

## Discussion

Response to the commentary by X. Artru about the article *Interpretation of the results of the experiment on generation of parametric X-radiation by relativistic electrons in a single-crystal target*, by S.V. Blazhevich and A.V. Noskov

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## A B S T R A C T

An analysis of the comment by X. Artru on the results of a theoretical interpretation of the experiment devoted to measuring the angular density of parametric X-ray radiation (PXR) generated by relativistic electrons in a diamond crystal is carried out. We have pointed out the incorrectness of the arguments of the author put forward against the results of the interpretation. It is shown that even in the case of the absorption absence, the sum of angular densities of usual PXR and forward PXR (FPXR) depends on the orientation of the surface of the monocrystalline target relative to a system of diffracting atomic planes, contrary to the opinion of the author of the criticism.

The author of comments [1] on article [2] has expressed his doubts about the correctness of the dynamical formula for the angular density of the parametric X-ray radiation (PXR), which was used in [2] (formula (6)) to interpret the experimental results of [3], and presented several facts, in his opinion, confirming his point of view.

The main objection of the author [1] is related to the presence in the article [2] of the well-known asymmetry parameter  $\varepsilon = \frac{\sin(\delta + \theta_B)}{\sin(\delta - \theta_B)}$  as the general coefficient in formula (6), where  $\delta$  is the angle of orientation of diffracting atomic planes in a single crystal target relative to the target boundary and  $\theta_B$  is the angle of Bragg scattering.

This objection is formulated as the statement that the boundaries of the target cannot affect the PXR, since this radiation occurs on the atomic planes in the volume of the target.

As a confirmation of his opinion the author of the comment [1] gives a dynamic formula obtained by V.G. Baryshevsky et al. in [4]:

$$\frac{dN_s^{PXR}}{d\Omega} = \frac{\alpha\omega_B T}{4\pi\sin^2\theta_B} \frac{|\chi'_g|^2 C_s^2 \theta^{(s)2}}{(\gamma^{-2} + \theta^2 - \chi'_0)^2 + \varepsilon^{-1} \chi_g'^2 C_s^2}, \quad (1)$$

in which the indicated common factor  $\varepsilon$  is absent and which turns into the kinematic formula in the limit  $\chi_g \rightarrow 0$ . (The notation used in (1) are presented in [2]).

As an additional confirmation for his statement, X. Artru demonstrates the independence of the sum of the angular densities PXR and FPXR on the asymmetry parameter  $\varepsilon$ , and therefore on the conditions at

the crystal boundaries. This sum has been obtained by putting together formula (1) and the formula for FPXR derived by V.G. Baryshevsky in [5]:

$$\frac{dN_s^{FPXR}}{d\Omega} = \frac{\alpha\chi_g'^2 C_s^2 \omega_B T}{4\pi\sin^2\theta_B} \frac{\varepsilon^{-1} \theta_s^2}{(\gamma^{-2} + \theta^2 - \chi'_0)^2 \cdot [(\gamma^{-2} + \theta^2 - \chi'_0)^2 + \varepsilon^{-1} \chi_g'^2 C_s^2]}. \quad (2)$$

The result is the expression

$$\frac{dN^{PXR}(\theta)}{d\Omega} + \frac{dN^{FPXR}(\theta')}{d\Omega'} = \frac{\alpha\omega_B T}{4\pi\sin^2\theta_B} \left\{ \sum_{s=x,y} \frac{|C_s \chi'_g|^2 \cdot \theta_s^2}{(\gamma^{-2} + \theta^2 - \chi'_0)^2} \right\}, \quad (3)$$

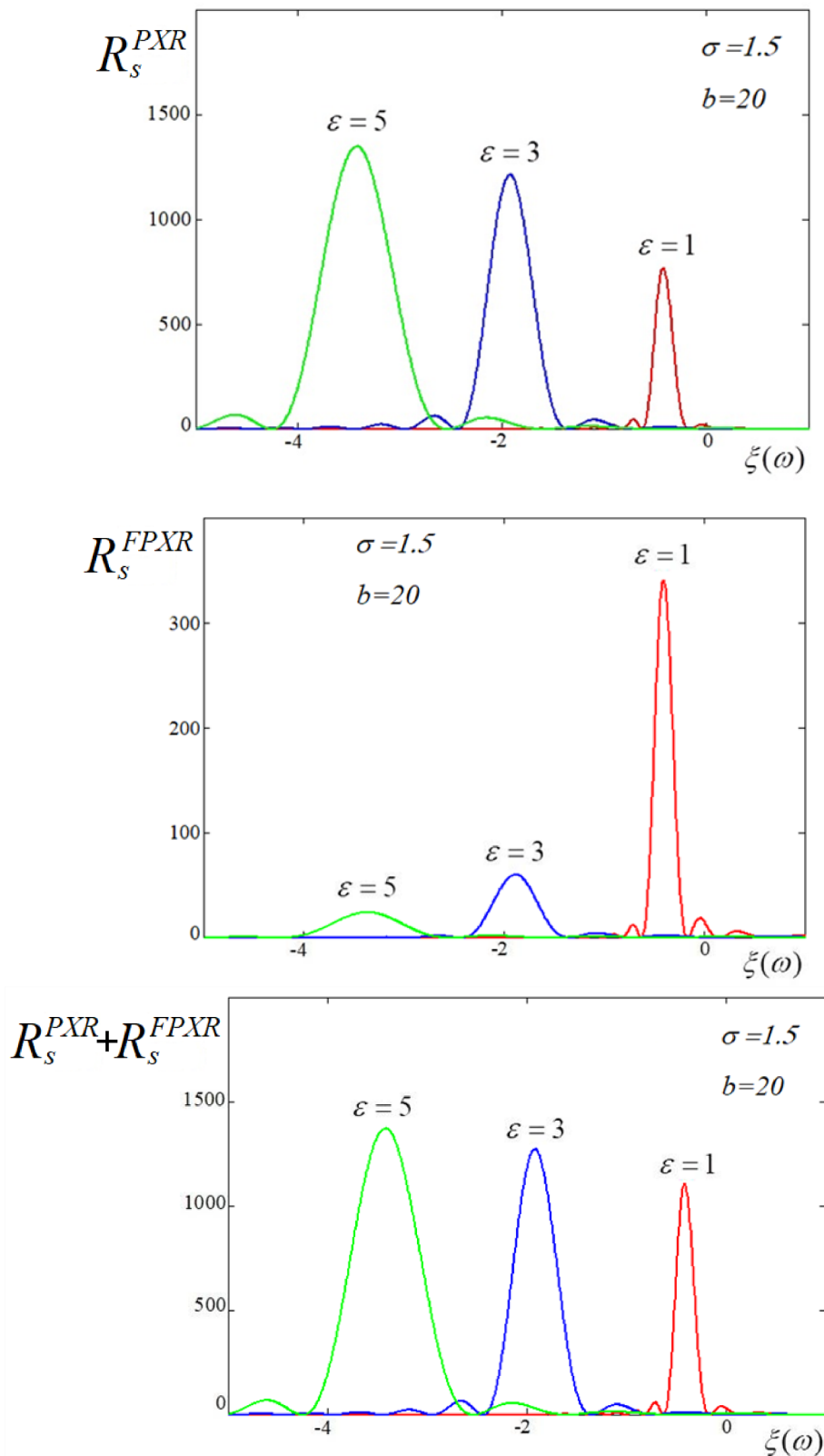
which can be correct if we consider that there is a typographic error in expression (2), namely, an additional factor  $\chi_g'^2 C^{(s)2}$  in the numerator is missing. The correct formula is presented by the expression (10) in the work of O.M. Lugovskaya [6].

However, the fact that there is no dependence on  $\varepsilon$  in formula (3) shows only that the model in which the sum of the angular densities PXR and FPXR given by formulas (1) and (2) does not take into account the conditions of the radiation passage through the crystal boundaries, and does not prove at all that this model correctly reflects the reality. In any case, the fact of independence of the sum of two quantities from a parameter does not prove the absence of such a dependence in each of these quantities separately. One can see that in formula (1) the angular density of the PXR depends on  $\varepsilon$  that originally points to influence of

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Fig. 1. PXR and FPXR spectra under different values of reflection asymmetry (parameter  $\epsilon$ ).



the boundary condition

For comparison with (1) we use the expression for angular density of PXR excited in a crystal by a single electron obtained in our work [7]. For the conditions under which the expression (1) was obtained in [4] by Baryshevsky V.G., our expression will take the following form

$$\begin{aligned} \frac{dN_s^{PXR}}{d\Omega} &= \frac{\alpha\omega_B P_s^2 \epsilon^2}{4\pi \sin^2 \theta_B} \cdot \frac{L}{\sin^2 \theta_B \epsilon (\gamma^{-2} + \theta^2 - \chi'_0)^2 + \chi'_g{}^2 C_s^2} = \\ &= \frac{\alpha\omega_B T \epsilon}{4\pi \sin^2 \theta_B (\gamma^{-2} + \theta^2 - \chi'_0)^2 + \epsilon^{-1} \chi'_g{}^2 C_s^2} \end{aligned} \quad (4)$$

which indeed differs from (1) only by the presence of a total factor  $\epsilon$ ,  $L$

is the target thickness. This additional common factor  $\varepsilon$  in the formula for the PXR angular density appeared in our PXR dynamic theory developed to take into account the influence of the asymmetry of the radiation wave reflection on the width of the PXR spectrum peak. After integrating the spectral-angular density of the PXR over frequency  $\omega$ , the changes of the peak width manifest themselves in the angular density of PXR. The dependence of spectral distribution of the FPXR on the reflection asymmetry parameter  $\varepsilon$  was investigated in detail and presented in our works [8,9]. The expression obtained for FPXR angular density in our dynamic theory completely coincides with the expression (2) obtained by V.G. Baryshevsky. It is evident, that sum of (4) and (2) will depend on the asymmetry parameter  $\varepsilon$  and therefore on target boundary conditions.

To demonstrate the dependence of the spectral-angular densities of PXR and FPXR on the reflection asymmetry we calculated the spectral distributions of PXR and FPXR generated by a single relativistic electron in the monocrystalline target for different values of asymmetry parameter  $\varepsilon$  under the other fixed parameters of the generation process. The results are presented in Fig. 1. The curves in Fig. 1 are calculated for the case when photo absorption is absent using the following expressions derived in the frame of our dynamical theory:

$$\omega \frac{d^2 N_s^{PXR, FPXR}}{d\omega d\Omega} = \frac{e^2}{4\pi^2} P_s^2 \frac{\theta^2}{(\theta^2 + \gamma^{-2} - \chi'_0)} R_s^{PXR, FPXR}, \quad (5)$$

where

$$R_s^{PXR} = 4 \left( 1 - \frac{\xi^{(s)}}{\sqrt{\xi^{(s)2} + \varepsilon}} \right)^2 \frac{\sin^2 \left[ \frac{b^{(s)}}{2} \left( \sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right) \right]}{\left( \sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right)^2} \quad (6)$$

and

$$R_s^{FPXR} = \frac{4}{\xi^{(s)2} + \varepsilon} \frac{\sin^2 \left[ \frac{b^{(s)}}{2} \left( \sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right) \right]}{\left( \sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)2} + \varepsilon}}{\varepsilon} \right)^2} \quad (7)$$

are the functions of the PXR and FPXR spectral distributions. The parameter  $b^{(s)} = L/2 \sin(\delta - \theta_B) L_s^{ext}$  is half the thickness of the single crystal target expressed in extinction lengths  $L_s^{ext} = \frac{1}{\omega |\chi'_g C_s|}$ ,  $\omega_B$  is Bragg frequency,  $L$  is the target thickness,  $\sigma^{(s)} = \frac{1}{|\chi'_g C_s|} (\gamma^{-2} + \theta^2 - \chi'_0)$ . The parameter  $\xi^{(s)}(\omega)$  is a linear function  $\xi^{(s)}(\omega) = \frac{2 \sin^2 \theta_B}{\nu^2 |\chi'_g C_s|} \left( \frac{\omega(1 - \theta_{//} \cot \theta_B)}{\omega_B} - 1 \right) + \frac{1 - \varepsilon}{2\nu^{(s)}}$ ,  $\xi^{(s)}(\omega_B) = 0$  if  $\theta_{//} = 0$  and  $\varepsilon = 1$ . The parameter  $\nu^{(s)} = \frac{\chi'_g C^{(s)}}{\chi'_0}$ ,  $0 \leq \nu \leq 1$ .

The most important argument confirming the correctness of our theory is the result of the comparison with experiment [3], which

showed excellent agreement in absolute units of the results of our calculations with the data on measuring the angular distribution and the total yield of PXR in contrast to the results of calculations using the dynamic Nitta formula (used in [3;2]) similar with formula (1) which shows the large discrepancy ( $\varepsilon$  times) with the experiment in the condition of reflection asymmetry. But for symmetric reflection ( $\varepsilon = 1$ ) the results of calculations by both formulas coincide.

In spite of the fact that the operation of summing the formulas (1) and (2) is correct, the formula (1) was obtained in [4] without use of the integration of any concrete expression for PXR spectrum and thus cannot be considered to be correct in our case of asymmetric reflection.

The dynamical theory of coherent radiation excited in single crystal targets describing the effect of the radiation waves reflection asymmetry on the PXR spectrum width was developed in our works [7,10] (for a single relativistic electron) and [11] (for a beam of the relativistic electrons) in the two-wave approach of dynamic theory of diffraction [12].

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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