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DESCRIPTION OF INPUT SIGNALS CLASS OF CONTROL SYSTEM

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Abstract
The paper considers the method for determination of an input signals class of the continuous-time linear control system. The class of input signals of a continuous-time linear control system is specified by a heterogeneous differential equation. An estimate is obtained for the steady-state error of control. An example of solving a problem is provided.
Keywords: control system; input signals; differential equation; steady-state error.

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ОПИСАНИЕ КЛАССА ВХОДНЫХ СИГНАЛОВ СИСТЕМЫ УПРАВЛЕНИЯ

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Аннотация
В статье рассматривается метод определения класса входных сигналов линейной системы управления с непрерывным временем. Класс входных сигналов линейной системы управления с непрерывным временем описывается обыкновенным дифференциальным уравнением. Получена оценка установившейся ошибки управления. Приводится пример решения задачи.
Ключевые слова: системы управления; входные сигналы; дифференциальное уравнение; установившаяся ошибка.

Introduction

The quality of the synthesized control system depends on utilization of information about input signals. Therefore, the so-called absorption principle [7] in the automatic control theory is widespread. The absorption principle is built on class description of the input signals by homogeneous differential or difference equations with arbitrary initial conditions [1, 3-5, 8-11]. In this paper, the larger class of input signals of a continuous-time linear control system is specified by the stationary heterogeneous differential equation with arbitrary initial conditions and a restricted right member [2, 6]. This approach can be easily extended to discrete systems.

1. Statement of the problem

Let

$$W_d(s)=\frac{R_d(s)}{Q_d(s)},\ W_s(s)=\frac{R_s(s)}{Q_s(s)}, \qquad (1)$$

be transfer functions corresponding to a desirable control system and a synthesized control system, where $D(s)=Q_d(s)Q_s(s)$ is a stable polynomial.

In this paper, we shall determine the signals class $V_\delta = V_\delta[t_0,+\infty)$ such that

$$\forall\delta>0\ \ \forall f\in V_\delta\ \ \exists t^*>t_0\ \ \forall t>t^*\quad (|\varepsilon(t)|<\delta). \qquad (2)$$

here

$$\varepsilon(t) \div E(s) = (W_d(s) - W_s(s))F(s), \; F(s) \div f(t). \quad (3)$$

2. Main results

Consider the error E(s) determined by (3). Using (1), we get

$$E(s) = \frac{R_d(s)Q_s(s) - R_s(s)Q_d(s)}{D(s)} F(s) \quad (4)$$

or

$$E(s) = \frac{L(s)}{D(s)} F(s), \quad (5)$$

where $\deg D = n$, $\deg L = m$, and

$$L(s) = R_d(s)Q_s(s) - R_s(s)Q_d(s). \quad (6)$$

We show that the input signals class V_δ is defined by the set of solutions of the linear differential equation

$$L(p)f(t) = \varphi(t), \; p = \frac{d}{dt}, \quad (7)$$

with arbitrary initial conditions and a piecewise continuous restricted right member, i.e.,

$$|\varphi(t)| \le M(\delta) \; \forall t \ge t_0, \; M(\delta) \ge 0. \quad (8)$$

Here $f \in C^{m-1}[t_0, +\infty)$ and $f^{(m)}$ is piecewise continuous. Next we define $M = M(\delta)$. The application of the Laplace transformation to equation (7) yields

$$L(s)F(s) = \Phi(s) + L_0(s), \quad (9)$$

where L_0 is a polynomial of degree $m-1$. Note that the Laplace transformation of the functions $f^{(i-1)}(t_0)$, $i = 1, \ldots, m$ and φ is existed. Combining (5) and (9), we obtain

$$E(s) = \frac{L_0(s)}{D(s)} + \frac{\Phi(s)}{D(s)}. \quad (10)$$

Since $D(s)$ is the Hurwitz polynomial, it follows that

$$\lim_{t\to\infty} \varepsilon_1(t) = 0. \quad (11)$$

Here

$$\varepsilon_1(t) \div \frac{L_0(s)}{D(s)} \quad (12)$$

is a transient components of the error $\varepsilon(t)$. We can therefore write

$$\forall \gamma > 0 \; \exists t^* > t_0 \; \forall t > t^* \quad (|\varepsilon_1(t)| < \gamma). \quad (13)$$

Let us consider now a steady-state components

$$\varepsilon_2(t) \div \frac{\Phi(s)}{D(s)} \quad (14)$$

— of the error $\varepsilon(t)$. Let

$$D(s) = \prod_{i=1}^{\nu} (s + \lambda_i)^{k_i}, \; \sum_{i=1}^{\nu} k_i = n, \quad \lambda_i \in \mathrm{C}, \; \operatorname{Re} \lambda_i > 0. \quad (15)$$

Then

$$E_2(s) = \sum_{i=1}^{\nu} \sum_{j=1}^{k_i} \frac{c_{ij}}{(s+\lambda_i)^j} \Phi(s), \quad (16)$$

where

$$c_{ij} = \frac{1}{(j-1)!} \lim_{s\to-\lambda_i} \frac{\mathrm{d}^{j-1}}{\mathrm{d}s^{j-1}} \left[\frac{(s+\lambda_i)^{k_i}}{D(s)} \right] = \frac{1}{(j-1)!} \frac{\mathrm{d}^{j-1}}{\mathrm{d}s^{j-1}} [\prod_{\substack{r=1 \\ r\ne i}}^{n} (s+\lambda_r)^{k_r}]^{-1}_{s=-\lambda_i}. \quad (17)$$

By the convolution theorem, we have

$$\varepsilon_2(t) = \sum_{i=1}^{\nu} \sum_{j=1}^{k_i} \frac{c_{ij}}{(k_i - j)!} \int_{t_0}^{t} (t-\tau)^{k_i - j} e^{-\lambda_i (t-\tau)} \varphi(\tau) \, d\tau. \quad (18)$$

Taking into account (8), we get

$$|\varepsilon_2(t)| \le M(\delta) \sum_{i=1}^{\nu} \sum_{j=1}^{k_i} \frac{|c_{ij}|}{(k_i - j)!} \int_{t_0}^{t} (t-\tau)^{k_i - j} \operatorname{Re} e^{-\lambda_i (t-\tau)} \, d\tau. \quad (19)$$

It can easily be checked that

$$\int_{t_0}^{t} (t-\tau)^{k_i - j} \operatorname{Re} e^{-\lambda_i (t-\tau)} \, d\tau < \int_{t_0}^{\infty} \tau^{k_i - j} \operatorname{Re} e^{\lambda_i \tau} \, d\tau = \frac{(k_i - j)!}{(\operatorname{Re} \lambda_i)^{k_i - j + 1}}. \quad (20)$$

Finally, we obviously obtain the estimate

$$|\varepsilon_2(t)| \le M(\delta) \sum_{i=1}^{\nu} \sum_{j=1}^{k_i} \frac{|c_{ij}|}{(\operatorname{Re} \lambda_i)^{k_i - j + 1}} = M(\delta)\Delta \quad \forall t \ge 0, \quad (21)$$

where c_{ij} is given by (17) and

$$\Delta=\sum_{i=1}^{\nu}\sum_{j=1}^{k_i}\frac{|c_{ij}|}{(\operatorname{Re}\lambda_i)^{k_i-j+1}}\,. \qquad (22)$$

Combining (21) and (13), we get the following proposition:

$$\forall\gamma>0\ \exists t^*>t_0\ \exists t>t^*\quad (|\varepsilon(t)|<\gamma+M(\delta)\Delta)\,. \quad (23)$$

Let $\gamma=\delta/2$ and

$$M(\delta)=\frac{\delta}{2\Delta}\,. \qquad (24)$$

Thus proposition (2) is executed for the input signals satisfying conditions (7), (8), and (24).

Notice that relationships (17) and (22) indicate the practical methods for the decrease of the steasy-state error. If $\varphi(t)\equiv 0$, then the steady-state error is equal to 0.

3. Example

Let us consider

$$W_d(s)=s\,,\quad W_s(s)=\frac{s}{\tau s+1}\,,\quad \tau=\text{const}>0\,. \quad (25)$$

Therefore,

$$L(s)=s^2\,,\quad D(s)=s+\frac{1}{\tau}\,. \qquad (26)$$

From (22), we obtain $\Delta=\tau$. Thus the signals class $V_\delta[0,+\infty)$ is defined by the following conditions:

$$\ddot f(t)=\varphi(t)\,, \qquad (27)$$

$$|\varphi(t)|\le M(\delta)\quad \forall t\ge 0,\ M(\delta)=\frac{\delta}{2\tau}\,. \qquad (28)$$

This implies that

$$f(t)=C_0+C_1t+\int_0^t(t-\tau)\varphi(\tau)\,d\tau\,,\quad \forall\,C_0,C_1\in\mathrm{R}\,. \ (29)$$

Clearly, algebraic polynomials of degree 1, trigonometric polynomials, decreasing exponents, and logarithmic functions belong to the selected class V_δ. Note also that the class V_δ are not exhausted the listed functions.

Let us consider now

$$W_s(s)=\frac{-2\alpha^3}{(s+\alpha)^2}\,,\ \alpha=\text{const}>0\,. \qquad (30)$$

Hence,

$$L(s)=s(s+\alpha)^2+2\alpha^3=(s+2\alpha)(s^2+\alpha^2)\,,$$

$$D(s)=(s+\alpha)^2\,. \qquad (31)$$

Using (22), we get $\Delta=1/\alpha$. Consequently the signals class $V_\delta[0,+\infty)$ is determined by the equation

$$(p+2\alpha)(p^2+\alpha^2)f(t)=\varphi(t)\,,\quad p=\frac{d}{dt}\,, \qquad (32)$$

where

$$|\varphi(t)|\le M(\delta)\quad \forall t\ge 0,\ M(\delta)=\frac{\alpha\delta}{2}\,. \qquad (33)$$

If $\varphi(t)\equiv 0$, then the element with transfer function (30) realizes the asymptotically fine noise-proof differentiating of amplitude modulated signals

$$f(t)=C_1e^{-2\alpha t}+C_2\sin\alpha t+C_3\cos\alpha t\ \ \forall C_1,C_2,C_3\in\mathrm{R}\,. \ (34)$$

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