ON THE FORCE ACTING ON A HEATED SPHERICAL DROP MOVING IN A GASEOUS MEDIUM

N. V. Malai,[1] K. S. Ryazanov,[1] E. R. Shchukin,[2] A. A. Stukalov[1]

UDC 533.72

The flow around a heated spherical drop in a viscous non-isothermal gaseous medium with uniformly distributed constant-power heat sources (sinks) acting inside is theoretically described in the Stokes approximation. It is assumed that the mean temperature of the drop surface can differ substantially from the temperature of the ambient gaseous medium. An analytical expression for the drag force and drift velocity in the gravity field is derived by solving hydrodynamic equations with allowance for the temperature dependence of viscosity, thermal conductivity, and density of the gaseous medium.

Keywords: *motion of heated drops in a gas, gravitational motion, Stokes approximation, drag force.*

1. Formulation of the Problem. In some cases of both theoretical and practical importance, it is necessary to study the influence of heating of the drop surface and circulation motion of the liquid inside the drop on its drag force and velocity, for instance, in designing experimental facilities with directed motion of particles, in development of methods of fine cleaning of gases from aerosol particles, in mathematical modeling of particle deposition in plane-parallel channels with different temperatures of the medium inside, etc. In addition, there are problems of waste cleaning from aerosol particles, whose solution involves estimation of the drag force of particle motion in the ambient medium.

Analytical expressions for the drag force of a heated drop with uniformly distributed heat sources inside the drop and for its drift velocity in the gravity field with arbitrary temperature differences between the drop surface and the area far from the drop, which are generalizations of the Hadamard–Rybchinskii formulas [1], are derived in this work in the Stokes approximation.

Let us consider a steady flow around a heated spherical drop of radius R and density ρ_l by a gas flow moving along the Oz axis; the flow velocity far from the drop is $\boldsymbol{U}_\infty$. Uniformly distributed heat sources (sinks) with a constant power q_l act inside the drop, which results in drop surface heating. The action of these sources may be caused, for instance, by a volume chemical reaction [2], by radioactive decomposition of the particle substance, by absorption of electromagnetic radiation, etc. The thus-induced increase in temperature of the drop surface affects thermophysical characteristics of the gaseous medium and, therefore, may exert a pronounced effect on the velocity and pressure fields in the neighborhood of the drop and, hence, on the value of its drag force.

The drop is assumed to have a rather large size ($R \lesssim 100$ μm), its thermal conductivity is much greater than the thermal conductivity of the gas ($\lambda_l \gg \lambda_g$), and it has a spherical shape (surface tension acting on the liquid–gas interface counteracts against shear stresses, which tend to deform the spherical shape of the drop). As $\lambda_l \gg \lambda_g$, the thermocapillary effect (Marangoni effect) is ignored in the problem [3] (the temperature over the drop surface is constant), and the drop radius remains almost unchanged during the characteristic time.

---

[1]Belgorod State University, Belgorod 308015. [2]Joint Institute of High Temperatures, Russian Academy of Sciences, Moscow 127412; malay@bsu.edu.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 52, No. 4, pp. 63–71, July–August, 2011. Original article submitted March 11, 2009; revision submitted July 22, 2010.

The motion of drops with small relative differences in temperature in their neighborhood was studied in much detail in known works dealing with the motion of spherical drops in viscous liquid and gaseous media [4–6]. The temperature difference is understood as the difference between the drop surface temperature and the temperature of the ambient medium far from the drop. The relative temperature difference is assumed to be small if the inequality $(T_{ls} - T_{g\infty})/T_{g\infty} \ll 1$ is satisfied and to be considerable if $(T_{ls} - T_{g\infty})/T_{g\infty} \sim O(1)$ (T_{ls} is the mean temperature of the drop surface and $T_{g\infty}$ is the temperature of the gaseous medium far from the particle; the subscripts g and l refer to the gaseous medium and the heated liquid drop, respectively; the gas parameters at infinity, in the undisturbed flow, are marked by the subscript "∞," and the subscript s refers to the values of physical quantities at the mean temperature of the drop surface equal to T_{ls}).

The motion of heated solid particles (with considerable temperature differences) in viscous liquid and non-isothermal gaseous media was considered in various papers (see, e.g., [7–10]) where surface heating was demonstrated to exert a significant effect on the drag force and velocity of particles.

In considering the flow around a heated drop with an arbitrary relative difference in temperature, it is necessary to take into account the temperature dependence of the coefficients of molecular transport (viscosity and thermal conductivity) and density of the gaseous medium in gas-dynamic equations. In this work, the power-law dependences of these coefficients on temperature are used to describe the properties of the gaseous medium:

$$\mu_g = \mu_{g\infty} t_g^{\beta}, \qquad \lambda_g = \lambda_{g\infty} t_g^{\alpha}, \qquad \rho_g = \rho_{g\infty}/t_g.$$

Here, $\mu_{g\infty} = \mu_g(T_{g\infty})$, $\lambda_{g\infty} = \lambda_g(T_{g\infty})$, $\rho_{g\infty} = \rho_g(T_{g\infty})$, $t_g = T_g/T_{g\infty}$, $0.5 \leqslant \alpha \leqslant 1.0$, and $0.5 \leqslant \beta \leqslant 1.0$ ($\alpha = 0.81$ and $\beta = 0.72$ for air; $\alpha = 0.71$ and $\beta = 0.69$ for nitrogen; the relative error of approximation is within 4%) [11].

The equations for the particle velocity $\boldsymbol{U}$, pressure P, and temperature T inside and outside the heated drop are written in the following form in the Stokes approximation under the assumptions made [1, 12, 13]:

$$\frac{\partial P_g}{\partial x_k} = \frac{\partial}{\partial x_j}\Big[\mu_g\Big(\frac{\partial U_j^g}{\partial x_k} + \frac{\partial U_k^g}{\partial x_j} - \frac{2}{3}\,\delta_{jk}\,\frac{\partial U_m^g}{\partial x_m}\Big)\Big], \qquad \operatorname{div}(\rho_g \boldsymbol{U}_g) = 0; \tag{1.1}$$

$$\frac{\partial P_l}{\partial x_k} = \frac{\partial}{\partial x_j}\Big[\mu_l\Big(\frac{\partial U_j^l}{\partial x_k} + \frac{\partial U_k^l}{\partial x_j} - \frac{2}{3}\,\delta_{jk}\,\frac{\partial U_m^l}{\partial x_m}\Big)\Big], \qquad \operatorname{div}(\rho_l \boldsymbol{U}_l) = 0; \tag{1.2}$$

$$\operatorname{div}(\lambda_g \nabla T_g) = 0, \qquad \operatorname{div}(\lambda_l \nabla T_l) = -q_l, \qquad P_g = n_g k T_g. \tag{1.3}$$

Problem (1.1)–(1.3) is solved in the drop-fitted coordinate system (in a spherical coordinate system (r, θ, φ) where the coordinate r is counted from the drop center and the angle θ is counted from the free-stream velocity direction) under the boundary conditions

$$r = R, \quad U_r^g = U_r^l = 0, \quad U_\theta^g = U_\theta^l, \quad T_g = T_l, \quad \lambda_g\,\frac{\partial T_g}{\partial r} = \lambda_l\,\frac{\partial T_l}{\partial r} + \sigma_0\sigma_1(T_l^4 - T_{g\infty}^4),$$

$$\mu_g\Big[r\,\frac{\partial}{\partial r}\Big(\frac{U_\theta^g}{r}\Big) + \frac{1}{r}\,\frac{\partial U_r^g}{\partial \theta}\Big] = \mu_l\Big[r\,\frac{\partial}{\partial r}\Big(\frac{U_\theta^l}{r}\Big) + \frac{1}{r}\,\frac{\partial U_r^l}{\partial \theta}\Big]; \tag{1.4}$$

$$r \to \infty, \quad \boldsymbol{U}_g \to U_\infty \cos\theta\,\boldsymbol{e}_r - U_\infty \sin\theta\,\boldsymbol{e}_\theta, \quad P_g \to P_{g\infty}, \quad T_g \to T_{g\infty}; \tag{1.5}$$

$$r \to 0, \quad |\boldsymbol{U}_l| \neq \infty, \quad P_l \neq \infty, \quad T_l \neq \infty. \tag{1.6}$$

Here, U_r and U_θ are the radial and tangential components of particle velocity, μ, λ, and ρ are the viscosity, thermal conductivity, and density, respectively, σ_0 is the Stefan–Boltzmann constant, σ_1 is the integral emissivity, $q_l = \text{const}$ is the constant density of heat sources acting inside the drop [14], and $\boldsymbol{e}_r$ and $\boldsymbol{e}_\theta$ are the unit orths of the spherical coordinate system.

The boundary conditions on the drop surface (1.4) take into account the conditions of impermeability and continuity for the normal and tangential velocity components, equality of temperatures, and continuity of heat fluxes and tangential components of the stress tensor. The boundary conditions (1.5) are valid at a large distance from the drop, and Eq. (1.6) is the condition of finiteness of physical quantities.

The force acting on the particle from the flow is determined by the formula [1, 12]

$$F_z = \int\limits_S (-P_g \cos\theta + \sigma_{rr}\cos\theta - \sigma_{r\theta}\sin\theta) r^2 \sin\theta\, d\theta\, d\varphi, \tag{1.7}$$

where σ_{rr} and $\sigma_{r\theta}$ are the components of the total stress tensor, which are described in the spherical coordinate system as

$$\sigma_{rr} = \mu_g\Big(2\,\frac{\partial U_r^g}{\partial r} - \frac{2}{3}\,\mathrm{div}\,U_g\Big), \qquad \sigma_{r\theta} = \mu_g\Big(\frac{\partial U_\theta^g}{\partial r} + \frac{1}{r}\,\frac{\partial U_r^g}{\partial \theta} - \frac{U_\theta^g}{r}\Big).$$

The governing parameters of the problem are the coefficients $\mu_{g\infty}$, $\rho_{g\infty}$, and $\lambda_{g\infty}$, and also the quantities R, $T_{g\infty}$, and U_∞, which remain unchanged in the course of the flow around a spherical drop. Using these parameters, we can compose a dimensionless complex, namely, the Reynolds number $\mathrm{Re}_\infty = \varepsilon = \rho_{g\infty}U_\infty R/\mu_{g\infty} \ll 1$.

At $\varepsilon \ll 1$, the incoming flow induces only a minor perturbation; therefore, we can use the zeroth approximation in terms of ε in solving gas-dynamic equations.

The expressions for the velocity, pressure, and temperature fields are sought in the form

$$U_r(r,\theta) = U_\infty G(y)\cos\theta, \qquad U_\theta(r,\theta) = -U_\infty g(y)\sin\theta, \qquad P(r,\theta) = h(y)\cos\theta, \tag{1.8}$$

where $G(y)$, $g(y)$, and $h(y)$ are arbitrary functions depending on the coordinate $y = r/R$.

2. Velocity Feld and Temperature Distribution. Determination of the Drag Force and Drift Velocity of the Drop. To find the force acting from the gas onto the drop and the drift velocity of the drop in the gravity field, we have to know the distributions of temperature, particle velocity, and pressure in the neighborhood of the drop. Integrating Eqs. (1.3), we obtain

$$t_{g0}(y) = \Big(1 + \frac{\Gamma_0}{y}\Big)^{1/(1+\alpha)}, \qquad t_{l0}(y) = B_0 + \frac{D_0}{y} + \frac{1}{y}\int\limits_y^1 \psi_0\,dy - \int\limits_y^1 \frac{\psi_0}{y}\,dy. \tag{2.1}$$

Here, B_0 and D_0 are constants determined from the appropriate boundary conditions on the drop surface (1.4), $\Gamma_0 = t_{is}^{1+\alpha} - 1$ is the dimensionless parameter characterizing the difference in temperature between the drop surface and the medium far from the drop, $\psi_0 = -\dfrac{R^2}{2\lambda_{g\infty}T_{g\infty}}\,y^2 \displaystyle\int\limits_{-1}^{+1} q_l\,dx$, $x = \cos\theta$, $t_{ls} = T_{ls}/T_{g\infty}$, and T_{ls} is the mean temperature of the heated drop surface, which is found by solving the system of equations

$$t_{ls} = t_{gs}, \qquad \frac{l_s}{1+\alpha}\,\frac{\lambda_{gs}}{\lambda_{ls}}\,t_{gs} = \frac{R^2}{3\lambda_{ls}T_{g\infty}}\,q_l - \sigma_0\sigma_1\,\frac{RT_{l\infty}^3}{\lambda_{ls}}\,(t_{ls}^4 - 1),$$
$$\lambda_{gs} = \lambda_{g\infty}t_{gs}^{\alpha}, \quad \lambda_{ls} = \lambda_{l\infty}t_{ls}^{\omega}, \quad t_{ls} = t_{l0}\Big|_{y=1}, \quad t_{gs} = t_{g0}\Big|_{y=1}, \quad l_s = \Gamma_0/(1+\Gamma_0). \tag{2.2}$$

In view of Eq. (2.1), the expression for dynamic viscosity can be presented as

$$\mu_g = \mu_{g\infty}t_{g0}^{\beta}. \tag{2.3}$$

In what follows, Eq. (2.3) is used to find the velocity and pressure fields in the neighborhood of the heated drop.

Substituting Eq. (1.8) into the continuity equation (1.1) and taking into account the dependence of density on temperature $\rho_g = \rho_{g\infty}/t_{g0}$, we find the relation between the functions $G(y)$ and $g(y)$:

$$g(y) = G(y) + \frac{1}{2}\,y\Big(\frac{dG(y)}{dy} - fG(y)\Big). \tag{2.4}$$

Here,

$$f = \frac{1}{t_{g0}}\,\frac{dt_{g0}}{dy} = -\frac{l}{y(1+\alpha)}.$$

Let us find the expressions for the components of velocity and pressure of the gaseous medium. Substituting Eqs. (2.3), (1.8), and (2.4) into the Navier–Stokes equations (1.1) linearized in terms of velocity, separating the variables, and applying certain transformations, we obtain an inhomogeneous third-order differential equation for the function $G(y)$; the solution of this equation is sought in the form of generalized power series (see [10]). The recurrent formulas for the power series coefficients are determined by the method of undetermined coefficients. The general expressions for the particle velocity components satisfying the solution boundedness condition at $y \to \infty$ have the form

$$U_r^g(y,\theta) = U_\infty \cos\theta\,(A_1G_1(y) + A_2G_2(y) + A_3G_3(y)),$$

$$U_\theta^g(y,\theta) = -U_\infty \sin\theta\,(A_1G_4(y) + A_2G_5(y) + A_3G_6(y)),$$

where

$$G_1 = \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)} l^n, \qquad G_2 = \frac{1}{y}\sum_{n=0}^{\infty} C_n^{(2)} l^n + \frac{\omega_2}{y^3}\ln y \sum_{n=0}^{\infty} C_n^{(1)} l^n,$$

$$G_3 = \sum_{n=0}^{\infty} C_n^{(3)} l^n + \omega_3 \ln y\,\frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)} l^n, \qquad G_k = \Big(1 + \frac{l}{2(1+\alpha)}\Big)G_{k-3} + \frac{1}{2}\,yG_{k-3}^{\mathrm{I}} \quad (k = 4, 5, 6),$$

G_1^{I}, G_2^{I}, and G_3^{I} are the first derivatives of the functions G_1, G_2, and G_3 with respect to y, respectively.

The expressions for the coefficients $C_n^{(1)}$ $(n \geqslant 1)$, $C_n^{(3)}$ $(n \geqslant 4)$, and $C_n^{(2)}$ $(n \geqslant 3)$ found by the method of undetermined coefficients are written as

$$C_n^{(1)} = \frac{1}{n(n+3)(n+5)}\,\{[(n-1)(3n^2+13n+8) + \gamma_1(n+2)(n+3) + \gamma_2(n+2)]C_{n-1}^{(1)}$$

$$- [(n-1)(n-2)(3n+5) + 2\gamma_1(n^2-4) + \gamma_2(n-2) + \gamma_3(n+3)]C_{n-2}^{(1)}$$

$$+ (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3]C_{n-3}^{(1)}\},$$

$$C_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)}\Big\{[(n-1)(3n^2+n-6) + \gamma_1 n(n+1) + n\gamma_2]C_{n-1}^{(2)}$$

$$- [\gamma_3(n+1) + (n-1)(n-2)(3n-1) + 2\gamma_1 n(n-2) + \gamma_2(n-2)]C_{n-2}^{(2)}$$

$$+ (n-2)[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)]C_{n-3}^{(2)}$$

$$+ \frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)\Delta_k - 6\,\frac{\omega_0(\omega_0-1)\cdots(\omega_0-n+1)}{n!}\Big\},$$

$$C_n^{(3)} = \frac{1}{n(n+2)(n-3)}\Big\{(n-1)(3n^2-5n-4+\gamma_1 n+\gamma_2)C_{n-1}^{(3)}$$

$$- [(n-1)(n-2)(3n-4) + 2\gamma_1(n-1)(n-2) + \gamma_2(n-2) + n\gamma_3]C_{n-2}^{(3)}$$

$$+ (n-2)[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3]C_{n-3}^{(3)}$$

$$+ \frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)\Delta_k\Big\},$$

$$\Delta_k = (3k^2+16k+15)C_k^{(1)} - [(k-1)(6k+13) + \gamma_1(2k+5) + \gamma_2]C_{k-1}^{(1)}$$

$$+ [3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3]C_{k-2}^{(1)}.$$

In calculating the coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ by the recurrent formulas, one should take into account that

$$C_0^{(1)} = 1, \quad C_0^{(3)} = 1, \quad C_1^{(3)} = 0, \quad C_2^{(3)} = \gamma_3/4, \quad C_3^{(3)} = 1,$$

$$\frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}\,(10 + 3\gamma_1 + \gamma_2), \quad C_0^{(2)} = 1, \quad C_2^{(2)} = 1, \quad \omega_0 = \frac{\beta}{1+\alpha},$$

$$\frac{\omega_2}{\Gamma_0^2} = \frac{1}{15}\Big(\frac{1}{4}\,(2\gamma_1 + \gamma_2 + 6\omega_0)(4 + 3\gamma_1 + \gamma_2) + 3\gamma_3 + 3\omega_0(\omega_0 - 1)\Big),$$

$$C_1^{(2)} = -\frac{1}{8}\,(2\gamma_1 + \gamma_2 + 6\omega_0).$$

At $n < 0$, $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ have zero values.

To take into account the influence of circulation motion of the liquid inside the heated drop, we find the solution of Eqs. (1.2). The solution of the hydrodynamic part of the equations for the domain inside the drop is the Hill's spherical vortex [1]. The general solution of these equations has the form [1]

$$U_r^l(y,\theta) = U_\infty \cos\theta\,(A_4 + A_5 y^2), \qquad U_\theta^l(y,\theta) = -U_\infty \sin\theta\,(A_4 + 2A_5 y^2),$$

$$P_l(y,\theta) = P_0 + 10\frac{\mu_{ls}U_\infty}{R}\cos\theta\, y^2 A_5.$$

The integration constants $A_1, A_2, \ldots, A_5$ involved into the formulas for the velocity fields are determined from the corresponding boundary conditions on the particle surface:

$$A_1 = -A_2\,\frac{G_2}{G_1} - \frac{G_3}{G_1}, \qquad A_2 = -\frac{N_2 + \mu_{gs}N_3/(3\mu_{ls})}{N_1 + \mu_{gs}N_4/(3\mu_{ls})},$$

$$A_3 = 1, \qquad A_4 = -A_5, \qquad A_5 = \frac{1}{2G_1}\,(A_2 N_1 + N_2). \tag{2.5}$$

Here,

$$N_1\Big|_{y=1} = G_1(1)G_2^{\rm I}(1) - G_2(1)G_1^{\rm I}(1), \qquad N_2\Big|_{y=1} = G_1(1)G_3^{\rm I}(1) - G_3(1)G_1^{\rm I}(1),$$

$$N_3\Big|_{y=1} = G_3(1)G_1^{\rm II}(1) - G_1(1)G_3^{\rm II}(1) + (2 + l_s/(1+\alpha))(G_3(1)G_1^{\rm I}(1) - G_1(1)G_3^{\rm I}(1)),$$

$$N_4\Big|_{y=1} = G_2(1)G_1^{\rm II}(1) - G_1(1)G_2^{\rm II}(1) + (2 + l_s/(1+\alpha))(G_2(1)G_1^{\rm I}(1) - G_1(1)G_2^{\rm I}(1)),$$

$G_1^{\rm II}$, $G_2^{\rm II}$, and $G_3^{\rm II}$ are the second derivatives of the functions G_1, G_2, and G_3 with respect to y, respectively.

The total force acting on the heated drop is determined by integrating the stress tensor over the drop surface [see Eq. (1.7)]. Substituting the above-obtained expressions into Eq. (1.7) and integrating, we obtain

$$\boldsymbol{F}_z = -4\pi R\mu_{g\infty}U_\infty A_2 \boldsymbol{n}_z,$$

where $\boldsymbol{n}_z$ is the unit vector in the direction of the Oz axis.

Taking into account the constant A_2 in Eq. (2.5), we obtain the following expression for the force acting on the heated spherical drop:

$$\boldsymbol{F}_\mu = 6\pi R\mu_{g\infty} f_\mu U_\infty \boldsymbol{n}_z, \qquad f_\mu = \frac{2}{3}\,\frac{N_2 + \mu_{gs}N_3/(3\mu_{ls})}{N_1 + \mu_{gs}N_4/(3\mu_{ls})}. \tag{2.6}$$

Formula (2.6) allows us to estimate the drag force of the heated drop, i.e., the drop with uniformly distributed constant-power heat sources acting inside the drop. Formula (2.6) is valid for arbitrary relative differences in temperature in the neighborhood of the drop with allowance for the power-law dependence of the coefficients of molecular transport (viscosity and thermal conductivity) and density of the gaseous medium on temperature.

As an example, we find the drift velocity of the heated drop in the gravity field. For this purpose, Eq. (2.6) should be equated to the expression for the gravity force, which takes into account the buoyancy force. As a result, we obtain the formula

$$\boldsymbol{U}_p = h_\mu \boldsymbol{n}_z, \qquad h_\mu = \frac{2}{9}\,R^2\,\frac{\rho_{ls} - \rho_{gs}}{\mu_{g\infty}}\,\frac{N_1 + \mu_{gs}N_4/(3\mu_{ls})}{N_2 + \mu_{gs}N_3/(3\mu_{ls})}\,g, \tag{2.7}$$

where g is the free-fall acceleration.

Formula (2.7) generalizes the Hadamard–Rybchinskii formula [1] and allows calculating the drift velocity of the drop in the gravity field with arbitrary temperature differences in the neighborhood of the drop.

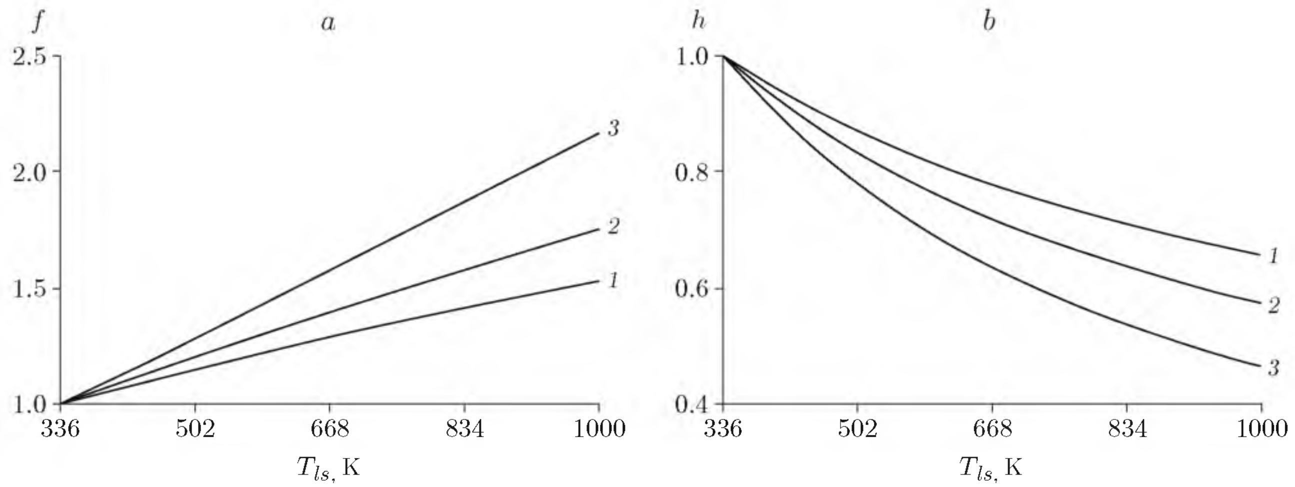

Fig. 1. Functions f (a) and h (b) versus the mean temperature of the drop surface T_{ls}: $\alpha = \beta = 0.5$ (1), $\alpha = \beta = 0.7$ (2), and $\alpha = \beta = 1.0$ (3).

Passing to the limit $\mu_{ls} \to \infty$ in Eqs. (2.6) and (2.7) (the dynamic viscosity of the drop is extremely large), we obtain the formulas

$$F_\mu = 6\pi R \mu_{g\infty} \frac{2N_2}{3N_1} U_\infty, \qquad U_p = \frac{2}{9} R^2 \frac{\rho_{ls} - \rho_{gs}}{\mu_{g\infty}} \frac{N_1}{N_2} g,$$

which coincide with the expressions for the force and velocity of gravitational motion of a uniformly heated spherical solid particle [10].

If the degree of heating of the drop surface is rather small, i.e., the mean temperature of the drop surface is only slightly different from the ambient temperature far from the drop ($\Gamma_0 \to 0$), then the dependence of density and molecular transport coefficients on temperature can be neglected. Then, for $y = 1$, we obtain $G_1 = 1$, $G_1^{\rm I} = -3$, $G_1^{\rm II} = 12$, $G_2 = 1$, $G_2^{\rm I} = -1$, $G_2^{\rm II} = 2$, $G_3 = 1$, $G_3^{\rm I} = 0$, $G_3^{\rm II} = 0$, $G_4 = -1/2$, $G_5 = 1/2$, $G_6 = 1$, $N_1 = 2$, $N_2 = 3$, $N_3 = 6$, and $N_4 = 6$. In this case, Eq. (2.7) takes the form

$$U = \frac{2}{9} R^2 \frac{\rho_l - \rho_g}{\mu_{g\infty}} \frac{1 + \mu_{g\infty}/\mu_{l\infty}}{1 + 2\mu_{g\infty}/(3\mu_{l\infty})} g$$

and transforms to the Hadamard–Rybchinskii formula [1], i.e., the limiting transition is performed.

Figure 1 shows the drag force and velocity of gravitational drift of a potassium drop moving in a nitrogen flow as functions of the temperature T_{ls}. The numerical estimates are obtained by Eqs. (2.6) and (2.7) relating the values of $f = f_\mu / f_\mu \big|_{T_{ls}}$ and $h = h_\mu / h_\mu \big|_{T_{ls}}$ ($T_{ls} = 336$ K) with the values of T_{ls} for the drop with the radius $R = 100$ μm at $T_{g\infty} = 336$ K and $P_g = 1$ atm. The curves $f(T_{ls})$ and $h(T_{ls})$ are constructed for $\alpha = \beta = 0.5$, 0.7, and 1.0.

It is seen in the figure that heating of the drop surface is insignificant in the case with a small difference in temperature in the neighborhood of the drop. This result agrees with experimental data [1]. As the mean temperature increases, heating of the drop surface exerts a significant effect on the velocity of gravitational motion of the drop and its drag force. A similar result is obtained in the case of a uniformly heated spherical solid particle. This fact was demonstrated in [10], where a comparison with experimental data [15] was performed.

Conclusions. Expressions for the drag force and drift velocity of a heated spherical drop in the gravity field in a viscous nonisothermal gaseous medium with arbitrary differences in temperature between the particle surface and the domain far from it are derived with allowance for the dependence of the gaseous medium density and molecular transport coefficients (viscosity and thermal conductivity) on temperature. These expressions are generalizations of the Hadamard–Rybchinskii formulas.

Some authors believed that it is necessary to take into account the Barnett temperature stresses in solving problems with nonisothermal flows [16]. For instance, the problem of the flow around an intensely heated sphere was solved numerically with allowance for the Barnett stresses [17]. The Barnett temperature stresses can exert a significant effect on particle motion at Mach numbers substantially smaller than unity ($M \to 0$) and Reynolds numbers of the order of or smaller than unity [18]. In the present work, we consider the flow around a heated particle at Reynolds and Peclet number appreciably smaller than unity. In this case, the temperature stresses can be neglected even in the case with the relative temperature difference of the order of unity. Taking into account that $\mathrm{Re} = \mathrm{M}/\mathrm{Kn}$, one cane obtain estimates for the Mach and Knudsen numbers and, correspondingly, determine the area of applicability of the theory considered here.

This work was supported by and by the Federal Purpose Program "Science-Educational Manpower of Innovational Russia" in 2009–2013 (State Contract No. 1923 dated 26.10.09) and by the Federal Purpose Program of the Research and Education Center "Controlled electromagnetic processes in condensed media" (State Contract No. 02.740.11.0545).

REFERENCES

1. J. Happel and H. Brenner, *Low Reynolds Number Hydrodynamics*, Noordhoff, Leyden (1965, 1973).
2. L. K. Antanovskii and B. K. Kopbosynov, "Nonstationary thermocapillary drift of a drop of a viscous liquid," *J. Appl. Mech. Tech. Phys.*, **27**, No. 2, 208–213 (1986).
3. A. E. Rednikov and Yu. S. Ryazantsev, "Thermocapillary motion of a drop with uniform internal heat release," *Prikl. Mat. Mekh.*, **53**, No. 2, 271–277 (1989).
4. Yu. I. Yalamov and A. S. Sanasaryan, "Motion of drops in a temperature-inhomogeneous viscous medium," *Inzh.-Fiz. Zh.*, **48**, 2693–2696 (1974).
5. Yu. I. Yalamov and V. S. Galoyan, *Dynamics of Drops in Inhomogeneous Viscous Media* [in Russian], Luis, Erevan (1985).
6. Yu. P. Gupalo, A. E. Rednikov, and Yu. S. Ryazantsev, "Thermocapillary drift of a drop with a nonlinear dependence of surface tension on temperature," *Prikl. Mat. Mekh.*, **53**, No. 3, 433–442 (1989).
7. D. R. Kassoy, T. C. Adomcon, J. R. Messiter, and A. F. Messiter, "Compressible low Reynolds number flow around a sphere," *J. Phys. Fluids*, **9**, No. 4, 671–681 (1966).
8. E. R. Shchukin and N. V. Malai, "Photophoretic and thermodiffusion-phoretic motion of heated non-volatile particles," *Inzh.-Fiz. Zh.*, **54**, No. 4, 628–635 (1988).
9. N. V. Malai, E. R. Shchukin, and Yu. I. Yalamov, "Hydrodynamic resistance of a spheroidal particle with uniform internal heat release," *J. Appl. Mech. Tech. Phys.*, **42**, No. 6, 986–990 (2001).
10. N. V. Malai, E. R. Shchukin, A. A. Stukalov, and K. S. Ryazanov, "Gravity-induced motion of a uniformly heated solid particle in a gaseous medium," *J. Appl. Mech. Tech. Phys.*, **49**, No. 1, 58–63 (2008).
11. S. Bretschneider, *Properties of Gases and Liquids. Engineering Methods of Calculation* [in Russian], Khimiya, Moscow (1966).
12. L. D. Landau and E. M. Lifshits, *Course of Theoretical Physics*, Vol. 6: *Fluid Mechanics*, Pergamon Press, Oxford-Elmsford, New York (1987).
13. G. Z. Gershuni, E. M. Zhukhovitskii, and A. A. Nepomnyashchii, *Stability of Convective Flows* [in Russian], Nauka, Moscow (1989).
14. C. Bohren and D. Huffman, *Absorption and Scattering of Light by Small Particles*, Wiley, New York (1983).
15. V. I. Babii and I. P. Ivanov, "Aerodynamic drag of a particle under nonisothermal conditions," *Teploénergetika*, No. 9, 19–23 (1965).
16. M. N. Kogan, V. S. Galkin, and O. G. Fridlender, "On stresses arising in gases owing to inhomogeneity of temperature and concentrations. New types of free convection," *Usp. Fiz. Nauk*, **119**, No. 1, 111–124 (1976).
17. V. S. Galkin, M. N. Kogan, and O. G. Fridlender, "Low-Reynolds-number gas flow around a strongly heated sphere," *Prikl. Mat. Mekh.*, **36**, No. 5, 880–885 (1972).
18. V. Yu. Aleksandrov and O. G. Fridlender, "Slow gas flows and effect of negative drag of a strongly heated spherical particle," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 168–177 (2008).