Quasi-localized States and Resonance Scattering of Particles by Defects in Semiconductor Crystals with Band Spectrum Structure

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Abstract—The interaction of particles with planar defects in semiconductor crystal was studied on the basis of a model of the band energy-spectrum structure. Generalized localized states are shown to exist in the vicinity of defects. It is also shown that nonquadratic dispersion law gives rise to quasi-localized states. Particle scattering by the semiconductor boundary was analyzed. The condition for the total reflection from the interface is formulated as follows: the incident-particle energy should be equal to the energy of a quasi-localized state. It was found that a weak dissipation of particle energy in the crystal causes instability of the resonance condition for the total reflection.

1. Recently, considerable attention has been focused on scattering of waves by planar and point defects in crystals. Elastic scattering of waves by a thin passive (devoid of internal degrees of freedom) layer in an isotropic medium was investigated elsewhere [1, 2]. It was demonstrated that, if the phase velocity of an incident wave falls within the range between longitudinal and transverse velocities of sound ($c_t < c < c_l$), the total transmission and the total reflection are both possible. A similar problem was studied previously [3] by the methods of the crystal-lattice dynamics on the basis of a simple model of planar defect in fcc crystal and in [4], where the effect of interaction between atoms (not necessary neighboring) in a linear chain was investigated. A theory providing explanation of the related resonance effects and, specifically, unexpected total reflection was proposed in [5].

The above-mentioned studies dealt with elastic-displacement vector fields containing two partial terms. If the phase velocity of a wave falls within the range $c_t < c < c_l$, one of the partial terms accounts for a longitudinal wave localized in the vicinity of a defect and the other, for a bulk transverse wave. This type of oscillations is referred to as quasi-localized [6, 7]. Resonance scattering is possible because of the interaction between transverse and longitudinal modes at a defect. These waves do not interact and propagate independently in the bulk of an ideal crystal. The resonance occurs provided the phase velocity (or frequency) of incident wave coincides with the phase velocity (or frequency) of quasi-localized oscillation.

Ambiguous frequency spectrum may arise for a field describing the electron and hole motion in special semiconductor crystals with band energy-spectrum structure. For example, the generalized model of energy spectrum was used [8] to explain a variety of CdSb and ZnSb properties. The key point of this model is the presence of two energy valleys near the Brillouin zone edge in the valence band and two valleys in the conduction band. Detailed study [9, 10] indicates that the dependence of energy ε on either electron or hole quasi-momentum $\mathbf{k}$ in nondegenerate bands of In_4Se_3 crystal can be described by the dispersion law

$$\varepsilon(\mathbf{k}) = E_0 - \alpha_x k_x^2 - \alpha_y k_y^2 - \alpha_z k_z^2 + \beta_x k_x^4 + \beta_y k_y^4 + \beta_z k_z^4 \tag{1}$$

in the vicinity of a band gap, which has multiple valleys and is non-parabolic with negative curvature at the minimal wave vectors; as $\mathbf{k}$ increases, the parabolicity is restored. Such an atypical dispersion law (1) obtained regardless of spin–orbit interaction is valid in the vicinity of the Brillouin zone center in In_4Se_3 and results from the interaction of close subbands in the valence and conduction bands.

In this paper, we analyze stationary eigenstates and scattering of quasiparticles by a planar defect; the particles are described by two-partial scalar field in semiconductor with the band spectrum structure. The assumptions that follow and concern the parameters of the dispersion law (1) actually allow us to treat the problem as one-dimensional, which makes it possible to obtain analytical results easily and reveal some special features of the two-partial field dynamics. In semiconductor crystal with band energy-spectrum structure, dispersion law is not quadratic but biquadratic, like (1). The wave function of such a state will consist of partial components. This may result in a total reflection from the planar interface between the media under the following nontrivial conditions: the parameter of defect (interface) is nonzero, and the incident particle energy does not coincide with the edge of the bulk state continuum.

In the studied system, both localized and quasi-localized states can exist. Spatial dispersion in the medium where the wave propagates leads to a nonquadratic dependence of energy on quasi-momentum. Consideration of the spatial dispersion will obviously lead to a change in the system properties. Specifically, the localized states will become extended [11, 12], which means that the amplitudes of their wave functions decrease with distance in oscillating manner (similar to the generalized Rayleigh waves). The localized states appearing in the vicinity of defects are also thoroughly studied for models implying biquadratic dispersion law [13]. However, no consideration has been given to quasi-localized states characterized by a scalar field, especially for nonsymmetrical stationary eigenstates with energy belonging to a defect-free medium continuum; therefore, these states are currently of interest in theory and applications.

It was recently shown [14] that, in a more realistic model, the anomalous total reflection from a passive defect is absent when dissipation in crystal is taken into account. It is of interest to ascertain the conditions under which the anomalous total reflection from interface in crystals is possible. In this paper, we analyze the effect of scattering-particle energy dissipation on the condition for the reflection resonance.

2. Let us consider a planar defect, for example, an interface between two similar semiconductor crystals with dispersion law given by (1), where $\alpha_j > 0$ and $\beta_j > 0$ ($j = x, y, z$). We assume that the crystal exhibits a strong spatial dispersion only in one direction, to be precise, in the direction perpendicular to the plane of the defect (this assumption can be validated by choosing an appropriate anisotropy type). Let us choose the coordinates in such a way that yOz plane coincides with the plane of defect and let us align the Ox axis with normal to this plane. Then, denoting $\alpha = \alpha_x$, $\beta = \beta_x$, $\alpha_\perp = \alpha_y = \alpha_z$ and assuming that $\beta_x \gg \beta_y$ and $\beta_x \gg \beta_z$, we obtain the dispersion law as

$$\varepsilon(\mathbf{k}) = E_0 - \alpha k^2 - \alpha_\perp k_\perp^2 + \beta k^4, \tag{2}$$

where $k = k_x$ and $k_\perp^2 = k_y^2 + k_z^2$. The dispersion law (2) corresponds to a time-independent Schrödinger equation:

$$\varepsilon\Psi = E_0\Psi + \alpha\frac{\partial^2\Psi}{\partial x^2} + \alpha_\perp\left(\frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right) + \beta\frac{\partial^4\Psi}{\partial x^4} + U(x)\Psi, \tag{3}$$

where the interface between crystal half-spaces is simulated by the potential $U(x) = U_0\delta(x)$. Assuming that crystal is uniform over the interface, let us seek a solution of (3) in the form $\Psi(x, y, z) = \psi(x)\exp(ik_y y + ik_z z)$. With this assumption, Eq. (3) is reduced to one-dimensional equation for ψ function:

$$E\psi = E_0\psi + \alpha\frac{\partial^2\psi}{\partial x^2} + \beta\frac{\partial^4\psi}{\partial x^4} + U(x)\psi, \tag{4}$$

where we introduced $E = \varepsilon + \alpha_\perp k_\perp^2$. As follows from (4), the stationary homogeneous states $\psi(x) = \psi_0\exp(ikx)$ are characterized by the dispersion law

$$E(k) = E_0 - \alpha k^2 + \beta k^4. \tag{5}$$

Integrating (2) in the vicinity of interface $x = 0$ yields the boundary condition

$$\alpha\left(\frac{\partial\psi(+0)}{\partial x} - \frac{\partial\psi(-0)}{\partial x}\right) + \beta\left(\frac{\partial^3\psi(+0)}{\partial x^3} - \frac{\partial^3\psi(-0)}{x^3}\right) + U_0\psi(0) = 0. \tag{6}$$

For $\beta = 0$, condition (6) coincides with the well-known boundary condition for the Schrödinger equation with a quadratic dispersion law. We will concentrate on the case $\beta \neq 0$. The system of boundary conditions should also include the requirement of continuity of the wave function $\psi(x)$ and its second derivative at $x = 0$, which implies the continuity of its first derivative as well. Then, we obtain the following system of boundary conditions:

$$\psi(+0) = \psi(-0), \quad \frac{\partial\psi(+0)}{\partial x} = \frac{\partial\psi(-0)}{\partial x},$$
$$\frac{\partial^2\psi(+0)}{\partial x^2} = \frac{\partial^2\psi(-0)}{\partial x^2}, \quad \frac{\partial^3\psi(+0)}{\partial x^3} - \frac{\partial^3\psi(-0)}{\partial x^3} = \eta\psi(0), \tag{7}$$

where $\eta = -U_0/\beta$.

Depending on energy E, the solutions to Eq. (4) correspond to homogeneous, localized, or quasi-localized states.

As follows from the dispersion law (5), each particular state is defined by two different quasi-momenta. Within the continuum range $E_m < E < E_0$, where $E_m = E_0 - \alpha^2/4\beta$, there are two pairs of real quasi-momenta:

$$k_{1,2}^2 = k_m^2 \pm \sqrt{\frac{E - E_m}{\beta}}. \tag{8}$$

Obviously, wave function of the homogeneous wave with quasi-momenta (8) comprises two terms with different amplitudes and oscillation frequencies:

$$\psi(x) = A_1 e^{ik_1x} + A_2 e^{ik_2x}. \tag{9}$$

The more interesting point, in the context of condensed-state physics, is the origin of localized states in the vicinity of a defect. Let us consider the case of $E < E_m$ in more detail. In this spectral range, each state is defined by two complex quasi-momenta: $\kappa_1 = \gamma - iq$ and $\kappa_2 = \gamma + iq$, where

$$q^2 = \frac{1}{2}\left\{\sqrt{\frac{E_0 - E}{\beta}} + k_m^2\right\},$$
$$\gamma^2 = \frac{1}{2}\left\{\sqrt{\frac{E_0 - E}{\beta}} - k_m^2\right\}, \quad (10)$$

and $k_m^2 = \alpha/2\beta$. With the boundary conditions (7), a solution to Eq. (4) can be written as

$$\psi(x) = A\sin(q|x| + \vartheta)e^{-\gamma|x|}, \quad (11)$$

where phase is given by $\sin 2\vartheta = \sqrt{(E_m - E)/(E_0 - E)}$, and A is an arbitrary constant. Wave function (11) describes the so-called generalized localized state [11, 12], which decays in oscillatory manner with increasing distance from defect. The boundary conditions (4) provide a dispersion relationship for localized energy levels

$$4\gamma(\gamma^2 + q^2) = \eta, \quad (12)$$

which exist only if the defect parameter satisfies the condition $U_0 < 0$.

The prerequisites for the formation of a localized state similar to (11) in the vicinity of planar defect were studied elsewhere [13] for domain wall in ferromagnetic superconductor. In uniform magnetic field, at sufficiently low temperature, an inhomogeneous superconducting state is possible. The wave function of this state has the meaning of the order parameter and obeys the linearized Ginzburg–Landau equation for the fourth space derivative, since the effective exchange field in the vicinity of domain wall is considerably decreased. Solution similar to (11) in the vicinity of point defect was obtained in [13] for three-dimensional spherically symmetric case of dependence of energy on wave vector.

3. Now we discuss the case of $E > E_0$, when one of the quasi-momenta is real $k_1 = k$ and the other is pure imaginary $k_2 = i\kappa$:

$$k^2 = \sqrt{\frac{E - E_0}{\beta} + k_m^4} + k_m^2,$$
$$\kappa^2 = \sqrt{\frac{E - E_0}{\beta} + k_m^4} - k_m^2. \quad (13)$$

The dependences $E(k) = E_0 - \alpha k^2 + \beta k^4$ and $E(\kappa) = E_0 + \alpha\kappa^2 + \beta\kappa^4$ show (Fig. 1) that each energy value has two corresponding quasi-momenta (13).

In the case under study, quasi-localized state [6, 7] is formed within the continuum; its wave function consists of two partial components, one of which governs a standing wave along the entire Ox axis and the other

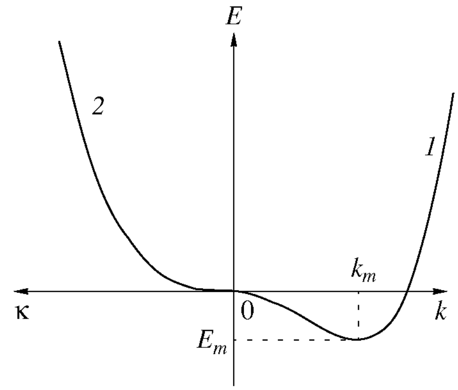

Fig. 1. Quasiparticle energy versus quasi-momentum (*1*) real and (*2*) imaginary components.

corresponds to an oscillation localized in the vicinity of defect:

$$\psi(x) = \begin{cases} A\sin(kx - \varphi_1) + Me^{-\kappa x}, & x < 0, \\ B\sin(kx - \varphi_2) + Me^{-\kappa x}, & x > 0. \end{cases} \quad (14)$$

Solution (14) incorporates five parameters: the amplitudes A, B, and M and phases φ_1 and φ_2. Substituting (14) into the boundary conditions (7), we obtain a set of algebraic homogeneous equations for quasi-localized state amplitudes A, B, and M:

$$\begin{cases} A\sin\varphi_1 - B\sin\varphi_2 = 0, \\ Ak\cos\varphi_1 - Bk\cos\varphi_2 + 2\kappa M = 0, \\ A(\eta\sin\varphi_1 + k^3\cos\varphi_1) - Bk^3\cos\varphi_2 - M(\eta + 2\kappa^3) = 0. \end{cases} \quad (15)$$

Equating the determinant of system (15) to zero, we obtain a relationship between φ_1 and φ_2, where one of the phases may be considered as a free parameter:

$$\{\eta + 2\kappa(k^2 + \kappa^2)\}k\sin(\varphi_1 - \varphi_2) = 2\kappa\eta\sin\varphi_1\sin\varphi_2. \quad (16)$$

The following special feature of quasi-localized states is worth noting. It turns out that there exists a stationary eigenstate $\psi_N = \psi_S + \psi_L$, such that standing wave exists only in one of half-spaces

$$\psi_S(x) = \begin{cases} A\sin kx, & x < 0, \\ 0, & x > 0, \end{cases} \quad (17)$$

whereas a localized state may exist on either side of the defect $\psi_L(x) = M\exp(-\kappa|x|)$. Actually, if $B = 0$ in (15), it follows from (16) that $\varphi_1 = 0$ and the localized-state amplitude is uniquely defined as $M = -(k/2\kappa)A$. The energy of these quasi-localized states is determined from the relationship

$$\eta = -2\kappa(k^2 + \kappa^2). \quad (18)$$

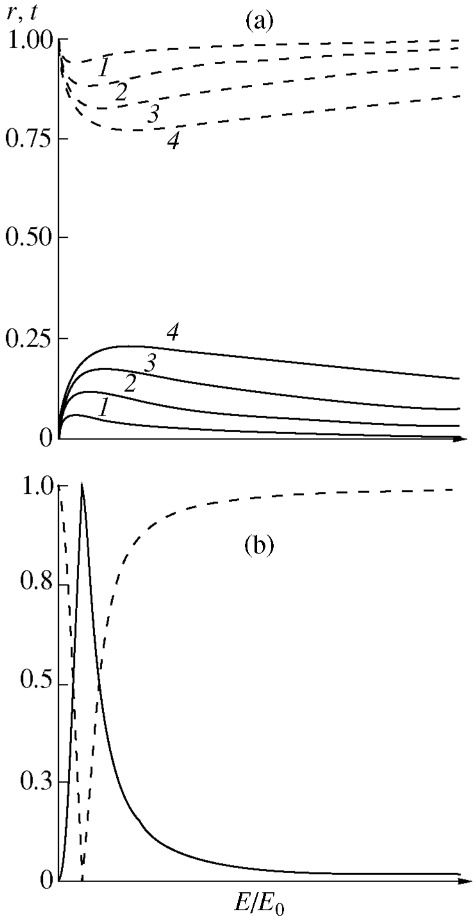

Fig. 2. Typical behavior of reflection coefficient $r = |R|^2$ (solid lines) and transmission coefficient $t = |T|^2$ (dashed lines) for (a) nonresonance case, $U_0 < 0$, for defect parameter $U_0 = U_j$ (j = 1, 2, 3, 4, curves *1–4*, respectively), where $|U_1| < |U_2| < |U_3| < |U_4|$; and (b) resonance case, $U_0 > 0$, where $\varepsilon_R = E_R/E_0$ is given by (18).

Quasi-localized state ψ_N exists only if $U_0 > 0$. In the next section we will show that the existence condition for nonsymmetrical ($B = 0$) quasi-localized state coincides with the condition for total resonance reflection from the defect.

We note that the quasi-localized state accounting for the single standing wave along the entire Ox axis ($A = B$) with non-zero local amplitude M is possible only for $\kappa = 0$, which corresponds to the edge of energy spectrum $E = E_0$.

4. Let us now consider scattering of a quasiparticle by crystal interface. We assume that the incident-particle energy falls within the range where quasi-localized states exist $E > E_0$. Then, we seek solution to Eq. (4) in the form

$$\psi(x) = \begin{cases} e^{ikx} + Re^{-ikx} + Me^{\kappa x}, & x < 0, \\ Te^{ikx} + Ne^{-\kappa x}, & x > 0, \end{cases} \tag{19}$$

where R, T, M, N stand for the amplitudes of the reflected, transmitted, and localized on either side of defect waves, and k and κ are defined by (13). Substituting (19) into the boundary conditions (7), we readily obtain the corresponding amplitudes. The reflection and transmission coefficients are of our prime interest:

$$|R(E)|^2 = \frac{|\Delta_R^0(E)|^2}{|\Delta_R^0(E)|^2 + |\Delta_T^0(E)|^2}, \tag{20}$$

$$|T(E)|^2 = \frac{|\Delta_T^0(E)|^2}{|\Delta_R^0(E)|^2 + |\Delta_T^0(E)|^2}, \tag{21}$$

where

$$|\Delta_R^0(E)|^2 = 4\eta^2\kappa^2, \tag{22}$$

$$|\Delta_T^0(E)|^2 = 4k^2\{\eta + 2\kappa(\kappa^2 + k^2)\}^2. \tag{23}$$

Clearly, the quantum-mechanics conservation law

$$|R(E)|^2 + |T(E)|^2 = 1 \tag{24}$$

holds in the medium, if the wave-energy absorption in the crystal volume is disregarded.

The reflection coefficient is proportional to the defect intensity $|R|^2 \sim U_0^2$, which leads to $|R|^2 = 0$ and $|T|^2 = 1$ for $U_0 \longrightarrow 0$; i.e., quasiparticle is insensitive to the interface and may pass freely from one half-space into another. For nonzero defect parameter U_0, total transmission may occur if the incident particle has the energy of the spectrum edge $E = E_0$. For high-energy quasiparticles, the interface is almost transparent (Fig. 2a). However, the discussed conditions are trivial, and they also hold in the case of quadratic dispersion law.

The fascinating fact is that, under nontrivial conditions, the total reflection of quasiparticle from interface is possible, i.e., $|R|^2 = 1$ and $|T|^2 = 0$ (Fig. 2b). Analysis of expressions (20) and (21) shows that the total reflection occurs if the incident quasiparticle energy $E_R(\eta)$ satisfies the condition $2\kappa(k^2 + \kappa^2) = -\eta$, which coincides with (18).

It is well known that the density of states exhibits sharp peaks for energies giving rise to singularities of the reflection or transmission coefficients. Because of this, the energy levels where the total reflection or transmission occurs are referred to as resonance levels.

In the system under study, a resonance (total reflection, to be precise) arises when the incident quasiparticle energy coincides with the energy level of a quasi-localized nonsymmetrical eigenstate. Resonance is conditioned by interaction at the interface of the media of wave-function partial components, one accounting for spatially localized state and the other, for traveling wave. These conclusions are in agreement with the results reported in publications [2–5], though they were concerned with resonance effects in quite different physical fields (the acoustic Rayleigh waves).

5. In order to find out if the anomalous reflection from the passive defect analyzed in the previous section is observable in experiment, we should account for dissipation in crystal. Following [14], we will show that the condition for total reflection may be unstable in relation to small perturbations. Energy absorption in medium can be treated as such a perturbation. To account for dissipation in crystal volume, we should add a term in the form $i\nu\psi$ to Eq. (4). Then, the equation for scalar field in the studied model becomes

$$E\psi = E_0\psi + \alpha\frac{\partial^2\psi}{\partial x^2} + \beta\frac{\partial^4\psi}{\partial x^4} - i\nu\psi + U(x)\psi. \quad (25)$$

Solution to the scattering problem based on Eq. (25) can be written in the form

$$\psi(x) = \begin{cases} e^{ipx} + Re^{-ipx} + Me^{\lambda x}, & x < 0, \\ Te^{ipx} + Ne^{-\lambda x}, & x > 0, \end{cases} \quad (26)$$

where the quasi-momenta p and λ are complex quantities. Assuming that dissipation of quasiparticle energy is rather weak, we obtain

$$p = k + \frac{i\nu}{2k\Omega} \quad \text{and} \quad \lambda = \kappa + \frac{i\nu}{2\kappa\Omega}, \quad (27)$$

where $\Omega(E) = 2\sqrt{\beta(E - E_m)}$, and k and κ are still defined by (13). Then, the reflection and transmission coefficients take the following forms:

$$|R(E, \nu)|^2 = \frac{|\Delta_R(E, \nu)|^2}{|\Delta(E, \nu)|^2}, \quad (28)$$

$$|T(E, \nu)|^2 = \frac{|\Delta_T(E, \nu)|^2}{|\Delta(E, \nu)|^2}, \quad (29)$$

where

$$|\Delta_R(E, \nu)|^2 = |\Delta_R^0(E)|^2 + \frac{\nu^2\eta^2}{\kappa^2\Omega^2}, \quad (30)$$

$$|\Delta_T(E, \nu)|^2 = |\Delta_T^0(E)|^2\left(1 + \frac{\nu^2}{16k^4\Omega^2}\right) + \frac{2\nu^2}{\kappa\Omega}\left\{\frac{2k^4(5\kappa^2 + k^2)^2}{\kappa\Omega} - |\Delta_T^0(E)|\frac{5(\kappa^2 + k^2)}{k}\right\}, \quad (31)$$

$$|\Delta(E, \nu)|^2 = |\Delta_R(E, \nu)|^2 + |\Delta_T(E, \nu)|^2 + \frac{\nu^2}{\kappa\Omega^2}|\Delta_T^0(E)|(5\kappa^2 + k^2) \quad (32)$$

$$+ \frac{2\nu\eta}{\Omega}\left\{|\Delta_T^0(E)|\frac{2k^2 - \kappa^2}{2\kappa k^2} - 4k(5\kappa^2 + k^2)\right\},$$

and the determinants $|\Delta_R^0(E)| = |\Delta_R(E, \nu = 0)$ and $|\Delta_T^0(E)| = |\Delta_T(E, \nu = 0)|$ are defined by (22) and (23). The consideration of dissipation in crystal naturally breaks the conservation law (24).

Let us consider the case when the incident-wave frequency is close to the resonance frequency E_R given by (18). In that event, $|\Delta_T^0(E_R)| = 0$, as was pointed above. The reflection and transmission (28), (29) coefficients can be written as

$$|R(E_R)|^2 = \frac{d^2}{d^2 + \gamma^2 + 2\gamma d}, \quad (33)$$

$$|T(E_R)|^2 = \frac{\gamma^2}{d^2 + \gamma^2 + 2\gamma d}, \quad (34)$$

where the following dimensionless quantities were introduced: $\gamma = \nu/\Omega\kappa_R^2 \ll 1$ is effective length of quasiparticle energy absorption, $d = U_0/\beta k_R(k_R^2 + 5\kappa_R^2)$ is an effective defect thickness, and $\kappa_R = \kappa(E_R)$ and $k_R = k(E_R)$.

If the effective defect thickness considerably exceeds the effective absorption length, $d \gg \gamma$, which is the case of a strong interaction between quasiparticle and interface, one can see from (33) and (34) that the total-reflection resonance is almost not disturbed.

In the case of a weak interaction between quasiparticle and interface, i.e., when $d \ll \gamma \ll 1$, expressions (33) and (34) yield $|T|^2 = 1$ and $|R|^2 = 0$, which indicates that the total transmission of wave through a defect is possible. If the energy of incident quasiparticle is close to the resonance energy of reflection in crystal without dissipation, then consideration of weak energy absorption leads to a reversed situation of resonance total transmission under the condition of weak interaction between quasiparticle and defect. Therefore, singularities in the reflection and transmission coefficients $|R|^2 = 1$ and $|T|^2 = 0$ prove to be unstable in relation to small perturbations, which can lead to $|T|^2 = 1$ and $|R|^2 = 0$ if quasiparticle interaction with defect is much weaker than the energy absorption in the crystal. These are the conclusions made in [14], where dissipation effect on elastic wave scattering in crystals was analyzed.

6. We considered the model of a semiconductor crystal with the band structure of quasiparticle spectrum when the dependence of energy on quasi-momentum is ambiguous. The main point of the model is consideration of spatial derivatives of the order higher than second in Schrödinger equation, i.e., inclusion of spatial dispersion. As a result, we managed to get insight into several fundamentally new effects conditioned by the fact that quasiparticles follow nonquadratic dispersion law, even in the simplest one-dimensional case. In conclusion, we would like to point out the role of spatial dispersion in studying the resonance wave scattering by crystal defects.

Consideration of spatial dispersion via nonquadratic dependence of energy on quasi-momentum enables, with nontrivial initial parameters, the total reflection of the wave $\psi(x) \sim \exp(ikx)$ (quasiparticle) from dispersive media interface. In the case of quadratic dispersion law $E(k) = E_0 + \alpha k^2$, the reflection and transmission coefficients have the forms

$$|R|^2 = \frac{U_0^2}{U_0^2 + 4\alpha(E-E_0)}, \quad |T|^2 = \frac{4\alpha(E-E_0)}{U_0^2 + 4\alpha(E-E_0)}. \tag{35}$$

The total reflection is evidently possible only if $E = E_0$. Therefore, as the analysis of the reflection (20) and transmission (21) coefficients indicates, resonance properties of interface in dispersive and dispersion-free media differ considerably.

Quasi-localized states also exhibit new features in dispersive medium. Specifically, spatial decay of the wave function of localized states is oscillatory. In contrast, for the quadratic dispersion law, at $E < E_0$, the wave function of a localized state decreases steadily $\psi(x) = \psi(0)\exp(-\kappa|x|)$, where $\kappa^2 = (E_0 - E)/\alpha > 0$. The energy of this localized state is defined by explicit formula $E_l = E_0 - (U_0^2/4\alpha)$.

We note that quasi-localized states governed by scalar field do not arise in media with quadratic dispersion law.

In media with spatial dispersion of the opposite sign ($\beta < 0$), the described above scattering features and quasi-localized states do not change. Substituting $\beta = -|\beta|$ in all the expressions above, one can see that the states interchange their energy-dependent types. Now, quasi-localized states exist for $E < E_0$ and localized, for $E > E_0$. Moreover, for $E_0 < E < E_0 + (\alpha^2/4\beta)$, localized states are conventional (their amplitudes decrease exponentially with distance from defect) and, for $E > E_0 + (\alpha^2/4\beta)$, localized states are generalized. Therefore, for $\beta < 0$, a change in the defect parameter may lead to a conversion of conventional localized states to generalized states. The effect is similar to that described in [11, 12].

It may be shown that the results obtained in Sections 1 and 2 depend only slightly on particular boundary conditions and equation of motion that cause nonquadratic dispersion of quasiparticles in many-valley (two-valley, in our case) semiconductor. Consequently, the main results are applicable not only to systems with uniformly distributed field, but to discrete models as well.

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REFERENCES

1. A. N. Darynskii and G. A. Maugin, Wave Motion **23**, 363 (1996).
2. A. M. Kosevich and A. V. Tutov, Phys. Lett. A **248**, 271 (1998).
3. A. M. Kosevich, D. V. Matsokin, and S. E. Savotchenko, Fiz. Nizk. Temp. **25**, 63 (1999) [Low Temp. Phys. **25**, 48 (1999)].
4. A. M. Kosevich and S. E. Savotchenko, Fiz. Nizk. Temp. **25**, 737 (1999) [Low Temp. Phys. **25**, 550 (1999)].
5. A. M. Kosevich, Zh. Éksp. Teor. Fiz. **115**, 306 (1999) [JETP **88**, 168 (1999)].
6. A. M. Kosevich and A. V. Tutov, Fiz. Nizk. Temp. **19**, 1273 (1993) [Low Temp. Phys. **19**, 905 (1993)].
7. A. M. Kosevich, D. V. Matsokin, and S. E. Savotchenko, Fiz. Nizk. Temp. **24**, 992 (1998) [Low Temp. Phys. **24**, 748 (1998)].
8. D. M. Bercha, O. B. Mitin, and I. M. Rarenko, Fiz. Tekh. Poluprovodn. (St. Petersburg) **28**, 1249 (1994) [Semiconductors **28**, 711 (1994)].
9. D. M. Bercha, O. B. Mitin, L. Yu. Kharkhalis, and A. I. Bercha, Fiz. Tverd. Tela (St. Petersburg) **37**, 3233 (1995) [Phys. Solid State **37**, 1778 (1995)].
10. D. M. Bercha, L. Yu. Kharkhalis, A. I. Bercha, and M. A. Sznajder, Phys. Status Solidi B **203**, 427 (1997).
11. S. V. Tarasenko, Fiz. Nizk. Temp. **24**, 219 (1998) [Low Temp. Phys. **24**, 164 (1998)].
12. S. V. Tarasenko, Fiz. Nizk. Temp. **24**, 832 (1998) [Low Temp. Phys. **24**, 624 (1998)].
13. A. I. Buzdin, V. N. Men'shov, and V. V. Tugushev, Zh. Éksp. Teor. Fiz. **91**, 2204 (1986) [Sov. Phys. JETP **64**, 1310 (1986)].
14. Yu. A. Kosevich and E. S. Syrkin, Phys. Lett. A **251**, 378 (1999).