INFLUENCE OF INTERNAL HEAT RELEASE ON THE THERMOPHORESIS OF A SOLID SPHEROIDALLY SHAPED AEROSOL PARTICLE

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The thermophoresis of a spheroidally shaped aerosol particle at small relative temperature differences has been considered with allowance for internal heat sources nonuniformly distributed in its volume.

Formulation of the Problem. We consider the steady-state motion of a solid aerosol particle of a spheroidal shape (oblate spheroid) with velocity $\mathbf{U}$ in the negative direction of the z axis inside which nonuniformly distributed internal heat sources of density q_p act. It is assumed that the gas is at rest at infinity and a small constant temperature gradient ∇T is maintained with the use of external sources. The motion of the particle occurs at small relative temperature differences in its vicinity, i.e., when $(T_s - T_\infty)/T_\infty << 1$. When this condition is fulfilled, the thermal conductivity and the coefficients of dynamic and kinematic viscosity can be assumed to be constants, whereas the gas can be considered as an incompressible medium. The particle size is much larger than the mean-free paths of the molecules of the gas mixture; therefore, we will disregard Knudsen corrections [1, 2].

Description of the thermophoretic motion of the particles is carried out in the Stokes approximation in a spheroidal coordinate system $(\varepsilon, \eta, \varphi)$ with its origin at the center of the spheroid, i.e., the origin of a fixed coordinate system in the instantaneous position of the center of the particle is selected. The curvilinear coordinates ε, η, φ are related to the Cartesian coordinates by the following relations [3]:

$$x = c \sinh \varepsilon \sin \eta \cos \varphi\,, \quad y = c \sinh \varepsilon \sin \eta \sin \varphi\,, \quad z = c \cosh \varepsilon \cos \eta\,, \tag{1}$$

$$x = c \cosh \varepsilon \sin \eta \cos \varphi\,, \quad y = c \cosh \varepsilon \sin \eta \sin \varphi\,, \quad z = c \sinh \varepsilon \cos \eta\,, \tag{2}$$

where we have $c = \sqrt{b^2 - a^2}$ in the case of a prolate spheroid ($a < b$, formula (1)) and $c = \sqrt{a^2 - b^2}$ in the case of an oblate spheroid ($a > b$, formula (2)); a and b are the semiaxes of the spheroid. The position of the Cartesian coordinate system is fixed relative to the particle so that the origin of coordinates is located at the center of the spheroid and the z axis coincides with the axis of symmetry of the spheroid.

With the constraints considered above, the velocity, pressure, and temperature distribution are described by the system of equations [4]

$$\nabla P_g = \mu_g \Delta U_g\,, \quad \text{div}\, U_g = 0\,, \tag{3}$$

$$\rho_g c_{pg} (\mathbf{U}_g \cdot \nabla)\, T_g = \lambda_g \Delta T_g\,, \quad \Delta T_p = -\, q_p / \lambda_p \tag{4}$$

with boundary conditions

$$\varepsilon = \varepsilon_0\,, \quad U_\varepsilon = -\frac{cU \cosh \varepsilon}{H_\varepsilon} \cos \eta\,, \quad U_\eta = \frac{cU \sinh \varepsilon}{H_\varepsilon} \sin \eta - K_{t.s} \frac{\nu_g}{T_g} (\nabla T_g \cdot \mathbf{e}_\eta)\,, \quad H_\varepsilon = \sqrt{\cosh^2 \varepsilon - \sin^2 \eta}\,,$$

$$T_g = T_p\,, \quad \lambda_g (\nabla T_g \cdot \mathbf{e}_\varepsilon) = \lambda_p (\nabla T_p \cdot \mathbf{e}_\varepsilon)\,; \tag{5}$$

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$$\varepsilon \to \infty, \quad U_g \to 0, \quad T_g \to T_\infty + |\nabla T|\, c \sinh\varepsilon \cos\eta, \quad P_g \to P_\infty; \tag{6}$$

$$\varepsilon \to 0, \quad T_p \neq \infty. \tag{7}$$

In boundary conditions (5) on the particle surface, we have allowed for: the slip condition for the tangential components of the mass velocity, the equality of temperatures, and the continuity of the heat fluxes on the particle surface. The particle surface corresponds to the coordinate surface with $\varepsilon = \varepsilon_0$. Boundary conditions (6) hold at a large distance from the particle ($\varepsilon \to \infty$), whereas the finiteness of the physical quantities characterizing the particle for $\varepsilon \to 0$ is allowed for in (7).

Temperature Distribution in the Vicinity of the Particle. Determination of the Force and the Thermophoresis Velocity. In the problem, in addition to the dimensionless Reynolds and Péclet numbers, there is another controlled small parameter $\xi = a|\nabla T|/T_\infty << 1$ characterizing the relative temperature difference on the particle size. Therefore, we will seek the solution of boundary-value problem (3), (4) in the form of an expansion in powers of ξ.

To find the force acting on the particle and the thermophoresis velocity in the prescribed external field of the temperature gradient it is necessary to know the velocity, pressure, and temperature distribution in the vicinity of the spheroid. In solution of the problem, we restrict ourselves to corrections of first order of smallness in ξ. Leaving the terms proportional to ξ in Eqs. (4) and solving the resulting systems of equations by the method of separation of variables, we finally obtain for the zero approximations ($\xi = 0$):

$$t_g^{(0)}(\lambda) = 1 + \gamma\lambda_0 \operatorname{arccot}\lambda, \tag{8}$$

$$t_p^{(0)}(\lambda) = D + \frac{\lambda_g}{\lambda_p}\gamma\lambda_0 \operatorname{arccot}\lambda + \int_{\lambda_0}^{\lambda} \operatorname{arccot}\lambda\, f\, d\lambda - \operatorname{arccot}\lambda \int_{\lambda_0}^{\lambda} f d\lambda, \tag{9}$$

where $\lambda = \sinh\varepsilon$, $\lambda_0 = \sinh\varepsilon_0$, and $\gamma = t_s - 1$ is the dimensionless parameter; $t_s = T_s/T_\infty$, and T_s is the average temperature of the spheroid surface, determined as

$$\frac{T_s}{T_\infty} = 1 + \frac{1}{4\pi\lambda_g c\lambda_0 T_\infty} \int_V q_p dV; \tag{10}$$

$$D = 1 + \left(1 - \frac{\lambda_g}{\lambda_p}\right)\gamma\lambda_0 \operatorname{arccot}\lambda_0; \quad f = -\frac{c^2}{2\lambda_p T_\infty} \int_{-1}^{+1} q_p (\lambda^2 + x^2)\, dx; \quad x = \cos\eta.$$

In formula (10), we integrate over the entire volume of the particle, and for the first approximations ($\sim\xi$) we have

$$t_g^{(1)}(\lambda, x) = \cos\eta \left\{ \frac{c\lambda}{a} + \Gamma c (\lambda \operatorname{arccot}\lambda - 1) + \omega \left[A_2 (\operatorname{arccot}\lambda - \frac{\lambda}{2}\operatorname{arccot}^2\lambda) + \right.\right.$$
$$\left.\left. + \frac{A_1}{2}(\operatorname{arccot}\lambda - \lambda \operatorname{arccot}^2\lambda) \right] \right\}, \tag{11}$$

$$t_p^{(1)}(\lambda, x) = \cos\eta \left\{ Bc\lambda + \frac{3(1 - \lambda \operatorname{arccot}\lambda)}{4\pi c^2 \lambda_p T_\infty} \int_V q_i z dV - \lambda \int_{\lambda_0}^{\lambda} (\lambda \operatorname{arccot}\lambda - 1) f_1 d\lambda + \right.$$

$$+ (\lambda \operatorname{arccot} \lambda - 1) \int_{\lambda_0}^{\lambda} \lambda f_1 d\lambda \Bigg\} . \tag{12}$$

Here $\omega = \Pr \gamma\lambda_0/(ac)$ and Pr is the Prandtl number.

The general solution of the Stokes equation in the oblate coordinate system has the following form (see, for example, [3]):

$$U_\varepsilon (\varepsilon, \eta) = \frac{U}{c \cosh \varepsilon H_\varepsilon} \cos \eta \left\{ \lambda A_2 + [\lambda - (1 + \lambda^2) \operatorname{arccot} \lambda] A_1 + c^2 (1 + \lambda^2) \right\}, \tag{13}$$

$$U_\eta (\varepsilon, \eta) = - \frac{U}{c H_\varepsilon} \sin \eta \left\{ \frac{A_2}{2} + (1 - \lambda \operatorname{arccot} \lambda) A_1 + c^2 \lambda \right\}, \tag{14}$$

$$P_{\mathrm{g}} (\varepsilon, \eta) = P_\infty + c \frac{\mu_{\mathrm{g}} U}{H_\varepsilon^4} x (\lambda^2 + x^2) A_2 . \tag{15}$$

The integration constants Γ, B, A_1, and A_2 appearing in (11)–(15) are determined from the boundary conditions on the spheroid surface (5). Integrating the stress tensor over the particle surface, we obtain the following general expression for the force, which is additively made up of the viscous force of the medium F_μ and the force $F^{(1)}$:

$$F = F_\mu + F^{(1)} , \tag{16}$$

where

$$F_\mu = - 8\pi\mu_{\mathrm{g}}^{\mathrm{s}} U \frac{c^2}{\beta [\lambda_0 + (1 - \lambda_0^2) \operatorname{arccot} \lambda_0]} ; \tag{17}$$

$$F^{(1)} = - 8\pi\mu_{\mathrm{g}}^{\mathrm{s}} c K_{\mathrm{t.s}} \frac{\nu_{\mathrm{g}}^{\mathrm{s}}}{t_{\mathrm{s}}} \frac{\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0}{\beta [\lambda_0 + (1 - \lambda_0^2) \operatorname{arccot} \lambda_0]} \frac{\delta}{(1 + \lambda_0^2) \Delta} \frac{|\nabla T|}{T_\infty} \times$$

$$\times \left[1 - \frac{3a (\lambda_0 \operatorname{arccot} \lambda_0 - 1)}{4\pi c^3 \lambda_{\mathrm{g}} T_\infty \lambda_0} \int_V q_{\mathrm{p}} z dV + \Pr \frac{1 + \lambda_0^2}{2} \lambda_0 \gamma \frac{1 + \lambda_0 \operatorname{arccot} \lambda_0 (2 - \lambda_0 \operatorname{arccot} \lambda_0)}{\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0} \right] ; \tag{18}$$

$$\beta = 1 - 2K_{\mathrm{t.s}} \frac{\lambda_0}{t_{\mathrm{s}}} \gamma \frac{\Pr}{\Delta} \frac{\delta}{1 + \lambda_0^2} \frac{\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0}{\lambda_0 + (1 - \lambda_0^2) \operatorname{arccot} \lambda_0} \left[1 - \left(\lambda_0 + \frac{1}{2} \operatorname{arccot} \lambda_0 \right) \operatorname{arccot} \lambda_0 - \right.$$

$$\left. - \frac{\lambda_0 - (2 - \lambda_0 \operatorname{arccot} \lambda_0) \lambda_0^2 \operatorname{arccot} \lambda_0}{\lambda_0 - (1 + \lambda_0^2) \operatorname{arccot} \lambda_0} \right] ; \quad \Delta = (1 - \delta) \operatorname{arccot} \lambda_0 + \delta \frac{\lambda_0}{1 + \lambda_0^2} - \frac{1}{\lambda_0} ; \quad \delta = \frac{\lambda_{\mathrm{g}}^{\mathrm{s}}}{\lambda_{\mathrm{p}}^{\mathrm{s}}} .$$

In the general case, the force $F^{(1)}$ consists of the sum of three forces stemming from respectively: a purely thermophoretic force (first term), a force proportional to the dipole moment of the density of the heat fluxes nonuniformly distributed in the particle's volume (second term), and a force due to the influence of the motion of the medium, i.e., allowing for the convective terms in the heat-conduction equation (third term).

Equating F to zero, we have the following expression for the velocity of motion of the solid spheroidal particle in the prescribed external field of the temperature gradient:

$$U_{\text{t.ph}} = -\frac{b}{a} K_{\text{t.s}} \frac{\nu_{\text{g}}^{\text{s}}}{t_{\text{s}}} \delta \frac{1 - (\lambda_0 + 1/\lambda_0) \operatorname{arccot} \lambda_0}{\sqrt{1+\lambda_0^2} \left[(1-\delta) \operatorname{arccot} \lambda_0 + \delta \dfrac{\lambda_0}{1+\lambda_0^2} - \dfrac{1}{\lambda_0} \right]} \left[1 - \frac{3a}{4\pi c^3 \lambda_{\text{g}} T_\infty} \left(\operatorname{arccot} \lambda_0 - \frac{1}{\lambda_0} \right) \times \right.$$

$$\left. \times \int_V q_{\text{p}} z dV + \Pr \frac{1+\lambda_0^2}{2} \lambda_0 \gamma \frac{1 + \lambda_0 \operatorname{arccot} \lambda_0 (2 - \lambda_0 \operatorname{arccot} \lambda_0)}{\lambda_0 - (1+\lambda_0^2) \operatorname{arccot} \lambda_0} \right] \frac{|\nabla T|}{T_\infty} . \tag{19}$$

To obtain the thermophoresis velocity for a prolate spheroid we must replace λ by $i\lambda$ and c by $-ic$ (i is the imaginary unit) in (19).

Analysis of the Results Obtained. If the influence of the medium's motion and of the internal heat sources is disregarded, expression (19) becomes an expression (coincident with formula (9) in [5]) for a pure velocity of thermophoresis of the spheroidal particle.

To evaluate the contribution of internal heat release to the velocity of thermophoresis of the spheroidal particle we must specify the nature of the heat sources nonuniformly distributed in its volume. We consider as an example the simplest case where the particle absorbs radiation as a black body. The radiation is absorbed in a thin layer of thickness $\delta\varepsilon << \varepsilon_0$ adjacent to the heated part of the particle surface. In this case the density of the heat sources inside the layer of thickness $\delta\varepsilon$ is equal to [6]

$$q_{\text{p}} (\varepsilon, \eta) = \begin{cases} -\dfrac{\cosh \varepsilon \cos \eta}{c (\cosh^2 \varepsilon - \sin^2 \eta)\, \delta\varepsilon} I_0 , & \dfrac{\pi}{2} \le \eta \le \pi , \quad \varepsilon_0 - \delta\varepsilon \le \varepsilon \le \varepsilon_0 ; \\ 0 , & 0 \le \eta \le \dfrac{\pi}{2} . \end{cases} \tag{20}$$

With account for (20) the expression for the thermophoresis velocity takes the form

$$U_{\text{t.ph}}^{*} = K_{\text{t.s}} \nu_{\text{g}}^{\text{s}} \delta f_{\text{t.ph}}^{*} \frac{|\nabla T|}{T_\infty} , \tag{21}$$

$$f_{\text{t.ph}}^{*} = -\frac{b}{a} \frac{1 - (\lambda_0 + 1/\lambda_0) \operatorname{arccot} \lambda_0}{\sqrt{1+\lambda_0^2}\, t_{\text{g}}^{\text{s}} \left[(1-\delta) \operatorname{arccot} \lambda_0 + \delta \dfrac{\lambda_0}{1+\lambda_0^2} - \dfrac{1}{\lambda_0} \right]} \left\{ 1 + \frac{\lambda_0^2 a}{2 \lambda_{\text{g}} T_\infty} I_0 \left(1 + \frac{1}{\lambda_0^2} \right) \times \right.$$

$$\left. \times \left[\lambda_0 \operatorname{arccot} \lambda_0 - 1 + \frac{\Pr}{4} \sqrt{1+\lambda_0^2} \frac{1 - \lambda_0 \operatorname{arccot} \lambda_0 (2 - \lambda_0 \operatorname{arccot} \lambda_0)}{\lambda_0 - (1+\lambda_0^2) \operatorname{arccot} \lambda_0} \right] \right\} ,$$

$$f_{\text{t.ph}}^{\text{w}} = -\frac{b}{a} \frac{1 - (\lambda_0 + 1/\lambda_0) \operatorname{arccot} \lambda_0}{\sqrt{1+\lambda_0^2}\, t_{\text{g}}^{\text{s}} \left[(1-\delta) \operatorname{arccot} \lambda_0 + \delta \dfrac{\lambda_0}{1+\lambda_0^2} - \dfrac{1}{\lambda_0} \right]} .$$

To illustrate the contribution of the shape factor (ratio of the semiaxes of the spheroid) and of the influence of the medium's motion and internal heat release on the thermophoresis velocity (21) Fig. 1 gives the curves relating the values $f = f_{\text{t.ph}}^{*}/f_{\text{t.ph}}^{\text{w}} \big|_{T_\infty = 300\text{K}}$ to the intensity of incident radiation for borated-graphite particles ($\lambda_{\text{p}}^{\text{s}}$ = 55 W/(m·deg) with spherical (curve 1) and spheroidal (curve 2) shapes of the surface with an equatorial radius of a = 35 μm for

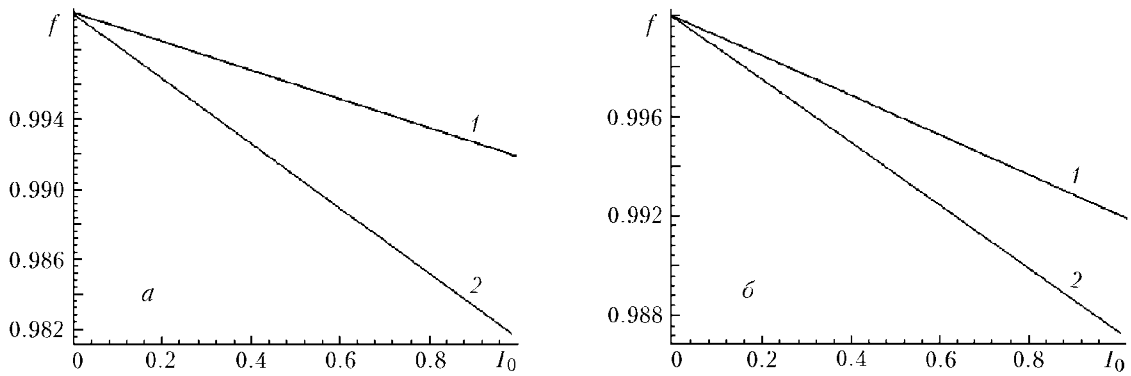

Fig. 1. Function f vs. incident-radiation intensity for the semiaxes ratio b/a = 0.2 (a) and b/a = 0.5 (b). I_0, W/cm^2.

different ratios of the spheroid semiaxes: b/a = 0.2 (see Fig. 1a) and b/a = 0.5 (see Fig. 1b) (the particles are suspended in air at T_∞ = 300 K and P_g = 10^5 Pa). A numerical analysis has shown that, when the ratio of the semiaxes is fixed, the total contribution of the medium's motion and the internal heat release leads to a monotonic decrease in the thermophoresis velocity (see Fig. 1) with increase in the incident-radiation intensity I_0, and this decrease substantially depends on the equatorial radius of the spheroid a.

NOTATION

a and b, semiaxes of the spheroid; c_{pg}, heat capacity of the gas; $\mathbf{e}_\eta$ and $\mathbf{e}_\varepsilon$, unit vectors of the spheroidal coordinate system; F_μ, viscous force of the medium; I_0, incident-radiation intensity; $K_{t.s}$, coefficient of thermal slip; q_p, density distribution of the heat sources inside the particle; Pr, Prandtl number; P_g, pressure of the gas; T_g and T_p, temperatures of the gas and the particle respectively; T_s, average temperature of the spheroid surface; T_∞ and P_∞, temperature and pressure of the gas at a large distance from the particle; U_ε and U_η, components of the mass velocity of the gas $\mathbf{U}_g$; $U = |\mathbf{U}|$, velocity; V, volume of the particle; λ_g and λ_p, thermal conductivities of the gas and the particle respectively; ν_g, μ_g, and ρ_g, kinematic and dynamic viscosities and density of the gaseous medium; $\xi = a|\nabla T|/T_\infty$, small parameter characterizing the relative temperature difference on the particle size; $\varepsilon, \eta, \varphi$, spheroidal coordinate system. Subscripts and superscripts: g, gas; 0, values of the quantities on the particle surface; p, particle; s, quantities at the average temperature of the spheroid surface; ∞, values of the physical quantities away from the particle (at infinity); w, quantities without allowance for internal heat release and the medium's motion; ε and η, components of the physical quantities in the spheroidal coordinate systems; t.s, thermal slip; t.ph, thermal phoresis (thermophoresis).

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