Contribution of incoherent effects to the orientation dependence of bremsstrahlung from rapid electrons in crystal

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Abstract. The bremsstrahlung cross section for relativistic electrons in a crystal is split into the sum of coherent and incoherent parts (the last is due to a thermal motion of atoms in the crystal). Although the spectrum of incoherent radiation in crystal is similar to one in amorphous medium, the incoherent radiation intensity could demonstrate substantial dependence on the crystal orientation due to the electrons' flux redistribution in the crystal. In the present paper we apply our method of the incoherent bremsstrahlung simulation developed earlier to interpretation of some recent experimental results obtained at the Mainz Microtron MAMI.

1. Introduction

It is well known (see, e.g. [1, 2, 3]) that high energy electron beam incident on an oriented single crystal produces the coherent radiation that is due to the spatial periodicity of the lattice atoms, and the incoherent one, that is due to the thermal spread of atoms from their positions of equilibrium in the lattice. For the first look, the incoherent part of radiation is similar to the last in amorphous medium (with Bethe-Heitler spectrum), and do not depend on the crystal orientation in relation to the particles beam.

However, in [4, 5] it was paid attention to the fact that some features of the particle's dynamics in the crystal (channeling effect etc.) could lead to various substantial orientation effects in the hard range of the spectrum, where (for $\varepsilon \sim 1$ GeV electrons) the incoherent part is predominant. The semi-numerical approach developed in [4, 5] was used for interpretation of early experimental data [6].

The ideass of [5] had been referred by the authors of recent experiments [7] to interpret some of their results. In our article we present the results of simulation of the incoherent radiation under the conditions of the experiment [7]. A good agreement with the experimental data confirms the interpretation given in [7].

For the reader convenience, in the next section we outline some theoretical ideas of our approach.

2. Bremsstrahlung in dipole approximation

Radiation of relativistic electron in matter develops in a large spatial region along the particle's momentum. This region is known as the coherence length (or formation length) [1, 2] $l_{\rm coh} \sim 2\varepsilon\varepsilon'/m^2c^3\omega$, where ε is the energy of the initial electron, ω is the radiated photon frequency, $\varepsilon' = \varepsilon - \hbar\omega$, m is the electron mass, c is the speed of light. In the large range of radiation frequencies the coherence length could exceed the interatomic distances in crystal:

$$l_{\rm coh} \gg a. \tag{1}$$

In this case the effective constant of interaction of the electron with the lattice atoms may be large in comparison with the unit, so we could use the semiclassical description of the radiation process. In the dipole approximation the spectral density of bremsstrahlung is described by the formula [2]

$$\frac{dE}{d\omega} = \frac{e^2\omega}{2\pi c^4}\int_\delta^\infty \frac{dq}{q^2}\left[1 + \frac{(\hbar\omega)^2}{2\varepsilon\varepsilon'} - 2\frac{\delta}{q}\left(1 - \frac{\delta}{q}\right)\right]|\mathbf{W}_q|^2, \tag{2}$$

where $\delta = m^2c^3\omega/2\varepsilon\varepsilon' \sim l_{\rm coh}^{-1}$, and the value

$$\mathbf{W}_q = \int_{-\infty}^{\infty} \dot{\mathbf{v}}_\perp(t)e^{icqt}dt \tag{3}$$

is the Fourier component of the electron's acceleration in the direction orthogonal to its initial velocity.

The main contribution to the integral in (2) is made by the small values of q, $q \sim \delta$. On the other hand, the characteristic distances on which the electron's acceleration in the field of the atom in (3) would substantially distinct from zero are equal by the order of magnitude to the atomic radius R. The corresponding time intervals in which the integrand in (3) is distinct from zero are $\Delta t \sim R/c$. But under the condition (1) in the frequency range of our interest $\delta \ll R^{-1}$. So we could present $\mathbf{W}_q$ in the form [8]

$$\mathbf{W}_q = c\sum_n \boldsymbol{\vartheta}_n e^{icqt_n}, \tag{4}$$

where $\boldsymbol{\vartheta}_n$ is the two-dimensional electron scattering angle under collision with the n-th atom, t_n is the time moment of the collision.

Consider now the radiation of the electron incident onto the crystal under small angle ψ to one of its crystallographic axes. It is known [1, 2] that averaging of the equation for $|\mathbf{W}_q|^2$ over the thermal vibrations of atoms in the lattice leads to the split of this value (and so the radiation intensity) into the sum of two terms describing coherent and incoherent effects in radiation:

$$\left\langle |\mathbf{W}_q|^2 \right\rangle = c^2\sum_{n,m} e^{icq(t_n - t_m)}\left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n)\right\rangle \left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_m + \mathbf{u}_m)\right\rangle \tag{5}$$

$$+c^2\sum_n \left\{\left\langle (\boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n))^2\right\rangle - (\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n + \mathbf{u}_n)\rangle)^2\right\}, \tag{6}$$

where $\boldsymbol{\rho}_n = \boldsymbol{\rho}(t_n) - \boldsymbol{\rho}_n^0$ is the impact parameter of the collision with the n-th atom in its equilibrium position $\boldsymbol{\rho}_n^0$, $\boldsymbol{\rho}(t)$ is the trajectory of the electron in the plane orthogonal to the crystallographic axis (which could be obtained by numerical integration of the equation of motion), and $\mathbf{u}_n$ is the thermal shift of the n-th atom from the position of equilibrium. In the range of radiation frequencies for which

$$l_{\rm coh} \ll a/\psi, \tag{7}$$

where a is the distance between two parallel atomic strings the closest to each other, the incoherent term (6) makes the main contribution into the bremsstrahlung intensity (2).

The radiation by the uniform beam of particles is characterized by the radiation efficiency, that is the radiation intensity (2) integrated over impact parameters of the particles' incidence onto the crystal in the limits of one elementary cell. So, the efficiency is the classical analog of the quantum cross section. In the further consideration we shall compare the radiation efficiency in the crystal to the Bethe-Heitler efficiency of bremsstrahlung in amorphous medium.

For further computational details see [4, 5].

3. Origin of the orientation dependence of the incoherent bremsstrahlung

When charged particles are incident onto the crystal under small angle θ to one of the atomic planes densely packed with atoms, the channeling phenomenon could takes the place (see, e.g., [2, 3]). Under planar channeling the electron moves in the potential well formed by the attractive continuum potential of the atomic plane (see figure 1, left panel). The largest incidence angle, for which the capture into the channel is possible, is called as the critical channeling angle θ_c [2, 3].

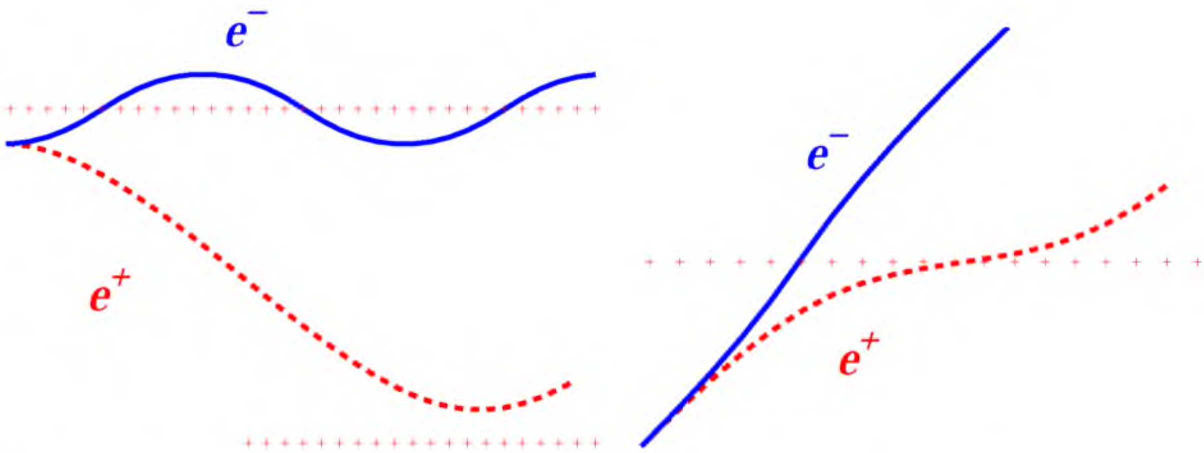

Figure 1. Typical trajectories of the electrons (——) and positrons (- - - -) under planar channeling (left) and above-barrier motion (right). Pluses mark the positions of atomic strings (perpendicular to the plane of the figure) forming the atomic planes of the crystal.

Under $\theta \ll \theta_c$ the most part of the incident electrons would move in the planar channeling regime. These electrons will collide with atoms at small impact parameters more frequently then in amorphous medium, that leads to the increase of the incoherent bremsstrahlung efficiency (see figure 2). For $\theta \sim \theta_c$ the above-barrier motion in the continuum potential takes the place for the most part of the particles (figure 1, right panel). Above-barrier electrons rapidly cross the atomic plane, with reduced number of close collisions with atoms comparing to the case of amorphous medium. This leads to the decrease of the incoherent bremsstrahlung efficiency (figure 2). For the positron beam the situation is opposite.

4. Results and discussion

In one of the experiments of [7] the integral intensity of radiation (with the photon energy $\hbar\omega \geq 15$ MeV, that eliminates the contribution of the channeling radiation) had been measured for different orientations of the silicon crystals in relation to 855 MeV electron beam. Note that under scanning of the goniometer angle ϕ both the angle of incidence to the [100] axis and the

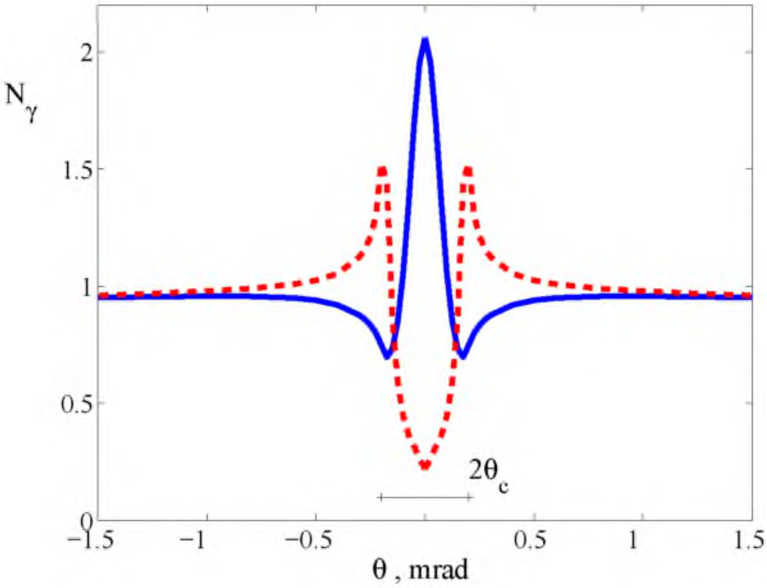

Figure 2. Incoherent bremsstrahlung efficiency (in ratio to the Bethe-Heitler efficiency in amorphous medium) from 1 GeV electrons (——) and positrons (- - - -) vs incidence angle θ to $(0\bar{1}1)$ plane of Si crystal, as a result of simulation.

angle to $(0\bar{1}1)$ plane (and other planes of the crystal) were changing. The zero angle between the plane $(0\bar{1}1)$ and the axis of the electron beam is achieved near $\phi = 26$ mrad.

The experimental data ([7], Fig. 14(b)) and the results of our simulation are presented on the figure 3.

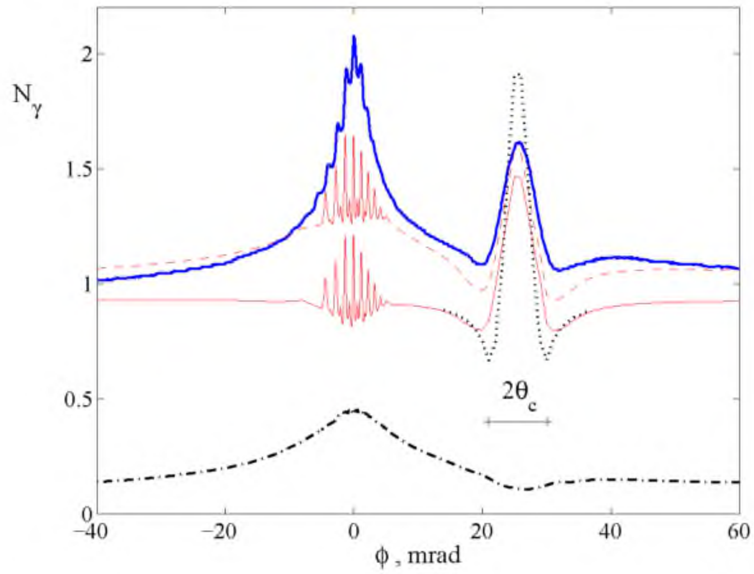

Figure 3. Coherent (— · —) and incoherent (——) contributions to the relative efficiency of radiation (- - - -) according to simulation, in comparison to the experimental results [7] (thick curve).

The right peak, and the gaps surrounding it, are caused by the contribution of incoherent radiation, as it was illustrated on the figure 2. Dotted curve presents the result of simulation without account of the incoherent multiple scattering of the electron on the thermal vibrations of the lattice atoms; taking that into account leads to some softening of the curve.

The left peak is caused by the effect of many crystallographic planes with common [100] axis. The fine structure of this peak is also determined by the orientation dependence of the *incoherent* radiation, as described above (see figure 4). The crystallographic planes responsible to the origin of specific local maxima in the incoherent radiation intensity are named on the figure.

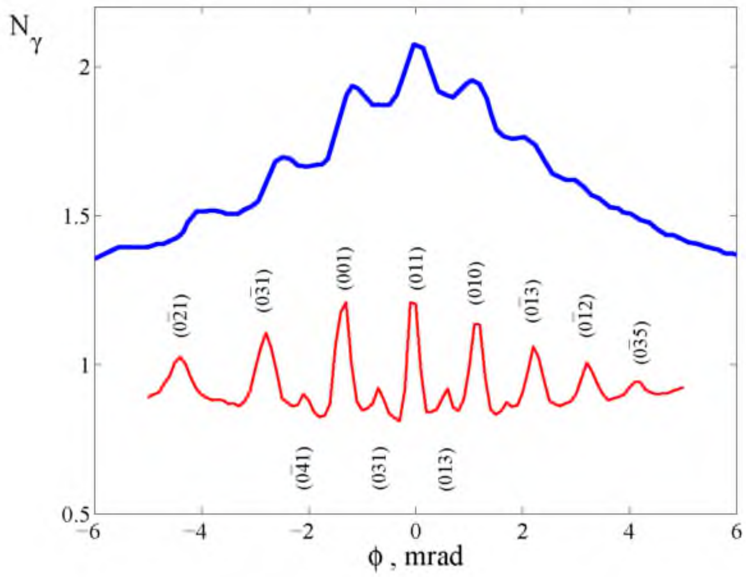

Figure 4. Region near $\phi = 0$ of the previous figure. Only experimental data (upper curve) and incoherent contribution (lower curve) are shown.

The coherent contribution was calculated using standard Born theory of the coherent bremsstrahlung [2]. We normalized the experimental curve to the right edge of the calculated one, far from the conditions of planar channeling, where Born theory of the coherent bremsstrahlung is surely valid. For better agreement with the experiment in the range near the left peak, it seems the most suitable to carry out the simulation of the coherent bremsstrahlung based on the same semiclassical approach, as used for the study of the incoherent one [8].

Acknowledgments

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