Transition radiation of fast charged particles on a fiber-like target, thin plates and atomic strings

N.F. Shul'ga [a,*], V.V. Syshchenko [b]

[a] National Science Center, Kharkov Institute of Physics and Technology, Akademicheskaya st., 1, Kharkov 61108, Ukraine
[b] Belgorod State University, Belgorod 308007, Russia

Abstract

The problem of transition radiation under impact of relativistic particles under small angle to the atomic string of a crystal is considered. The conditions under which the non-uniformity of the electron density along the string is not substantial are obtained. In this case the problem of transition radiation is reduced to the problem of the particle radiation on the thin fiber-like dielectric target. The formulae for the spectral–angular distribution of transition radiation under both regular and random collisions of a particle with the set of fiber-like targets are obtained. The radiation of the particle on a single atomic plane and on a set of planes in crystal is also considered. The consideration of the radiation process is carried out in the frames of perturbation theory on the interaction of the particle with a target.

Keywords: Transition radiation; Perturbation theory; Atomic strings

1. Introduction

The transition radiation arises under crossing by charged particle the boundary between two media with different dielectric properties (see [1–4] and references in them). For relativistic particle this radiation is concentrated in the region of small angles along the direction of the particle motion. The process of radiation develops in large spatial region along the particle velocity, that is called as the coherence length [2,5,6]. If the particle in the limits of this region crosses some boundaries of different media, the interference of radiation emitted under crossing of every boundary is substantial. It was shown in [7] that for long waves not only longitudinal, but also transverse dimensions of the region of radiation formation could have macroscopic sizes. If the transverse size of the target satisfies the condition $L_{\perp} \leqslant \gamma\lambda$, where λ is the length of the radiated wave and γ is the particle's Lorentz-factor, than the transverse sizes of the target and its geometrical shape make substantial influence on the transition radiation.

In the presented paper the problems on transition radiation by relativistic particle on dielectric targets of some particular geometries are

* Corresponding author. Tel.: +380-57-235-3420; fax: +380-57-235-2683.

E-mail address: shulga@kipt.kharkov.ua (N.F. Shul'ga).

considered. We consider the region of high frequencies of radiated waves, for which the dielectric function of the medium can be presented in the form

$$\varepsilon_\omega \approx 1 - \frac{\omega_p^2}{\omega^2}, \quad \omega > \omega_p, \tag{1}$$

where $\omega_p = (4\pi e^2 n_e(\vec{r})/m)^{1/2}$ is the plasma frequency, m is the electron mass, $n_e(\vec{r})$ is the electron density in the target.

We use the system of units, in which the velocity of light is equal to unit.

2. Transition radiation in the frames of perturbation theory

For the case of high frequencies of radiated waves, $\omega \gg \omega_p$, the second term in the dielectric function of the medium (1) can be considered as a small perturbation. In the frameworks of perturbation theory which was built, the spectral–angular density of radiation with given polarization is determined by relation

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = \omega^2 |\vec{e}\vec{I}|^2, \tag{2}$$

where $\vec{e}$ is the polarization vector, $\vec{e} \perp \vec{k}$, $|\vec{e}| = 1$, $\vec{k}$ is the wave vector of the radiated wave, and the value $\vec{I}$ in our case can be written in the form

$$\vec{I} \approx \frac{1}{4\pi\omega} \int \mathrm{d}^3 r \mathrm{e}^{\mathrm{i}\vec{k}\vec{r}} \omega_p^2(\vec{r}) \vec{E}_\omega^{(0)}(\vec{r}), \tag{3}$$

where $E_\omega^{(0)}(\vec{r})$ is the Fourier component of the non-disturbed Coulomb field of the uniformly moving relativistic particle,

$$\vec{E}_\omega^{(0)}(\vec{r}) = \int \frac{\mathrm{d}^3\kappa}{2\pi^2} \mathrm{i}e \frac{\vec{\kappa} - \omega\vec{v}}{\omega^2 - \vec{\kappa}^2} \delta(\omega - \vec{\kappa}\vec{v}) \mathrm{e}^{\mathrm{i}\vec{\kappa}\vec{r}}. \tag{4}$$

The spectral–angular distribution of radiation summed over polarizations is determined by equation

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = |\vec{k} \times \vec{I}|^2. \tag{5}$$

Using the Fourier transformation of the electron density distribution $n_e(\vec{r})$,

$$n_q = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 r n_e(\vec{r}) \mathrm{e}^{-\mathrm{i}\vec{q}\vec{r}},$$

we can rewrite (5) in the form

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = (2\pi)^6 \frac{e^6}{m^2} \left| \frac{\vec{k}}{\omega} \times \vec{J}_k \right|^2, \tag{6}$$

where

$$\vec{J}_k = \int \frac{\mathrm{d}^3 q}{2\pi^2} n_q \frac{\vec{k} - \vec{q} - \omega\vec{v}}{\omega^2 - (\vec{k} - \vec{q})^2} \delta(\omega - (\vec{k} - \vec{q})\vec{v}). \tag{7}$$

3. Transition radiation on a dielectric fiber and atomic strings

Let us consider the transition radiation by relativistic particle incident on a thin dielectric fiber under small angle $\psi \ll 1$ to its axis. The atomic string in crystal [6] or nanotube [8] can be treated as such fiber in the case when the length of radiation formation (the coherence length) exceeds in much the atomic string thickness along the particle motion direction,

$$l_{\mathrm{coh}} \sim \frac{2\gamma^2}{\omega} \frac{1}{1 + \gamma^2\theta^2} \gg \frac{2R}{\psi}, \tag{8}$$

where R is the screening radius of the atomic potential (Thomas–Fermi radius), $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz-factor of the particle, θ is the radiation angle (the angle between the wave vector $\vec{k}$ of the radiated wave and the particle velocity).

Let us take the electron density distribution in the fiber in Gaussian form,

$$n_e(\vec{r}) = \frac{n_e}{2\pi R^2} \exp\left[-\frac{(x - \psi z)^2 + (y - y_0)^2}{2R^2} \right], \tag{9}$$

where z-axis is directed along the particle velocity $\vec{v}$, the axis of the fiber is parallel to (x, z) plane, y_0 is the distance between the particle trajectory and the axis of the fiber, n_e is the electron density per unit length of the fiber. The Fourier transformation of this distribution has the form

$$n_q = (2\pi)^{-2} n_e \mathrm{e}^{\mathrm{i}q_y y_0} \delta(q_x \tan\psi + q_z) \quad \exp\left[-\frac{(q_x^2 + q_y^2)R^2}{2} \right]. \tag{10}$$

The spectral–angular distribution of radiation (6) must be averaged over all possible values of the impact parameter y_0,

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} \right\rangle = \frac{1}{a_y} \int_{-\infty}^{\infty} \mathrm{d}y_0 \frac{\mathrm{d}E(y_0)}{\mathrm{d}\omega\,\mathrm{d}o}, \tag{11}$$

where a_y is the distance between the atomic strings in the crystal along the y-axis. After substituting (7) with n_q determined by (10) into (6) and averaging according to (11), one can demonstrate that for small radiation angles and for small angles of incidence the condition (8) leads to the possibility to neglect the second exponential factor in (10), that corresponds to the case of the zero thickness of the fiber. In other words, the details of the electron density distribution in the fiber are not substantial under such conditions.

For the case of infinitely thin dielectric fiber we obtain

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} \right\rangle = \frac{e^6 n_e^2 \gamma}{a_\gamma m^2 \omega \psi^2} F(\theta, \varphi), \tag{12}$$

where

$$F(\theta, \varphi) = \frac{1 + 2\left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1 + \left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^{3/2}}, \tag{13}$$

φ is the azimuth angle (the angle between the x-axis and the projection of the wave vector $\vec{k}$ onto the plane (x, y)). This function is plotted on the Fig. 1. Let us outline the following features of the angular distribution of the radiation intensity obtained: (i) this distribution possesses the axial symmetry relatively to the axis of the fiber ($\theta = \psi$, $\varphi = 0$) (it can be easily shown analytically from (13)); (ii) the minimum of intensity takes place not in the direction of the particle motion ($\theta = 0$), but it is shifted a little from this direction; (iii) near the axis of symmetry of the angular distribution the intensity has no deep minimum, that frequently takes place in problems on transition radiation, but it has rather high level. Small changes of the incidence angle ψ do not change these qualitative results, but with increase of ψ the minimum in the center develops itself.

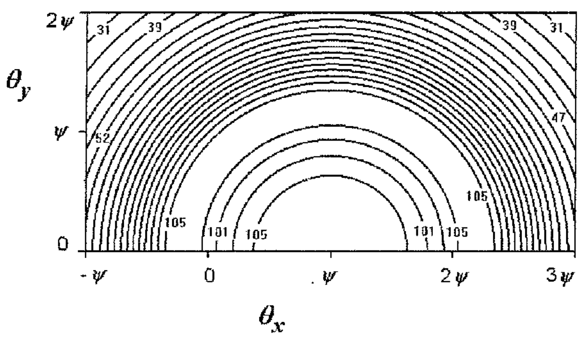

Fig. 1. Surface plot (view from above) of the function $100F(\theta, \varphi)$ for $\psi = 10^{-3}$, $\gamma = 2000$ ($\theta_x = \theta\cos\varphi, \theta_y = \theta\sin\varphi$). The lines of equal level of the surface are shown.

Choosing the polarization vectors in the form

$$\vec{e}^{(1)} = \frac{\vec{k} \times \vec{e}_x}{|\vec{k} \times \vec{e}_x|}, \quad \vec{e}^{(2)} = \frac{\vec{k} \times \vec{e}^{(1)}}{\omega},$$

we find that radiation is partially polarized in $\vec{e}^{(1)}$ direction with polarization

$$P = \frac{1}{1 + 2\left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}.$$

4. Transition radiation on a set of atomic strings in a crystal

Now let us consider, what will happen in the case when our particle collides with the set of parallel atomic strings in a crystal.

The motion of the particle in the periodical field of atomic strings in crystal can be both regular and chaotic [9]. If the particle moves chaotically in the plane perpendicular to atomic strings (under the constant incidence angle ψ), we can neglect the interference of radiation produced by interaction of the particle with different strings. In this case the symmetry of angular distribution of radiation intensity, described above, leads to the same form of the angular distribution (13), as in the case of the single string. The total intensity of radiation will be proportional to the number of strings under collision.

If the particle motion in the crystal is regular, the account of interference effects is necessary. For example, in the simplest case when the particle moves through the set of parallel string with the same impact parameter, the Eq. (12) transforms to

$$\left\langle \frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} \right\rangle = \frac{e^6 n_e^2 \gamma}{a_y m^2 \omega \psi^2} F(\theta, \varphi) \times N \frac{2\pi}{b} \sum_{n=-\infty}^{\infty} \delta\left(\frac{\omega}{2\gamma^2}(1+\gamma^2\theta^2) - \frac{2\pi}{b} n \right), \quad (14)$$

where b is the distance between atomic strings along the z-axis. The delta-function indicates that in this case the radiation under given angle θ can take place only with the frequency determined by relation

$$\omega_n = \frac{2\gamma^2}{1+\gamma^2\theta_n^2} \cdot \frac{2\pi}{b} n. \quad (15)$$

So the character of motion of the particle and the spatial arrangement of atomic strings can make substantial influence on the transition radiation.

5. Comparison with parametric X-ray radiation and coherent bremsstrahlung

In the case of 3-dimensional periodic medium (crystal, for instance) the integral over $\vec{q}$ in (7) changes to the summation over discrete set of values $\vec{g}_n = 2\pi(n_x/a_x, n_y/a_y, n_z/a_z)$ that is over reciprocal lattice vectors. Hence substitution (7) into (6) leads to the result

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = \frac{8\pi e^6}{m^2} \frac{T}{\omega^2} \sum_n n_g^2 |\vec{k} \times (\omega\vec{v} - \vec{g}_n)|^2 \times \frac{\delta(\omega - (\vec{k} - \vec{g}_n)\vec{v})}{[(\vec{k} - \vec{g}_n)^2 - \omega^2]^2}, \quad (16)$$

where n_g are the coefficients of expansion of electron density into Fourier series,

$$n(\vec{r}) = \sum_n n_g \mathrm{e}^{\mathrm{i}\vec{g}_n\vec{r}}$$

and T is the whole time of motion of the particle through non-uniform medium. This result coincides with the result for parametric X-ray radiation in 3-dimensional periodic medium (see [2], Eq. (28.160), or [10], Eq. (6)) with dielectric function of the medium in the form (1).

Thus our result (12) and (13) for the radiation intensity on a single atomic string corresponds to the case of infinitely outlying atomic strings from each other in crystal, so the sum over reciprocal lattice vectors in (16) can be changed to the integration over $\vec{q}$. In this case the main contribution to the integral will be made by the values $\vec{q}$ with zero component parallel to the particle velocity, so we obtain the condition $q_x\psi + q_z = 0$, that corresponds to the approximation of uniform electron density along the lattice (10).

The result (14) and (15), when the periodicity in arrangement of atomic strings leads to the connection between the angle and the frequency of radiation, can be interpreted in analogous way.

Note, however, that the results by Ter-Mikaelyan for parametric X-ray radiation relate only to the case of rectilinear trajectory of the particle. In the case of chaotic trajectory of the particle in crystal, the radiation produced under interaction with the set of atomic strings has the same structure as under interaction with the single string, as it was mentioned above.

Together with the effects considered above the coherent bremsstrahlung is possible under motion of relativistic electron in crystal under small angle to one of crystallographic axes. Due to this effect the spectral density of radiation of the electron in crystal can substantially exceed the spectral density of radiation of electrons in amorphous medium.

However, in region of low frequencies $\omega \leqslant \gamma\omega_p$ together with the coherent effect the Ter-Mikaelyan effect of influence of polarization of medium on bremsstrahlung become substantial. The last effect leads to substantial decrease of intensity of coherent radiation. If the condition (8) is satisfied together with the condition $\omega \ll \gamma\omega_p$, the main contribution to the radiation will be made by the effect of transition radiation of the particle on the fiber-like target considered above.

Furthermore, the coherent bremsstrahlung is concentrated mainly along the direction of the particle motion, and its intensity rapidly decreases under deflection from this direction. Thus in the region of angles $\theta \sim 2\psi \gg \gamma^{-1}$ the distribution of radiation intensity will be determined mainly by the effect of transition radiation considered above.

So, for the process of radiation of relativistic electrons in crystal one can find the conditions under which the contribution of coherent radiation mechanism will be negligible and the transition radiation on fiber-like target will be the main mechanism.

6. Transition radiation on atomic planes in a crystal

Now let us consider the transition radiation of the particle on the atomic plane in crystal in the case when this atomic plane can be treated as a uniform thin dielectric plate. The problem of radiation on a plate is not new in the theory of transition radiation [1–3], but our perturbation theory permits to obtain results in a rather simple way.

The Fourier transformation of the electron density distribution under condition (8) can be written in the form

$$n_q = (2\pi)^{-1} n_e \delta(q_y)\delta(q_x \tan\psi + q_z). \tag{17}$$

Here n_e denotes the electron density per unit area of the plane. Substitution (17) into general formulae (6) and (7) gives us the following result for the spectral–angular distribution of transition radiation:

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = \frac{4e^6 n_e^2 \gamma^2}{m^2\omega^2\psi^2} F_p(\theta,\varphi), \tag{18}$$

where

$$F_p(\theta,\varphi) = \frac{\gamma^2\theta^2 - \frac{\theta}{\psi}\cos\varphi(1+\gamma^2\theta^2) + \left(\frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1+\gamma^2\theta^2 - \frac{\theta}{\psi}\cos\varphi(1+\gamma^2\theta^2) + \left(\frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^2}. \tag{19}$$

The angular distribution (19) is plotted on the Fig. 2 for the case $\psi = 2\gamma^{-1}$. One can see that the distribution is symmetrical relatively to the reflection by the atomic plane.

Under increasing the angle of incidence ψ the distribution transforms to more common shape for the problems on transition radiation (Fig. 3). One can see two empty cones, one of which is directed almost along the particle velocity, and the second is "reflected" by the atomic plane.

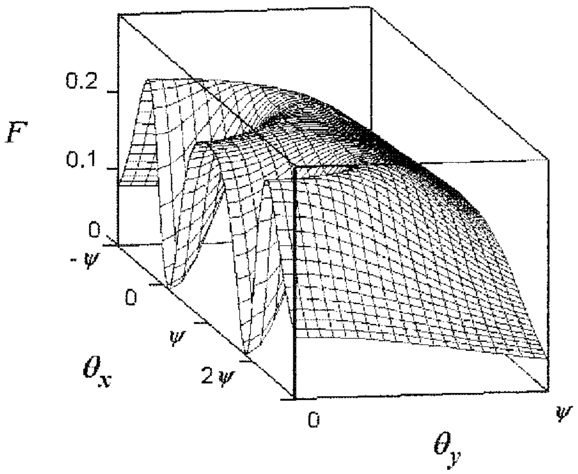

Fig. 2. 3-dimensional plot of the function $F_p(\theta,\varphi)$ (19) ($\theta_x = \theta\cos\varphi, \theta_y = \theta\sin\varphi$) for $\psi = 10^{-3}$, $\gamma = 2000$.

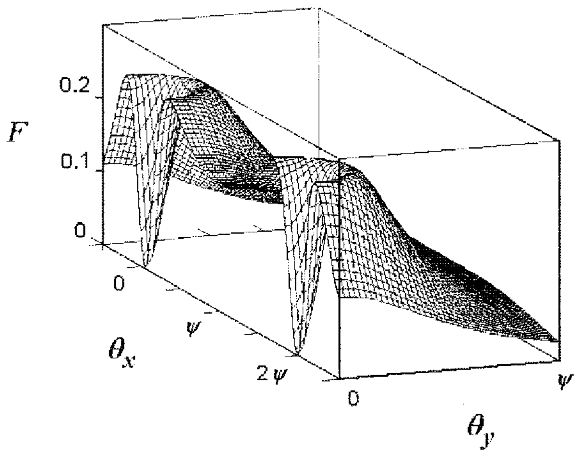

Fig. 3. The same as on Fig. 2 for $\psi = 2\times10^{-3}$.

For the periodic set of such parallel planes we obtain the result accounting interference effects,

$$\frac{\mathrm{d}E}{\mathrm{d}\omega\,\mathrm{d}o} = \frac{4e^6 n_e^2 \gamma^2}{m^2\omega^2\psi^2} F_p(\theta,\varphi)\cdot N\frac{2\pi}{b} \times \sum_{n=-\infty}^{\infty} \delta\left(\frac{\omega}{2\gamma^2}(1+\gamma^2\theta^2) - \frac{2\pi}{b}n\right). \tag{20}$$

This formula coincides with the corresponding result of the theory by Ter-Mikaelyan [2] of the parametric X-ray radiation by relativistic electrons in crystal on the set of periodically distributed crystallographic planes of atoms. Here we one more time obtain the connection between frequency of radiated wave and the radiation angle (15).

Acknowledgements

This work is supported in part by INTAS (project 97-30392) and Russian Foundation for Basic Research (project 00-02-16337).

References

[1] V.L. Ginzburg, V.N. Tsytovich, Transition Radiation and Transition Scattering, A. Hilger, Bristol, New York, 1990.
[2] M.L. Ter-Mikaelyan, High-Energy Electromagnetic Processes in Condensed Media, Wiley, New York, 1972.
[3] G.M. Garibian, C. Yang, X-ray Transition Radiation, Publ. Acad. of Science of Armenia, Yerevan, 1983 (in Russian).
[4] P. Rullhusen, X. Artru, P. Dhez, Sources Using Relativistic electrons, World Scientific, Singapore, 1998.
[5] B.M. Bolotovskij, Proc. P.N. Lebedev Phys. Inst. 140 (1982) 95 (in Russian).
[6] A.I. Akhiezer, N.F. Shul'ga, High-Energy Electrodynamics in Matter, Gordon and Breach, Amsterdam, 1996.
[7] S.N. Dobrovol'skii, N.F. Shul'ga, JETP 117 (2000) 668.
[8] J.W. Mintmire, B.I. Dunlap, C.I. White, Phys. Rev. Lett. 68 (1992) 631.
[9] A.I. Akhiezer, V.I. Truten', N.F. Shul'ga, Phys. Rep. 203 (1991) 289.
[10] M.L. Ter-Mikaelyan, Usp. Fiz. Nauk 171 (6) (2001) 597.