{\partial t^2}\Big) + \nabla(q_f+q_s) - \hat{\rho}\mathbf{F}\\ = \operatorname{div}\Big\{\lambda_0 A_0^s : \mathbb{D}(x,\mathbf{u}) + B_0^s\Big(\operatorname{div}\mathbf{u} - \frac{\beta_{0s}}{\eta_0}\vartheta\Big) + B_1^s q_f\Big\}\end{aligned} \tag{4.53}$$

and the continuity equation (4.27) for the solid component, where $\mathbb{A}_0^s$, B_0^s and B_1^s are the same as in (4.25), the continuity equation

$$\frac{1}{\eta_0}p_s + \operatorname{div}\mathbf{w}^f = (m-1)\operatorname{div}\mathbf{u}, \tag{4.54}$$

the state equations (4.43), the heat equation (4.44), and the relation

$$\begin{aligned}\frac{\partial\mathbf{w}^f}{\partial t} &= m\frac{\partial\mathbf{u}}{\partial t}(\mathbf{x},t) + \int_0^t B_1(\mu_1, t-\tau)\cdot\mathbf{z}(\mathbf{x},\tau)\,d\tau,\\ \mathbf{z}(\mathbf{x},t) &= -\frac{1}{m}\nabla q_f(\mathbf{x},t) + \rho_f\mathbf{F}(\mathbf{x},t) - \tau_0\rho_f\frac{\partial^2\mathbf{u}}{\partial t^2}(\mathbf{x},t),\end{aligned} \tag{4.55}$$

for $\tau_0 > 0$ and $\mu_1 > 0$, or Darcy's law

$$\frac{\partial\mathbf{w}^f}{\partial t} = m\frac{\partial\mathbf{u}}{\partial t} + B_2(\mu_1)\cdot\Big(-\frac{1}{m}\nabla q_f + \rho_f\mathbf{F}\Big), \tag{4.56}$$

for $\tau_0 = 0$, or finally, the momentum balance equation for the fluid component in the following form:

$$\tau_0\rho_f\frac{\partial^2\mathbf{w}^f}{\partial t^2} = \tau_0\rho_f B_3\cdot\frac{\partial^2\mathbf{u}}{\partial t^2} + (m\mathbb{I} - B_3)\cdot\Big(-\frac{1}{m}\nabla q_f + \rho_f\mathbf{F}\Big), \tag{4.57}$$

for $\mu_1 = 0$.

The problem is given the initial and boundary conditions (4.30), (4.32) and (4.49) for the displacement $\mathbf{u}$ and the temperature ϑ of the solid component, and the boundary condition

$$\mathbf{w}^f(\mathbf{x},t)\cdot\mathbf{n}(\mathbf{x}) = 0, \qquad (\mathbf{x},t)\in S, \quad t>0, \tag{4.58}$$

and the initial condition

$$\tau_0 \mathbf{w}^f = \tau_0 \frac{\partial \mathbf{w}^f}{\partial t} = 0 \tag{4.59}$$

for the displacement $\mathbf{w}^f$ *of the fluid component.*

In (4.53)–(4.59) $\mathbf{n}(\mathbf{x})$ *is the unit normal at* $\mathbf{x} \in S$, *and the matrices* $B_1(\mu_1, t)$, $B_2(\mu_1)$ *and* B_3 *are defined below by formulae* (4.60)–(4.65).

Proof. We deduce homogenized equations expressing the momentum balance (4.53) and the homogenized continuity equation (4.54) similarly to (4.25) and (4.29). For example, to obtain (4.54) it is sufficient to express $\operatorname{div}\mathbf{w}$ in (4.10) using homogenization in the first equation in (4.6): $\mathbf{w} = \mathbf{w}^f + (1-m)\mathbf{u}$. Initial conditions (4.30) and (4.59) are easy consequences of conditions (4.16) and (4.24). The derivation of boundary condition (4.58) is standard (see [8]).

Thus, the proof of the lemma reduces to the derivation of a homogenized equation for the velocity $\mathbf{v}$ of the fluid component in the form of Darcy's law or the momentum balance law.

a) If $\mu_1 > 0$ and $\tau_0 > 0$, then the solution of the microscopic equations (4.5), (4.18), and (4.20) with homogeneous initial conditions (4.24) is expressed by the following formulae:

$$\mathbf{V} = \frac{\partial \mathbf{u}}{\partial t} + \int_0^t \mathbf{B}_1^f(\mathbf{y}, t-\tau) \cdot \mathbf{z}(\mathbf{x}, \tau)\, d\tau,$$

$$R = \int_0^t R_f(\mathbf{y}, t-\tau) \cdot \mathbf{z}(\mathbf{x}, \tau)\, d\tau,$$

where

$$\mathbf{B}_1^f(\mathbf{y}, t) = \sum_{i=1}^{3} \mathbf{V}^i(\mathbf{y}, t) \otimes \mathbf{e}_i, \qquad R_f(\mathbf{y}, t) = \sum_{i=1}^{3} R^i(\mathbf{y}, t)\mathbf{e}_i,$$

and the functions $\mathbf{V}^i(\mathbf{y}, t)$ and $R^i(\mathbf{y}, t)$ are defined by means of the periodic boundary-value problem

$$\begin{gathered}\tau_0 \rho_f \frac{\partial \mathbf{V}^i}{\partial t} = \mu_1 \Delta \mathbf{V}^i - \nabla R^i, \quad \operatorname{div}_y \mathbf{V}^i = 0, \qquad \mathbf{y} \in Y_f, \quad t > 0; \\ \mathbf{V}^i = 0, \quad \mathbf{y} \in \gamma, \quad t > 0; \qquad \tau_0 \rho_f \mathbf{V}^i(\mathbf{y}, 0) = \mathbf{e}_i, \quad \mathbf{y} \in Y_f.\end{gathered} \tag{4.60}$$

In (4.60) $\mathbf{e}_i$ is a unit vector along the x_i-axis. Therefore,

$$B_1(\mu_1, t) = \langle \mathbf{B}_1^f \rangle_{Y_f}(t). \tag{4.61}$$

b) If $\tau_0 = 0$ and $\mu_1 > 0$, then the solution of the stationary microscopic equations (4.5), (4.18) and (4.20) is given by the formula

$$\mathbf{V} = \frac{\partial \mathbf{u}}{\partial t} + \mathbf{B}_2^f(\mathbf{y}) \cdot \left(-\frac{1}{m} \nabla q + \rho_f \mathbf{F} \right)$$

in which

$$\mathbf{B}_2^f(\mathbf{y}) = \sum_{i=1}^{3} \mathbf{U}^i(\mathbf{y}) \otimes \mathbf{e}_i,$$

and the functions $\mathbf{U}^i(\mathbf{y})$ are determined by the periodic boundary-value problem

$$\begin{aligned} -\mu_1 \Delta \mathbf{U}^i + \nabla R^i = \mathbf{e}_i, \quad \operatorname{div}_y \mathbf{U}^i = 0, \qquad \mathbf{y} \in Y_f; \\ \mathbf{U}^i = 0, \qquad \mathbf{y} \in \gamma. \end{aligned} \tag{4.62}$$

Thus,

$$B_2(\mu_1) = \langle \mathbf{B}_2^f(\mathbf{y}) \rangle_{Y_s}. \tag{4.63}$$

The matrix $B_2(\mu_1)$ is symmetric and positive definite (see [8], Ch. 8).

c) Finally, if $\tau_0 > 0$ and $\mu_1 = 0$, then to solve the microscopic equations (4.5) and (4.21) together with (4.23) and (4.24), we first find the pressure $R(\mathbf{x}, t, \mathbf{y})$ as a solution of the periodic Neumann problem for Laplace's equation in the domain Y_f. If

$$R(\mathbf{x}, t, \mathbf{y}) = \sum_{i=1}^{3} R_i(\mathbf{y}) \mathbf{e}_i \cdot \mathbf{z}(\mathbf{x}, t).$$

where $R^i(\mathbf{y})$ is the solution of the problem

$$\Delta R_i = 0, \quad \mathbf{y} \in Y_f; \qquad \nabla R_i \cdot \mathbf{n} = \mathbf{n} \cdot \mathbf{e}_i, \quad \mathbf{y} \in \gamma, \tag{4.64}$$

then (4.57) occurs as a result of integration of the homogenized equations (4.21) and

$$B_3 = \sum_{i=1}^{3} \langle \nabla R_i(\mathbf{y}) \rangle_{Y_s} \otimes \mathbf{e}_i \tag{4.65}$$

with respect to time, where $m\mathbb{I} - B_3$ is a symmetric positive definite matrix (see [8], Ch. 8).

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Received 11/DEC/06 and 13/JUL/07
Translated by A. MEIRMANOV