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On the Ratio of the Relativistic Electron PXR in the Bragg Direction and in the Forward Direction in Laue Geometry

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Abstract—Parametric X-ray radiation and parametric X-ray radiation at a small angle with respect to the velocity of a relativistic electron crossing a single crystal plate in Laue scattering geometry are studied based on the dynamic X-ray diffraction theory. Analytical expressions for the spectral–angular density of radiations are derived in the general case of asymmetric reflection. The ratio of the contributions of these radiation mechanisms is considered.

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INTRODUCTION

When a fast charged particle crosses a single crystal, its Coulomb field is scattered on the system of parallel atomic crystal planes, generating parametric X-ray radiation (PXR) [1–3]. The theory of parametric X-ray radiation (PXR) of a relativistic particle in the crystal predicts radiation not only in the Bragg scattering direction, but also along the emitting particle velocity (FPXR) [4–6]. This radiation results from dynamical diffraction effects in PXR. There have been attempts to experimentally study FPXR [7–11]. Only in the experiment in [11] were X-rays of relativistic electrons from a thick absorbing single-crystal target detected under conditions of FPXR generation; however, the sought reflection was very weak against the background of radiation generated by electrons on structural elements of the experimental setup. Thus, the theoretical study of PXR properties in the direction of the relativistic electron velocity and the search for conditions for more prominent experimental observations of this dynamic effect remain urgent.

The dynamic FPXR effect was theoretically described in detail in the symmetric case in [12–14]. PXR and transition radiation (TR) in the general case of asymmetric reflection and FPXR in the Bragg geometry were theoretically described in [15–17] and [18], respectively. In these papers, it was shown that the spectral–angular density of these emission mechanisms depends strongly on reflection asymmetry and the effects associated with this density were determined.

In this paper, based on the two-wave approximation of the dynamic diffraction theory [19], analytical expressions for PXR and FPXR amplitudes are derived in the general case of asymmetric reflection, as well as expressions describing the spectral–angular density of radiations. Based on the derived expressions, the ratio of radiation yields is studied depending on reflection asymmetry.

RADIATION AMPLITUDE

Let us consider radiation of a fast charged particle crossing a single-crystal plate with constant velocity **V** (Fig. 1). In this case, we will consider equations for the

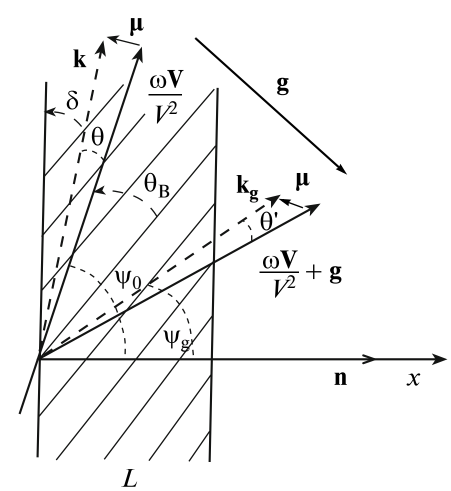

Fig. 1. Emission geometry and the system of notations of the used quantities: θ and θ' are emission angles, θ_B is the Bragg angle (the angle between the electron velocity **V** and atomic planes), δ is the angle between the surface and atomic planes under consideration, and **k** and $\mathbf{k_g}$ are the wave vectors of incident and diffracted photons.

Fourier transform of the electromagnetic field

$$\mathbf{E}(\mathbf{k},\omega) = \int dt\, d^3\mathbf{r}\, \mathbf{E}(\mathbf{r},t)\exp(i\omega t - i\mathbf{kr}). \qquad (1)$$

Since the relativistic particle field can be considered as transverse, the incident $\mathbf{E}_0(\mathbf{k}, \omega)$ and diffracted $\mathbf{E}_\mathbf{g}(\mathbf{k}, \omega)$ electromagnetic waves are defined by two amplitudes with different transverse polarizations,

$$\begin{aligned} \mathbf{E}_0(k,\omega) &= E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)} + E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)}, \\ \mathbf{E}_\mathbf{g}(k,\omega) &= E_\mathbf{g}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)} + E_\mathbf{g}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}. \end{aligned} \qquad (2)$$

Polarization unit vectors $\mathbf{e}_0^{(1)}$, $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(1)}$, and $\mathbf{e}_1^{(2)}$ are chosen such that $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_0^{(2)}$ are perpendicular to the vector $\mathbf{k}$, and the vectors $\mathbf{e}_1^{(1)}$ and $\mathbf{e}_1^{(2)}$ are perpendicular to the vector $\mathbf{k}_\mathbf{g} = \mathbf{k} + \mathbf{g}$. The vectors $\mathbf{e}_0^{(2)}$ and $\mathbf{e}_1^{(2)}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ (π polarization), and vectors $\mathbf{e}_0^{(1)}$ and $\mathbf{e}_1^{(1)}$ are perpendicular to it (σ polarization); $\mathbf{g}$ is the reciprocal lattice vector which defines the system of reflecting atomic crystal planes.

The system of equations for the Fourier transform of the electromagnetic field in the two-wave approximation in the dynamic diffraction theory is written as [20]

$$\begin{cases} (\omega^2(1+\chi_0) - k^2)E_0^{(s)} + \omega^2\chi_{-\mathbf{g}}C^{(s)}E_\mathbf{g}^{(s)} \\ = 8\pi^2 ie\omega\theta VP^{(s)}\delta(\omega - \mathbf{kV}), \\ \omega^2\chi_\mathbf{g}C^{(s)}E_0^{(s)} + (\omega^2(1+\chi_0) - k_\mathbf{g}^2)E_\mathbf{g}^{(s)} = 0, \end{cases} \qquad (3)$$

where $\chi_\mathbf{g}$ and $\chi_{-\mathbf{g}}$ are the coefficients of the Fourier expansion of the dielectric susceptibility of the crystal in reciprocal lattice vectors $\mathbf{g}$,

$$\begin{aligned} \chi(\omega,\mathbf{r}) &= \sum_\mathbf{g}\chi_\mathbf{g}(\omega)\exp(i\mathbf{gr}) \\ &= \sum_\mathbf{g}\left(\chi'_\mathbf{g}(\omega) + i\chi''_\mathbf{g}(\omega)\right)\exp(i\mathbf{gr}). \end{aligned} \qquad (4)$$

Let us consider a crystal with central symmetry ($\chi_\mathbf{g} = \chi_{-\mathbf{g}}$). In expression (4), the quantities $\chi'_\mathbf{g}$ and $\chi''_\mathbf{g}$ are given by

$$\begin{aligned} \chi'_\mathbf{g} &= \chi'_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp\left(-\frac{1}{2}g^2u_\tau^2\right), \\ \chi''_\mathbf{g} &= \chi''_0\exp\left(-\frac{1}{2}g^2u_\tau^2\right), \end{aligned} \qquad (5)$$

where $\chi_0 = \chi'_0 + i\chi''_0$ is the average dielectric susceptibility, $F(g)$ is the form factor of an atom containing Z electrons, $S(\mathbf{g})$ is the structure factor of the unit cell containing N_0 atoms, and u_τ is the root-mean-square amplitude of thermal vibrations of crystal atoms. The X-ray frequency region ($\chi'_\mathbf{g} < 0$, $\chi'_0 < 0$) is considered.

The quantities $C^{(s)}$ and $P^{(s)}$ in system (3) are defined as

$$\begin{aligned} &C^{(s)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)},\ C^{(1)} = 1,\ C^{(2)} = \cos 2\theta_\mathrm{B}, \\ &P^{(s)} = \mathbf{e}_0^{(s)}(\boldsymbol{\mu}/\mu),\ P^{(1)} = \sin\varphi,\ P^{(2)} = \cos\varphi, \end{aligned} \qquad (6)$$

where $\boldsymbol{\mu} = \mathbf{k} - \omega\mathbf{V}/V^2$ is the component of the virtual photon momentum, perpendicular to the particle velocity $\mathbf{V}$ ($\mu = \omega\theta/V$, where $\theta \ll 1$ is the angle between the vectors $\mathbf{k}$ and $\mathbf{V}$); θ_B is the angle between the electron velocity and the system of crystallographic planes (Bragg angle), φ is the azimuthal angle of radiation, measured from the plane formed by vectors $\mathbf{V}$ and $\mathbf{g}$, the reciprocal lattice vector is given by $g = 2\omega_\mathrm{B}\sin\theta_\mathrm{B}/V$, and ω_B is the Bragg frequency. The angle between the vector $\frac{\omega\mathbf{V}}{V^2}$ and the incident wave vector $\mathbf{k}$ is denoted by θ, and the angle between the vector $\frac{\omega\mathbf{V}}{V^2} + \mathbf{g}$ and the diffracted wave vector $\mathbf{k}_\mathbf{g}$ is denoted by θ'. System of equations (3) at the parameter $s = 1$ and 2 describes the σ and π-polarized fields.

Let us solve the dispersion relation following from system (3) for X-ray waves in the crystal,

$$(\omega^2(1+\chi_0) - k^2)(\omega^2(1+\chi_0) - k_\mathbf{g}^2) - \omega^4\chi_{-\mathbf{g}}\chi_\mathbf{g}C^{(s)2} = 0 \qquad (7)$$

by standard methods of the dynamic theory [19].

Let us search for the projections of wave vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ onto the X axis coinciding with the vector $\mathbf{n}$ (Fig. 1) in the form

$$\begin{aligned} k_x &= \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\psi_0} + \frac{\lambda_0}{\cos\psi_0}, \\ k_{\mathbf{g}x} &= \omega\cos\psi_\mathbf{g} + \frac{\omega\chi_0}{2\cos\psi_\mathbf{g}} + \frac{\lambda_\mathbf{g}}{\cos\psi_\mathbf{g}}. \end{aligned} \qquad (8)$$

Let us use the known expression relating the dynamic components λ_0 and $\lambda_\mathbf{g}$ for X-ray waves [19]

$$\lambda_\mathbf{g} = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_\mathbf{g}}{\gamma_0}, \qquad (9)$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_\mathbf{g}}{\gamma_0}\right)$; $\alpha = \frac{1}{\omega^2}(k_\mathbf{g}^2 - k^2)$; $\gamma_0 = \cos\psi_0$; $\gamma_\mathbf{g} = \cos\psi_\mathbf{g}$; ψ_0 is the angle between the wave vector $\mathbf{k}$ of the incident wave and the normal vector $\mathbf{n}$ to the plate surface, and $\psi_\mathbf{g}$ is the angle between the wave vector $\mathbf{k}_\mathbf{g}$ and the vector $\mathbf{n}$ (Fig. 1). The vector $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ magnitudes are given by

$$k = \omega\sqrt{1+\chi_0} + \lambda_0,\ k_\mathbf{g} = \omega\sqrt{1+\chi_0} + \lambda_\mathbf{g}. \qquad (10)$$

Substituting (8) into (7) and taking into account (9), $k_{\parallel} \approx \omega\sin\psi_0$, and $k_{\mathbf{g}\parallel} \approx \omega\sin\psi_{\mathbf{g}}$, we obtain the expressions for dynamic components

$$\lambda_{\mathbf{g}}^{(1,2)} = \frac{\omega}{4}\left(\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right),$$
$$\lambda_0^{(1,2)} = \omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta \pm \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right). \quad (11)$$

Since $|\lambda_0| \ll \omega$ and $|\lambda_{\mathbf{g}}| \ll \omega$, it can be shown that $\theta \approx \theta'$ (Fig. 1); therefore, in what follows, we denote the angle θ' as θ.

We write the solution to the system of equations (3) for incident and diffracted fields in the crystal as

$$E_0^{(s)\mathrm{cr}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{-\omega^2\beta - 2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0}{4\frac{\gamma_{\mathbf{g}}}{\gamma_0}\left(\lambda_0 - \lambda_0^{(1)}\right)\left(\lambda_0 - \lambda_0^{(2)}\right)} \quad (12a)$$
$$\times\delta(\lambda_0 - \lambda_0^*) + E_0^{(s)^{(1)}}\delta(\lambda_0 - \lambda_0^{(1)}) + E_0^{(s)^{(2)}}\delta(\lambda_0 - \lambda_0^{(2)}),$$

$$E_{\mathbf{g}}^{(s)\mathrm{cr}} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(1)}\right)\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(2)}\right)} \quad (12b)$$
$$\times\delta\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^*\right) + E_{\mathbf{g}}^{(s)^{(1)}}\delta(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(1)}) + E_{\mathbf{g}}^{(s)^{(2)}}\delta(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(2)}),$$

respectively, where $\lambda_0^* = \omega\left(\frac{\gamma^{-2} + \theta^2 - \chi_0}{2}\right)$; $\lambda_{\mathbf{g}}^* = \frac{\omega\beta}{2} + \frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$; $\gamma = \sqrt{1 - V^2}$ is the particle Lorentz factor; and $E_0^{(s)^{(1)}}$ and $E_0^{(s)^{(2)}}$, $E_{\mathbf{g}}^{(s)^{(1)}}$ are the free incident and diffracted fields in the crystal, respectively.

For the field in vacuum in front of the crystal, the solution to system (3) is given by

$$E_0^{(s)\mathrm{vac\,I}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2}{\omega}\lambda_0}\delta\left(\lambda_0 - \lambda_0^*\right)$$
$$= \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(-\chi_0 - \frac{2\gamma_0}{\omega\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}} + \beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^*\right). \quad (13)$$

where the relation $\delta\left(\lambda_0 - \lambda_0^*\right) = \frac{\gamma_{\mathbf{g}}}{\gamma_0}\delta\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^*\right)$ is used.

The incident and diffracted fields in vacuum behind the crystal are given by

$$E_0^{(s)\mathrm{vac\,II}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{1}{-\chi_0 - \frac{2\lambda_0}{\omega}}\delta(\lambda_0 - \lambda_0^*)$$
$$+ E_0^{(s)\mathrm{Rad}}\delta\left(\lambda_0 + \frac{\omega\chi_0}{2}\right), \quad (14)$$
$$E_{\mathbf{g}}^{(s)\mathrm{vac}} = E_{\mathbf{g}}^{(s)\mathrm{Rad}}\delta\left(\lambda_{\mathbf{g}} + \frac{\omega\chi_0}{2}\right),$$

where e $E_0^{(s)\mathrm{Rad}}$ and $E_{\mathbf{g}}^{(s)\mathrm{Rad}}$ are the fields of coherent radiations along the electron velocity and near the Bragg direction, respectively.

From the second equation of system (3), the expression relating the diffracted and incident fields in the crystal follows,

$$E_0^{(s)\mathrm{cr}} = \frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_g C^{(s)}}E_{\mathbf{g}}^{(s)\mathrm{cr}}. \quad (15)$$

To determine the incident $E_0^{(s)\mathrm{Rad}}$ and diffracted $E_{\mathbf{g}}^{(s)\mathrm{Rad}}$ fields, we use ordinary boundary conditions on the input and output surfaces of the crystal plate, respectively,

$$\int E_0^{(s)\mathrm{vac\,I}}d\lambda_0 = \int E_0^{(s)\mathrm{cr}}d\lambda_0, \quad \int E_{\mathbf{g}}^{(s)\mathrm{cr}}d\lambda_0 = 0,$$
$$\int E_0^{(s)\mathrm{cr}}\exp\left(i\frac{\lambda_0}{\gamma_0}L\right)d\lambda_0 = \int E_0^{(s)\mathrm{vac\,II}}\exp\left(i\frac{\lambda_0}{\gamma_0}L\right)d\lambda_0, \quad (16a)$$

$$\int E_0^{(s)\mathrm{vac\,I}}d\lambda_{\mathbf{g}} = \int E_0^{(s)\mathrm{cr}}d\lambda_{\mathbf{g}}, \quad \int E_{\mathbf{g}}^{(s)\mathrm{cr}}d\lambda_{\mathbf{g}} = 0,$$
$$\int E_{\mathbf{g}}^{(s)\mathrm{cr}}\exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right)d\lambda_{\mathbf{g}} = \int E_{\mathbf{g}}^{(s)\mathrm{vac}}\exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right)d\lambda_{\mathbf{g}}. \quad (16b)$$

and obtain the expression for radiation fields,

$$E_{\mathbf{g}}^{(s)\mathrm{Rad}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s)}\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right]}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)}\right)}$$
$$\times\left[\left(\frac{\omega}{-\chi_0\omega - 2\lambda_0^*} + \frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}\right)}\right)\right. \quad (17a)$$
$$\times\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)\right) - \left(\frac{\omega}{-\chi_0\omega - 2\lambda_0^*}\right.$$
$$\left.\left. + \frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}\right)}\right)\left(1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)\right],$$

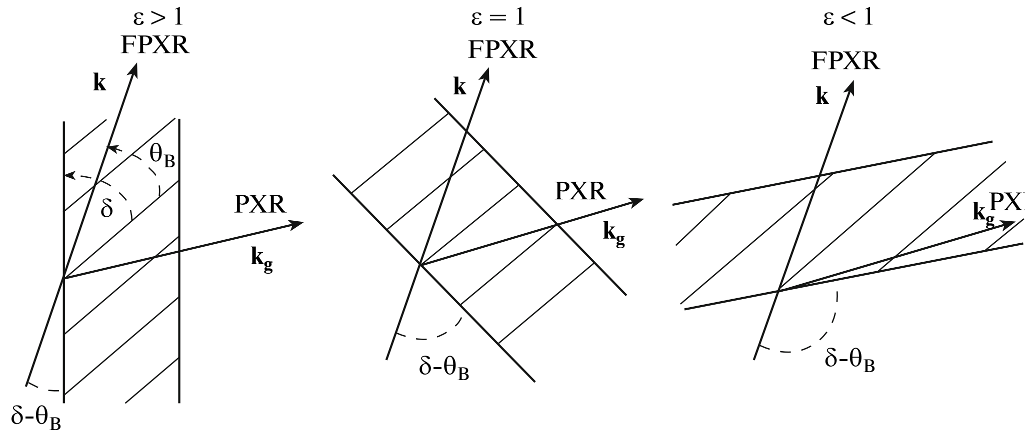

Fig. 2. Symmetric ($\varepsilon = 1$) and asymmetric ($\varepsilon > 1$, $\varepsilon < 1$) reflections of the particle field.

$$E_0^{(s)\mathrm{Rad}} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_g^*\right)\frac{L}{\gamma_g}\right]}{2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\left(\lambda_0^{(1)}-\lambda_0^{(2)}\right)}\times\left[\left(-\omega^2\beta-2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^{(2)}\right)\left(\frac{\omega}{-\omega\chi_0-2\lambda_0^*}+\frac{\omega}{2(\lambda_0^*-\lambda_0^{(2)})}\right)\right.$$
$$\left.\times\left(\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)-1\right)-\left(-\omega^2\beta-2\omega\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^{(1)}\right)\times\left(\frac{\omega}{-\omega\chi_0-2\lambda_0^*}+\frac{\omega}{2(\lambda_0^*-\lambda_0^{(1)})}\right)\left(\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)-1\right)\right]. \tag{17b}$$

The bracketed terms in expressions (17a) and (17b) correspond to two different branches of X-ray waves excited in the crystal.

For further analysis of radiation, we write the dynamic components in (11) as follows:

$$\lambda_0^{(1,2)} = \frac{\omega\left|\chi_{\mathbf{g}}'\right|C^{(s)}}{2\varepsilon}\left(-\xi^{(s)}+\frac{i\rho^{(s)}(1-\varepsilon)}{2}\pm\sqrt{\xi^{(s)^2}+\varepsilon-2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}+\kappa^{(s)}\varepsilon\right)-\rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4}+\kappa^{(s)^2}\varepsilon\right)}\right), \tag{18a}$$

$$\lambda_g^{(1,2)} = \frac{\omega\left|\chi_{\mathbf{g}}'\right|C^{(s)}}{2}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1-\varepsilon)}{2}\pm\sqrt{\xi^{(s)^2}+\varepsilon-2i\rho^{(s)}\left(\frac{(1-\varepsilon)}{2}\xi^{(s)}+\kappa^{(s)}\varepsilon\right)-\rho^{(s)^2}\left(\frac{(1-\varepsilon)^2}{4}+\kappa^{(s)^2}\varepsilon\right)}\right), \tag{18b}$$

where

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega)+\frac{1-\varepsilon}{2\nu^{(s)}},$$

$$\eta^{(s)}(\omega) = \frac{\alpha}{2\left|\chi_{\mathbf{g}}'\right|C^{(s)}} = \frac{2\sin^2\theta_B}{V^2\left|\chi_{\mathbf{g}}'\right|C^{(s)}}\left(1-\frac{\omega(1-\theta\cos\varphi\,\mathrm{ctg}\,\theta_B)}{\omega_B}\right), \tag{19}$$

$$\varepsilon = \frac{\gamma_{\mathbf{g}}}{\gamma_0} = \frac{\cos\psi_{\mathbf{g}}}{\cos\psi_0},\ \nu^{(s)} = \frac{\chi_{\mathbf{g}}'C^{(s)}}{\chi_0'},\ \rho^{(s)} = \frac{\chi_0''}{\left|\chi_{\mathbf{g}}'\right|C^{(s)}},\ \kappa^{(s)} = \frac{\chi_{\mathbf{g}}''C^{(s)}}{\chi_0''}.$$

Since the inequality $2\sin^2\theta_B/V^2\left|\chi_{\mathbf{g}}'\right|C^{(s)} \gg 1$ is valid in the X-ray frequency range, $\eta^{(s)}(\omega)$ is a rapid function of frequency ω; therefore, for further analysis of FPXR and TR spectra, it is very convenient to consider $\eta^{(s)}(\omega)$ as a spectral variable characterizing the frequency ω. An important parameter in expression (19) is the parameter ε, which defines the degree of asymmetry of field reflection with respect to the plate surface. We note that wave vectors of incident and diffracted photons in the symmetric case form equal angles with the surface plate (Fig. 2); in the case of asymmetric reflection, the angles are unequal. In the symmetric case, $\varepsilon = 1$ and $\delta = \pi/2$; in the asymmetric case, $\varepsilon \neq 1$ and $\delta \neq \pi/2$.

Let us write the asymmetry parameter in the form

$$\varepsilon = \frac{\sin(\delta+\theta_B)}{\sin(\delta-\theta_B)}, \tag{20}$$

where θ_B is the angle between the electron velocity and the system of crystallographic planes. We note that the angle $\delta - \theta_B$ of electron incidence onto the plate surface increases with decreasing parameter ε and vice versa (Fig. 2).

SPECTRAL-ANGULAR RADIATION DENSITY IN THE BRAGG SCATTERING DIRECTION

Let us write expression (17a) for the radiation field amplitude in the Bragg direction in the form of two terms,

$$E_{\mathbf{g}}^{(s)\mathrm{Rad}} = E_{\mathrm{PXR}}^{(s)} + E_{\mathrm{DTR}}^{(s)}, \tag{21a}$$

$$E_{\mathrm{PXR}}^{(s)} = -\frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\omega^2 \chi_{\mathbf{g}} C^{(s)}}{8\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}} \frac{1}{\lambda_0^*} \times \left[\left(\beta + \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}} L\right)}{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}\right)\right.$$

$$\left. - \left(\beta - \sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right)\left(\frac{1 - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}} L\right)}{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}\right)\right] \times \exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right], \tag{21b}$$

$$E_{\mathrm{DTR}}^{(s)} = \frac{8\pi^2 ieV\theta P^{(s)}}{\omega} \frac{\chi_{\mathbf{g}} C^{(s)}}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\sqrt{\beta^2 + 4\chi_{\mathbf{g}}\chi_{-\mathbf{g}} C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}$$
$$\times \left(\frac{\omega}{-\omega\chi_0 - 2\lambda_0^*} + \frac{\omega}{2\lambda_0^*}\right)\left[\exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}} L\right)\right. \tag{21c}$$
$$\left. - \exp\left(-i\frac{\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}} L\right)\right]\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right].$$

Expression (21b) describes the PXR field amplitude, and (21c) describes the amplitude of the DTR field induced due to diffraction of transition radiation generated on the input surface at the system of atomic planes of the crystal.

Substituting expressions (18) into (21b) and (21c), we write them in the form

$$E_{\mathrm{PXR}}^{(s)} = \frac{4\pi^2 ieV}{\omega} \frac{\theta P^{(s)}}{\theta^2 + \gamma^{-2} - \chi_0}\left(\frac{\xi^{(s)} + \sqrt{\xi^{(s)^2} + \varepsilon}}{\sqrt{\xi^{(s)^2} + \varepsilon}} \frac{1 - \exp\left[-ib^{(s)}\left(\sigma^{(s)} + \frac{\xi^{(s)} + \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon}\right) - b^{(s)}\rho^{(s)}\Delta^{(2)}\right]}{\sigma^{(s)} + \frac{\xi^{(s)} + \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon} - i\rho^{(s)}\Delta^{(2)}}\right.$$

$$\left. - \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}}{\sqrt{\xi^{(s)^2} + \varepsilon}} \frac{1 - \exp\left[-ib^{(s)}\left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon}\right) - b^{(s)}\rho^{(s)}\Delta^{(1)}\right]}{\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon} - i\rho^{(s)}\Delta^{(1)}}\right)\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right], \tag{22a}$$

$$E_{\mathrm{DTR}}^{(s)} = \frac{4\pi^2 ieV}{\omega}\theta P^{(s)}\left(\frac{1}{\theta^2 + \gamma^{-2}} - \frac{1}{\theta^2 + \gamma^{-2} - \chi_0}\right)\frac{\varepsilon}{\sqrt{\xi^{(s)^2} + \varepsilon}}\left(\exp\left[-ib^{(s)}\left(\sigma^{(s)} + \frac{\xi^{(s)} - \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon}\right) - b^{(s)}\rho^{(s)}\Delta^{(1)}\right]\right.$$
$$\times \tag{22b}$$
$$\left. - \exp\left[-ib^{(s)}\left(\sigma^{(s)} + \frac{\xi^{(s)} + \sqrt{\xi^{(s)^2} + \varepsilon}}{\varepsilon}\right) - b^{(s)}\rho^{(s)}\Delta^{(2)}\right]\right)\exp\left[i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{L}{\gamma_g}\right],$$

where

$$\Delta^{(2)} = \frac{\varepsilon+1}{2\varepsilon} + \frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} + \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}},$$

$$\Delta^{(1)} = \frac{\varepsilon+1}{2\varepsilon} - \frac{1-\varepsilon}{2\varepsilon}\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}} - \frac{\kappa^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}},$$

$$\sigma^{(s)} = \frac{1}{|\chi'_{\mathbf{g}}|C^{(s)}}\left(\theta^2+\gamma^{-2}-\chi'_0\right) \equiv \frac{1}{\nu^{(s)}}\left(\frac{\theta^2}{|\chi'_0|}+\frac{1}{\gamma^2|\chi'_0|}+1\right), \tag{23}$$

$$b^{(s)} = \frac{\omega|\chi'_{\mathbf{g}}|C^{(s)}}{2}\frac{L}{\gamma_0}.$$

The parameter $b^{(s)}$ can be written as

$$b^{(s)} = \frac{1}{2\sin(\delta-\theta_B)}\frac{L}{L_{\text{ext}}^{(s)}}, \tag{24}$$

from which it follows that $b^{(s)}$is equal to half the electron path in the plate, expressed in terms of the extinction length $L_{\text{ext}}^{(s)} = \dfrac{1}{\omega|\chi'_{\mathbf{g}}|C^{(s)}}$.

The PXR yield is formed for the most part by only one branch, i.e., the first one in expression (18a), corresponding to the second term in (22a). As is easy to check, the real denominator part vanishes only in this term. The solution to the corresponding equation,

$$\sigma^{(s)} + \frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon} = 0 \tag{25}$$

defines the frequency ω_* in the vicinity of which the spectrum of PXR photons emitted at a fixed observation angle is concentrated.

Substituting (21a), (22a), and (22b) into the well-known [20] expression for the spectral-angular density of X-ray radiation,

$$\omega\frac{d^2N}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E^{(s)\text{Rad}}\right|^2, \tag{26}$$

we find the expressions describing the contributions of PXR and DTR mechanisms to the spectral-angular radiation density and the term resulting from interference of these mechanisms,

$$\omega\frac{d^2N_{\text{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}P^{(s)^2}\frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi'_0)^2}R_{\text{PXR}}^{(s)}, \tag{27a}$$

$$R_{\text{ПРИ}}^{(s)} = \left(1-\frac{\xi}{\sqrt{\xi^2+\varepsilon}}\right)^2\frac{1+\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right)-2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)^2+\rho^{(s)^2}\Delta^{(1)^2}}, \tag{27b}$$

$$\omega\frac{d^2N_{\text{DTR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}P^{(s)^2}\theta^2 \times\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)^2 R_{\text{DTR}}^{(s)}, \tag{28a}$$

$$R_{\text{DTR}}^{(s)} = \frac{4\varepsilon^2}{\xi^2+\varepsilon}\exp\left(-b^{(s)}\rho^{(s)}\frac{1+\varepsilon}{\varepsilon}\right)$$
$$\times\left[\sin^2\left(b^{(s)}\frac{\left(\sqrt{\xi^2+\varepsilon}\right)}{\varepsilon}\right)+\text{sh}^2\left(b^{(s)}\rho^{(s)}\frac{(1-\varepsilon)\xi^{(s)}+2\varepsilon\kappa^{(s)}}{2\varepsilon\sqrt{\xi^2+\varepsilon}}\right)\right], \tag{28b}$$

$$\omega\frac{d^2N_{\text{INT}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}} -\frac{1}{\theta^2+\gamma^{-2}-\chi'_0}\right)\frac{1}{\theta^2+\gamma^{-2}-\chi'_0}R_{\text{INT}}^{(s)}, \tag{29a}$$

$$R_{\text{INT}}^{(s)} = -\frac{2\varepsilon}{\xi^{(s)^2}+\varepsilon}\text{Re}\left(\left(\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}\right)\right.$$
$$\times\frac{1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]}{\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}-i\rho^{(s)}\Delta^{(1)}}$$
$$\times\left(\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]\right.$$
$$\left.\left.-\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(2)}\right]\right)\right). \tag{29b}$$

Let us consider the case of a thin target $\left(b^{(s)}\rho^{(s)} \ll 1\right)$, where the absorbance $\rho^{(s)}$ can be neglected. To exhibit the dynamic effects, we will consider a crystal of such a thickness that the electron path length $L/\sin(\delta-\theta_B)$ in the plate would be much longer than the extinction length $L_{\text{ext}}^{(s)} = \frac{1}{\omega\left|\chi'_{\mathbf{g}}C^{(s)}\right|}$ of X-ray waves in the crystal, i.e., $b^{(s)} \gg 1$. In this case, the expressions describing the spectra of PXR, DTR radiations, and their interference take the form

$$R_{\text{PXR}}^{(s)} = 4\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)^2 \times\frac{\sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)^2}, \tag{30}$$

$$R_{\text{DTR}}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)^2}+\varepsilon}\sin^2\left(b^{(s)}\frac{\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right), \tag{31}$$

$$R_{\text{INT}}^{(s)} = -4\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\xi^{(s)^2}+\varepsilon}\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right) \times\frac{\sin\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}}{\varepsilon}\right)\right)-\sin\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)}{\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}}. \tag{32}$$

The expressions derived make it possible to study all PXR spectral-angular characteristics.

SPECTRAL-ANGULAR RADIATION DENSITY ALONG THE EMITTING PARTICLE VELOCITY

From the expression for the radiation field amplitude along the emitting particle velocity, we separate the two terms

$$E_0^{(s)\text{Rad}} = E_{\text{FPXR}}^{(s)} + E_{\text{TR}}^{(s)}, \tag{33a}$$

$$E_{\text{FPXR}}^{(s)} = \frac{4\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}}{2\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\frac{1}{\lambda_0^*} \times\left(\frac{1-\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}\right)}{\lambda_0^*-\lambda_0^{(2)}}-\frac{1-\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}\right)}{\lambda_0^*-\lambda_0^{(1)}}\right) \times\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_0^*\right)\frac{L}{\gamma_0}\right], \tag{33b}$$

$$E_{\text{TR}}^{(s)} = \frac{4\pi^2 ieV\theta P^{(s)}}{\omega}\left(\frac{\omega}{\omega\chi_0+2\lambda_0^*}-\frac{\omega}{2\lambda_0^*}\right) \times\left[\left(1-\frac{\beta}{\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(1-\exp\left(i\frac{\lambda_0^{(2)}-\lambda_0^*}{\gamma_0}L\right)\right) +\left(1+\frac{\beta}{\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}}\right)\left(1-\exp\left(i\frac{\lambda_0^{(1)}-\lambda_0^*}{\gamma_0}L\right)\right)\right] \times\exp\left[i\left(\frac{\omega\chi_0}{2}+\lambda_0^*\right)\frac{L}{\gamma_0}\right]. \tag{33c}$$

Expressions (33b) and (33c) describe the FPXR and TR field amplitudes, respectively.

The appearance of the FPXR reflection requires satisfying at least one of the following equalities

$$\operatorname{Re}\left(\lambda_0^*-\lambda_0^{(1)}\right)=0,\ \operatorname{Re}\left(\lambda_0^*-\lambda_0^{(2)}\right)=0, \tag{34}$$

i. e., the denominator real part of at least one of bracketed terms in expression (33b) should be zero.

Substituting (18a) into (33b) and (33c), we write the expressions for the amplitude of PXR along the electron velocity and TR in the form

$$E_{\text{FPXR}}^{(s)} = \frac{4\pi^2 ieV}{\omega}\frac{\theta P^{(s)}}{\gamma^{-2}+\theta^2-\chi_0}\frac{1}{\sqrt{\xi^{(s)^2}+\varepsilon}} \times\left(\frac{1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(2)}\right]}{\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}-i\rho^{(s)}\Delta^{(2)}} -\frac{1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]}{\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}-i\rho^{(s)}\Delta^{(1)}}\right)\times\exp\left[i\omega\left(\frac{\gamma^{-2}+\theta^2}{2}\right)\frac{L}{\gamma_0}\right], \tag{35a}$$

$$E_{\rm TR}^{(s)}=\frac{4\pi^2 ieV\theta P^{(s)}}{\omega}\left(\frac{1}{\gamma^{-2}+\theta^2}-\frac{1}{\gamma^{-2}+\theta^2-\chi_0}\right)\left(\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\left(1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right.\right.\right.$$
$$\left.\left.-\,b^{(s)}\rho^{(s)}\Delta^{(2)}\right]\right)+\left(1+\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\left(1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]\right)\Bigg)\exp\left[i\omega\left(\frac{\gamma^{-2}+\theta^2}{2}\right)\frac{L}{\gamma_0}\right]. \tag{35b}$$

As for PXR, the FPXR yield is formed for the most part only due to one of the branches, which corresponds to the second term (35a), since the denominator real part vanishes only in this term,

$$\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}=0. \tag{36}$$

Substituting (33a), (35a), and (35b) into (26), we obtain expressions describing the contributions of FPXR and TR mechanisms to the spectral-angular radiation density and the term resulting from interference of these mechanisms,

$$\omega\frac{d^2N_{\rm FPXR}^{(s)}}{d\omega d\Omega}=\frac{e^2}{4\pi^2}P^{(s)^2}\frac{\theta^2}{(\theta^2+\gamma^{-2}-\chi_0')^2}R_{\rm FPXR}^{(s)}, \tag{37a}$$

$$R_{\rm FPXR}^{(s)}=\frac{1}{\xi^{(s)^2}+\varepsilon}\frac{1+\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right)-2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)^2+\rho^{(s)^2}\Delta^{(1)^2}}, \tag{37b}$$

$$\omega\frac{d^2N_{\rm TR}^{(s)}}{d\omega d\Omega}=\frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\times\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)^2R_{\rm TR}^{(s)}, \tag{38a}$$

$$R_{\rm TR}^{(s)}=\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)^2\Bigg(1+\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(2)}\right)$$
$$-2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(2)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)\Bigg)+\left(1+\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)^2\Bigg(1+\exp\left(-2b^{(s)}\rho^{(s)}\Delta^{(1)}\right)$$
$$-2\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)\Bigg)$$
$$+\frac{2\varepsilon}{\xi^{(s)^2}+\varepsilon}\Bigg(1+\exp\left(-b^{(s)}\rho^{(s)}\frac{\varepsilon+1}{\varepsilon}\right)\cos\left(\frac{2b^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)-\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(2)}\right)\times\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)$$
$$-\exp\left(-b^{(s)}\rho^{(s)}\Delta^{(1)}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)\Bigg), \tag{38b}$$

$$\omega\frac{d^2N_{\rm INT}^{(s)}}{d\omega d\Omega}=\frac{e^2}{4\pi^2}P^{(s)^2}\theta^2\left(\frac{1}{\theta^2+\gamma^{-2}}-\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}\right)\frac{1}{\theta^2+\gamma^{-2}-\chi_0'}R_{\rm INT}^{(s)}, \tag{39a}$$

$$R_{\text{INT}}^{(s)} = \frac{-2}{\sqrt{\xi^{(s)^2}+\varepsilon}}\operatorname{Re}\left(\frac{1-\exp\left[-ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]}{\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}-i\rho^{(s)}\Delta^{(1)}}\right.$$

$$\times\left(\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\left(1-\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(2)}\right]\right)\right. \quad (39b)$$

$$\left.\left.+\left(1+\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\left(1-\exp\left[ib^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)-b^{(s)}\rho^{(s)}\Delta^{(1)}\right]\right)\right)\right).$$

In the case of a thin nonabsorbing target, expressions (37b), (38b), and (39b) describing the spectra take the form

$$R_{\text{FPXR}}^{(s)} = \frac{4}{\xi^{(s)^2}+\varepsilon}\frac{\sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)^2}, \quad (40)$$

$$R_{\text{TR}}^{(s)} = 4\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)^2\times\sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi^{(s)}+\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)$$

$$+4\left(1+\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)^2\sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)+\frac{4\varepsilon}{\xi^{(s)^2}+\varepsilon}\left(\cos^2\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right. \quad (41)$$

$$\left.-\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}}{\varepsilon}\right)\right)\cos\left(b^{(s)}\left(\frac{\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)\right),$$

$$R_{\text{INT}}^{(s)} = -\frac{4}{\sqrt{\xi^{(s)^2}+\varepsilon}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)}\times\left[\left(1-\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\left(\cos^2\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right.\right.$$

$$\left.\left.-\cos\left(\frac{b^{(s)}\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\cos\left(b^{(s)}\left(\sigma^{(s)}+\frac{\xi^{(s)}}{\varepsilon}\right)\right)\right)+2\left(1+\frac{\xi^{(s)}}{\sqrt{\xi^{(s)^2}+\varepsilon}}\right)\sin^2\left(\frac{b^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi^{(s)}-\sqrt{\xi^{(s)^2}+\varepsilon}}{\varepsilon}\right)\right)\right]. \quad (42)$$

RATIO OF PXR AND FPXR YIELDS

Since the PXR and FPXR spectra described by the expressions $R_{\text{PXR}}^{(s)}$ and $R_{\text{FPXR}}^{(s)}$ depend on the parameter ε, it is of interest to consider the effect of asymmetry on the ratio of yields of these radiations derived from expressions (30) and (40),

$$\frac{R_{\text{PXR}}^{(s)}}{R_{\text{FPXR}}^{(s)}} = \left(\sqrt{\xi^{(s)^2}+\varepsilon}-\xi^{(s)}\right)^2. \quad (43)$$

It follows from (43) that as the asymmetry parameter ε decreases, i.e., the angle $\delta-\theta_B$ of particle incidence on the target surface increases (Fig. 2), the intensity of PXR at small angles to the particle velocity

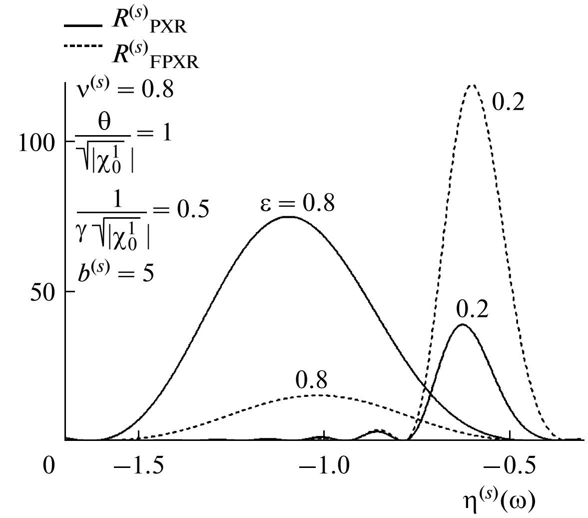

Fig. 3. Comparison of the PXR and FPXR spectra at different symmetries.

can significantly exceed the intensity of parametric radiation in the Bragg direction,

$$R^{(s)}_{\rm PXR} \ll R^{(s)}_{\rm FPXR}. \quad (44)$$

Let us independently consider the variation of the intensity of each radiation. Figure 3 shows the curves constructed by formulas (30) and (40). These curves show the increase in the FPXR peak amplitude and the decrease in the PXR peak amplitude with decreasing parameter ε. The curves were constructed for a fixed observation angle, path length, and energy of the emitting particle.

Ratio (43) will also depend on the observation angle θ, the emitting particle energy defined by the Lorentz factor γ, and the parameter $\nu^{(s)}$. The latter characterizes the degree of wave reflection from the system of parallel atomic planes in the crystal, caused by interference features of waves reflected from atoms of different planes (constructive $\nu^{(s)} \approx 1$) or destructive ($\nu^{(s)} \approx 0$). The effects of dynamical diffraction appear only in the case of constructive interference, i.e., for strong wave reflections from the system of diffracting atomic planes of the crystal. Using Eq. (36) the solution of which is the frequency in the vicinity of which the spectrum of parametric radiation photons concentrated and which corresponds to the spectrum maximum, we write ratio (43) in the form

$$\frac{R^{(s)}_{\rm PXR}}{R^{(s)}_{\rm FPXR}} = \frac{\varepsilon^2}{\nu^{(s)^2}}\left(\frac{\theta^2}{|\chi_0'|} + \frac{1}{\gamma^2|\chi_0'|} + 1\right)^2. \quad (45)$$

Considering this relation at the maximum of the angular density of parametric radiation $\theta = \sqrt{\gamma^{-2} + |\chi_0'|}$, we obtain

$$\frac{R^{(s)}_{\rm PXR}}{R^{(s)}_{\rm FPXR}} = \frac{2\varepsilon^2}{\nu^{(s)^2}}\left(\frac{1}{\gamma^2|\chi_0'|} + 1\right)^2. \quad (46)$$

Since the parameter $\nu^{(s)}$ is always less than unity, it follows from ratio (46) that the PXR yield will always exceed the FPXR yield if $\gamma^2|\chi_0'| \ll 1$ If $\gamma^2|\chi_0'| \geq 1$ or even slightly less than unity, the ratio of radiation yields depends strongly on asymmetry.

Let us consider the effect of reflection asymmetry on the ratio of angular densities of radiations. To this end, we integrate expressions (27a) and (37a) over the frequency function $\eta^{(s)}(\omega)$,

$$\frac{dN^{(s)}_{\rm PXR}}{d\Omega} = \frac{e^2P^{(s)^2}}{8\pi^2\sin^2\theta_{\rm B}}F^{(s)}_{\rm PXR}, \quad (47a)$$

$$F^{(s)}_{\rm PXR} = \nu^{(s)}\frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|} + \dfrac{1}{\gamma^2|\chi_0'|} + 1\right)^2}\int_{-\infty}^{+\infty} R^{(s)}_{\rm PXR}\,d\eta^{(s)}(\omega); \quad (47b)$$

$$\frac{dN^{(s)}_{\rm FPXR}}{d\Omega} = \frac{e^2P^{(s)^2}}{8\pi^2\sin^2\theta_{\rm B}}F^{(s)}_{\rm FPXR}, \quad (48a)$$

$$F^{(s)}_{\rm FPXR} = \nu^{(s)}\frac{\dfrac{\theta^2}{|\chi_0'|}}{\left(\dfrac{\theta^2}{|\chi_0'|} + \dfrac{1}{\gamma^2|\chi_0'|} + 1\right)^2}\int_{-\infty}^{+\infty} R^{(s)}_{\rm FPXR}\,d\eta^{(s)}(\omega). \quad (48b)$$

Figures 4 and 5 show the curves constructed by formulas (47b) and (48b) and describing the PXR and FPXR angular densities, respectively, in the case when $\gamma^2|\chi_0'| \geq 1$. It follows from the figures that parametric X-ray radiation along the emitting particle velocity at $\varepsilon \ll 1$ is characterized by an angular density significantly higher than the PXR density.

The FPXR and PXR angular densities as functions of the parameter $\nu^{(s)}$ are compared in Figs. 6 and 7, respectively. We can see that the FPXR angular density depends stronger on this parameter.

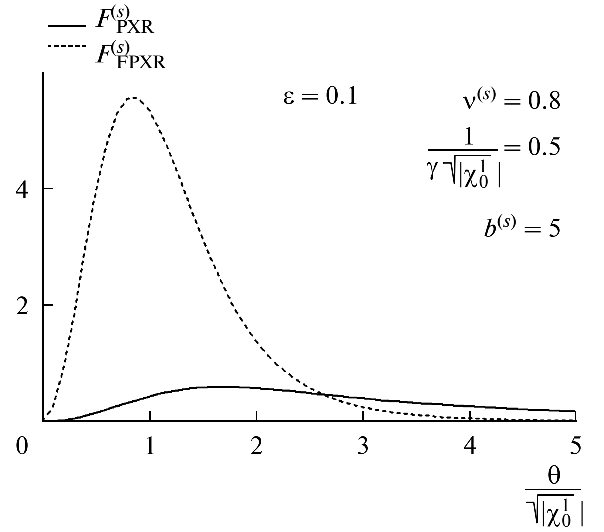

Fig. 4. Comparison of angular densities of radiations.

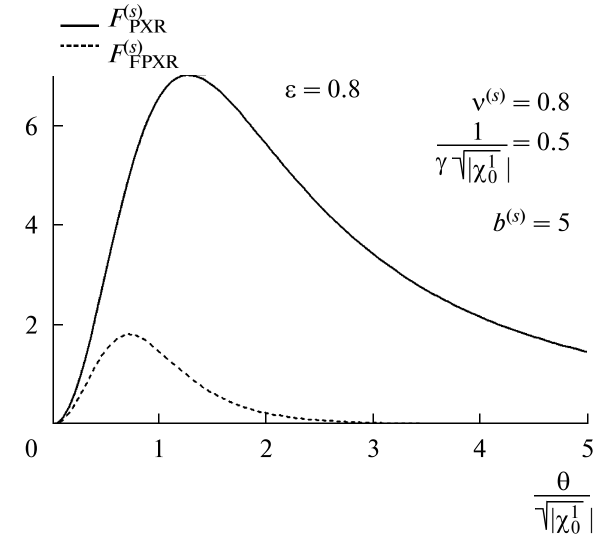

Fig. 5. The same as in Fig. 4, but for another asymmetry.

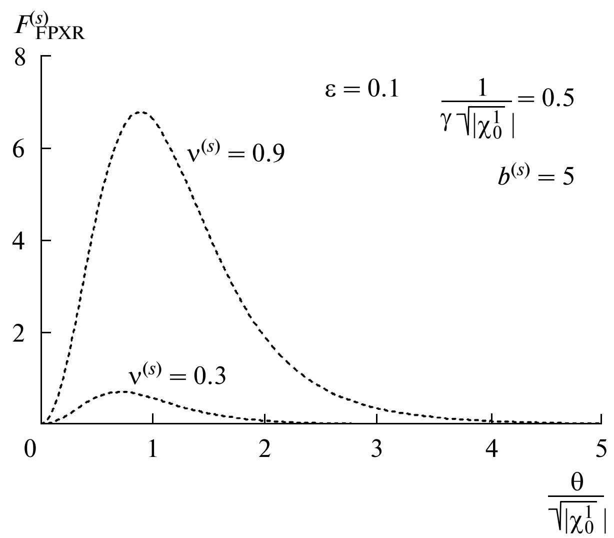

Fig. 6. Comparison of FPXR angular densities for different reflections.

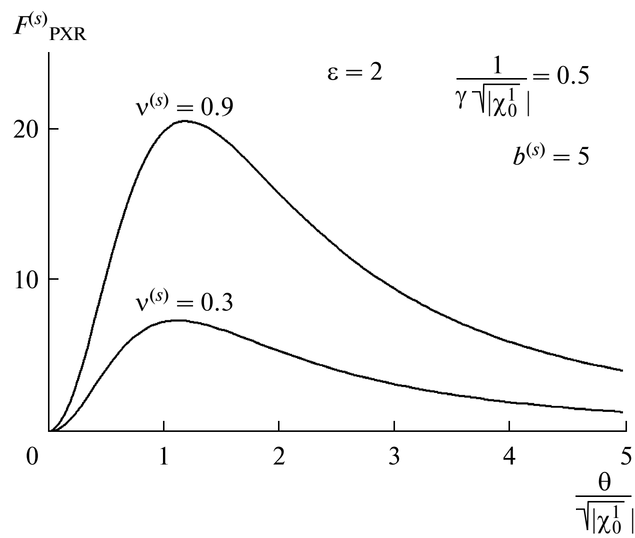

Fig. 7. Comparison of PXR angular densities for different reflections.

CONCLUSIONS

Let us formulate the main results obtained in this study.

Analytical expressions for the spectral-angular radiation density along the emitting particle velocity and in the Bragg direction were derived in the general case of asymmetric reflection.

Analysis of the derived expressions showed that, at the fixed angle θ_B between the electron velocity and the system of diffracting atomic planes of the crystal and the constant electron path length $2b^{(s)}$ in the crystal plate, the ratio of PXR and FPXR yields depends strongly on the angle δ between reflecting atomic planes and the crystal plate surface and, hence, on the reflection asymmetry. It was shown that a decrease in the parameter $\varepsilon = \sin(\delta + \theta_B)/\sin(\delta - \theta_B)$ and an increase in the angle $(\delta - \theta_B)$ of electron incidence on the crystal plate results in a decrease in the PXR spectral-angular density and an increase in the FPXR density that can even exceed the PXR density.

REFERENCES

1. M. L. Ter-Mikaelyan, *The Influence of the Medium on High-Energy Electromagnetic Processes* (AN ArmSSR, Yerevan, 1969) [in Russian].
2. G. M. Garibyan and Yan Shi, Zh. Eksp. Teor. Fiz. **61**, 930 (1971) [Sov. Phys. JETP **34**, 495 (1971)].
3. V. G. Baryshevskii and I. D. Feranchuk, Zh. Eksp. Teor. Fiz. **61**, 944 (1971) [Sov. Phys. JETP **34**, 502 (1971)].
4. G. M. Garibyan and Sh. Yan, Zh. Eksp. Teor. Fiz. **63**, 1198 (1972) [Sov. Phys. JETP **36**, 631 (1972)].
5. V. G. Baryshevsky and I. D. Feranchuk, Phys. Lett. A **57**, 183 (1976).
6. V. G. Baryshevsky and I. D. Feranchuk, J. Phys. (Paris) **44**, 913 (1983).
7. C. L. Yuan Luke, P. W. Alley, A. Bamberger, et al., Nucl. Instrum. Methods. Phys. Res. A **234**, 426 (1985).
8. B. N. Kalinin, G. A. Naumenko, D. V. Padalko, et al., Nucl. Instrum. Methods. Phys. Res. B **173**, 253 (2001).
9. G. Kube, C. Ay, H. Backe, N. Clawiter, et al., in *Proc. of the 5th Intern. Symp. on Radiation from Relativistic Electrons in Periodic Structures, Lake Aya, Altai Mountains, Russia, 2001*, p. 25.
10. H. Backe, N. Clawiter, Th. Doerk, et al., in *Proc. of the Intern. Symp. on Channeling-Bent Crystals-Radiation Processes* (Frankfurt am Main, Germany, 2003), p. 41.
11. A. N. Aleinik, A. N. Baldin, E. A. Bogomazova, et al., Pis'ma Zh. Eksp. Teor. Fiz. **80**, 447 (2004) [JETP Lett. **80**, 393 (2004)].
12. A. S. Kubankin, N. N. Nasonov, V. I. Sergienko, and I. E. Vnukov, Nucl. Instrum. Methods Phys. Res. A **201**, 97 (2003).
13. N. Nasonov and A. Noskov, Nucl. Instrum. Methods. Phys. Res. A **201**, 67 (2003).
14. A. Kubankin, N. Nasonov, and A. Noskov, in *Proc. of the 7th Intern. Russ.-Jpn. Symp. on Interaction of Fast Charged Particles with Solids* (Kyoto, Japan, 2002), p. 217.
15. S. Blazhevich and A. Noskov, Nucl. Instrum. Methods. Phys. Res. B **252**, 69 (2006).
16. S. V. Blazhevich and A. V. Noskov, Poverkhnost', No. 3, 62 (2008) [J. Surf. Invest. **2**, 225 (2008)].
17. S. V. Blazhevich and A. V. Noskov, Nucl. Instrum. Methods. Phys. Res. A **266**, 3777 (2008).
18. S. V. Blazhevich and A. V. Noskov, Izv. Vyssh. Uchebn. Zaved., Fiz. **50**, 48 (2007).
19. Z. G. Pinsker, *Dynamical Scattering of X-rays in Crystals* (Nauka, Moscow, 1974; Springer, Berlin, 1978).
20. V. A. Bazylev and N. K. Zhivago, *Radiation from Fast Particles in Matter and External Fields* (Nauka, Moscow, 1987) [in Russian].