Analysis of Thermophoretic Deposition of Particles from Laminar-Flow Gas Streams with Considerable Transversal Temperature Drops

E. R. Shchuckin, Z. L. Shulimanova,* M. O. Zakharchenko and N. V. Malay†

Institute for High Temperature Russian Academy of Science, Moscow, Russia

Abstract

A theoretical model was developed describing the thermophoretic deposition of aerosol particles in fully-developed laminar flow through channels with their walls at different fixed temperatures. Precipitation of particles occurs under the considerable transversal temperature drops. Gas flow takes place when the gas density, the dynamic viscosity and the thermal conductivity depend on the temperature of the gas. The nonlinear system of equations of gas dynamics describing the distribution of the temperature and the longitudinal component of the mass velocity was solved analytically. The expressions for the distributions of the temperature and the longitudinal component of the velocity in the gas flow enabled us to obtain formulae for direct estimation of the channel length at which the complete thermophoretic precipitation of particles occurs. Numerical evaluation has shown that the increase of the upper plate temperature causes a decrease in the length of complete thermophoretic entrapment of particles.

1. Introduction

Aerosol particles suspended in a non-isothermal gas acquire a mean velocity relative to the gas and move in the direction opposite to the temperature gradient. This effect, known as thermophoresis, occurs whenever the size of the particles is comparable to the mean-free-path of the background gas and is caused by the differential momentum transfer to the particles following collisions with molecules that originate in regions of the gas that differ in temperature. For particles with nearly spherical shape, the thermophoretic velocity is given by

$$U_T = -f_T \nu \nabla T / T, \tag{1}$$

where ν is the coefficient of kinematic viscosity and T is the gas temperature. The scalar coefficient value $f_T \leqslant 1$ depends on the particle and gas thermal conductivities and the Knudsen number (based on particle radius and gas mean free path). An empirical expression for f_T approximately valid for all Knudsen numbers has been given by Talbot *et al.* [1]. Theoretical expression for f_T for the large and moderately large solid spherical particles ($Kn \leqslant 0, 3$) are found in the works of Annis and Mason [2], Redchic and Gaydukov [3], Poddoskkin *et al.* [4]. For example, f_T of the moderately-large solid spherical particles can be evaluated by (Poddoskin *et al.* [4]).

$$f_T = 2K_{TS}^{(0)}\left\{\left(\frac{\kappa_e}{\kappa_i} + C_t Kn\right) \times [1 + Kn(\beta_B + \beta_R') - KnC_v^*(1 + 6C_m Kn)] + Kn(\beta_R - \beta_B)\left[1 - 2\frac{\kappa_e}{\kappa_i} C_q Kn\right]\right\} \Big/ (1 + 2C_m Kn) \times \left[1 + 2\frac{\kappa_e}{\kappa_i} + 2Kn\left(C_t - C_q \frac{\kappa_e}{\kappa_i}\right)\right], \tag{2}$$

where $Kn = \lambda/R$, κ_e and κ_i are the coefficients of thermal conductivity of the gas and the particle substance, and $C_v^* = C_v/K_{TS}^{(0)}$. Expressions for the gas-kinetic coefficients $K_{TS}^{(0)}$, C_T, β_R, β_R', β_B, C_m, C_v, C_q, provided by the Boltzmann equation in the Knudsen layer, are also given by Poddoskin *et al.* If the coefficients of accommodation for the tangential impulse and energy are equal to unity, gas-kinetic coefficients are as follows: $K_{TS}^{(0)} = 1.161$, $C_t = 2.179$, $\beta_R = 3.731$, $\beta_R' = -0.701$, $\beta_B = 3.651$, $C_m = 1.131$, $C_v = 0.971$, $C_q = 0.548$. The coefficient f_T of the small solid particles can be evaluated by Shchukin and Yalamov [5, 6])

$$f_T = \frac{3}{4}\frac{[1 + (5\pi/32)q_\tau(1 - q_E)]}{[1 + (\pi/64)q_\tau(9 - q_E)]}, \tag{3}$$

where q_τ and q_E are the coefficients of the accommodation of the tangential impulse and energy.

The thermophoretic force can have a considerable effect on the particle motion in the gas flows with non-uniform temperature distribution. Thus, for instance, this force causes accelerated deposition of aerosol particles on the cooler heat-exchange surfaces (Simon [7], Wisubenco *et al.* [8], Jia *et al.* [9]), formation and motion of soot in flames (Eisner and Rosner [10]), deposition soot on cooled gas turbine blades (Vermes G. [11]). The mechanism of thermophoretic transfer of particles can be used in the design of devices for the purification of small gas volumes (Fulford *et al.* [12], Moo-Young, Yamaguchi [13], Ostrovsky *et al.* [14], Shchukin *et al.* [15, 16]. In such devices the thermophoretic deposition of the particles can occur in the plane-parallel, coaxial and disk channels with different surface temperatures.

Experimental and theoretical studies have been performed concerning different aspects of thermophoretic depositions of particles between parallel plates (Fig. 1) with temperatures T_1 and T_2 (Fulfard *et al.* [12], Moo-Young [13], Shchukin *et al.* [15,16]). For example, in theoretical studies (Moo-Young *et al.* [13], Shchukin *et al.* [16]) a detailed analysis was made of the thermophoretic deposition of par-

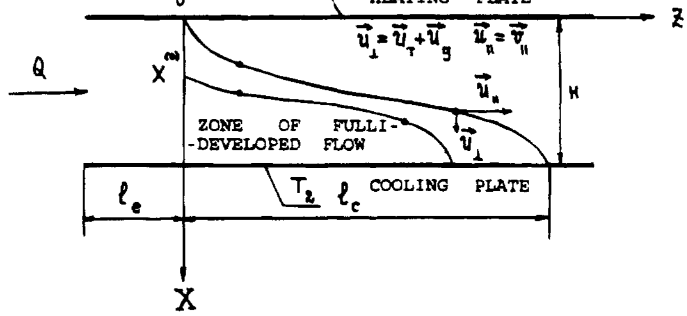

Fig. 1. Plane-parallel channel with different wall temperatures.

* Shadrinsk State Pedagogical Institute, Shadrinsk, Russia.
† Belgorod Pedagogical University, Belgorod, Russia.

ticles in a steady uni-component laminar gas flow with small transverse temperature drops, when thermal conductivity coefficients, dynamic viscosity and concentration of gas molecules are constant values.

In this paper, a theoretical study was made for thermophoretic deposition of aerosol particles from fully-developed laminar gas flow through channels with their walls having different fixed temperatures. Precipitation of particles occurs under arbitrary transversal temperature drops when the gas density, the dynamic viscosity and the thermal conductivity depend on the temperature of the gas. A numerical analysis of the dependence of the channel length at which complete thermophoretic precipitation of particles occurs at the temperature T_1 was carried out.

2. Formulation

We shall consider the thermophoretic deposition of particles in the horizontal plane-parallel channel (Fig. 1). The width and length of the plates in the channel are much greater than the distance H between the plates. The distance H is much larger than the molecular mean free path. The upper plate temperature T_1 may be much higher than the lower plate temperature T_2. We assume the aerosol to be sufficiently rarefied, so that the mutual effect of particles on their motion and on the distribution of the temperature and the gas velocity in the channel can be disregarded. The Brownian motion of the particles is considered to be insignificant, so that the vertical component U_x and the horizontal component U_z of the particle velocity take deterministic values. The particles are deposited on the lower plate due to the thermophoretic and gravitational forces.

Deposition occurs in the zone of the fully-developed laminar flow. In the zone of fully-developed flow the expressions for the gas temperature T and the horizontal component V_z of the gas velocity depend only on the transversal coordinate x and the vertical component $V_x = 0$. The zone of the fully-developed flow is in the region with

$$z > l_e, \quad l_e = 0.0657H(HV_{z\,\max}/\bar{\omega}), \tag{4}$$

where $\bar{\omega}$ is the average of the thermal diffusivity. The entrance length l_e is much smaller than the length l_c, on which all particles are deposited. The concentration of the gaseous molecules n and the coefficients of thermal conductivity κ and dynamic viscosity μ all depend on the gas temperature T.

In the laminar flow under consideration, the aerosol particles move to the lower plate along certain trajectories. Integration of the transport equation gives the coordinates of these trajectories

$$\mathrm{d}x/U_x = \mathrm{d}z/U_z, \tag{5}$$

where U_x and U_z are the vertical and horizontal components of the particle velocity

$$U_x = U_{Tx} + U_{gx}, \quad U_z = V_z. \tag{6}$$

In (6) U_{Tx}, U_{gx} are the thermophoretic and gravitational projections of the particle velocity. When the surface of the particle is near-spherical, the equation for U_{Tx} is

$$U_{Tx} = -f_T \nu \frac{1}{T}\frac{\partial T}{\partial x}. \tag{7}$$

3. Distribution of the gas-dynamic quantities in a fully-developed flow with arbitrary transversal temperature drops

In evaluation of the thermophoretic collection of the particles, it is necessary to know the distributions of the temperature T and the gas velocity components V_x, V_z. In the zone of fully-developed flow the component $V_x = 0$, and the expressions for T and V_z depend upon the transversal coordinate x. The stationary gas transport equations

$$\alpha_p \rho\left(V_x \frac{\partial T}{\partial x} + V_z \frac{\partial T}{\partial z}\right) = \frac{\partial}{\partial x}\left(\kappa \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial z}\left(\kappa \frac{\partial T}{\partial z}\right), \tag{8}$$

$$\frac{\partial}{\partial x}(nV_x) + \frac{\partial}{\partial z}(nV_z) = 0, \tag{9}$$

$$\rho\left(V_x \frac{\partial V_x}{\partial x} + V_z \frac{\partial V_x}{\partial z}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[\mu\left(\frac{4}{3}\frac{\partial V_x}{\partial x} - \frac{2}{3}\frac{\partial V_z}{\partial z}\right)\right] + \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial V_x}{\partial z} + \frac{\partial V_z}{\partial x}\right)\right], \tag{10}$$

$$\rho\left(V_x \frac{\partial V_z}{\partial x} + V_z \frac{\partial V_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \frac{\partial}{\partial z}\left[\mu\left(\frac{4}{3}\frac{\partial V_z}{\partial z} - \frac{2}{3}\frac{\partial V_x}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[\mu\left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z}\right)\right], \tag{11}$$

$$P = nkT \tag{12}$$

simplify under our condition to

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\kappa \frac{\mathrm{d}T}{\mathrm{d}x}\right) = 0, \tag{13}$$

$$\frac{\partial P}{\partial x} = 0, \tag{14}$$

$$\frac{\partial}{\partial z} P = \frac{\mathrm{d}}{\mathrm{d}x}\left(\mu \frac{\partial V_z}{\mathrm{d}x}\right), \tag{15}$$

where P is the pressure of the gas. The thermal conductivity κ and the gas viscosity μ depend only upon the temperature T. The boundary conditions are

$$T|_{x=0} = T_1, \quad T|_{x=H} = T_2, \tag{16}$$

$$P|_{z=0} = P_0, \tag{17}$$

$$V_z|_{x=0} = 0, \quad V_z|_{x=H} = 0. \tag{18}$$

First, integrating eq. (13) yields

$$\kappa \frac{\mathrm{d}T}{\mathrm{d}x} = A \tag{19}$$

where A is an integration constant. By integrating of eq. (19) together with the boundary condition (16), we obtain the following dependence of T on x:

$$\int_{T_1}^{T} \kappa \,\mathrm{d}T = t \int_{T_1}^{T_2} \kappa \,\mathrm{d}T, \tag{20}$$

where $t = x/H$.

From equation (14) it follows that the gas pressure p depends only upon the longitudinal coordinate z:

$$p = p(z). \tag{21}$$

After substituting eq. (21) into eq. (15), the following equation is obtained:

$$\frac{\mathrm{d}p}{\mathrm{d}z} = \frac{\mathrm{d}}{\mathrm{d}x}\left(\mu \frac{\mathrm{d}V_z}{\mathrm{d}x}\right). \tag{22}$$

The left part of this equation depends only on z and the right part depends on x. From that it follows:

$$\frac{\mathrm{d}}{\mathrm{d}z}p = -a, \tag{23}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mu \frac{\mathrm{d}V_z}{\mathrm{d}x}\right) = -a, \tag{24}$$

where a is a positive constant. By integration of eq. (23) together with the condition (17), we obtain the following expression for the gas pressure

$$p = p_0 - az. \tag{25}$$

Integrating eq. (24) together with the condition (18) yields

$$V_z = V^{(0)}G(x) \tag{26}$$

where $\mu^{(1)} = \mu|_{x=0}$ is the dynamic viscosity at the upper plate surface;

$$V^{(0)} = \frac{aH^2}{2\mu^{(1)}},$$

$$G = \frac{2\mu^{(1)}}{H^2}\left\{\frac{\int_0^H \frac{x}{\mu}\,\mathrm{d}x}{\int_0^H \frac{1}{\mu}\,\mathrm{d}x}\int_0^x \frac{1}{\mu}\,\mathrm{d}x - \int_0^x \frac{x}{\mu}\,\mathrm{d}x\right\}. \tag{27}$$

The equation for $V^{(0)}$ can be obtained by the following condition of flow continuity of the molecules in the cross-sections of the channel:

$$Q = b\int_0^H nV_z\,\mathrm{d}x, \tag{28}$$

where b is the width of the plates and n is the concentration of the molecules. Substituting the formula for V_z given by eq. (26) in (28) gives

$$V^{(0)} = \frac{Q}{bHn^{(1)}S}, \quad a = \frac{2Q\mu^{(1)}}{bH^3n^{(1)}S}, \quad S = \int_0^1 G(n/n^{(1)})\,\mathrm{d}t, \tag{29}$$

where $n^{(1)} = n|_{x=0}$. In the channels with laminar flow the longitudinal drops of the gas pressure are small:

$$p_0 - p \ll p_0.$$

Therefore, the value of n can be obtained by means of

$$n = p_0/kT, \quad \text{or } n = n^{(1)}/\theta, \tag{30}$$

where $n^{(1)} = p_0/kT_1$ is the concentration of molecules at the upper plate, k is the Boltzman constant, $\theta = T/T_1$ is nondimensional temperature. When the values of the function G (27) are determined eq. (20) is used.

From eqs (20), (26), (27), when the transversal temperature drops are very small and the coefficients κ and μ are constant, it follows:

$$T = T_1 + (T_2 - T_1)t, \quad V_z = V^{(0)}t(1-t), \quad V^{(0)} = \frac{6Q}{bHn^{(1)}}.$$

The expressions for T (20) and V_z (26) were obtained first in our paper. We can express κ and μ as

$$\kappa = \kappa^{(1)}\kappa^*, \quad \mu = \mu^{(1)}\mu^*, \tag{31}$$

where $\kappa^{(1)} = \kappa|_{T=T_1}$, $\mu^{(1)} = \mu|_{T=T_1}$; κ^* and μ^* are the nondimensional functions depending on $\theta = T/T_1$. Substituting the formulas for n, κ and μ given by eqs (30), (31) in (20), (27) and (29) gives the following nondimensional equations for the nondimensional temperature θ, the function G and coefficient S:

$$\int_1^\theta \kappa^*\,\mathrm{d}\theta = t\int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta, \tag{32}$$

$$G = 2\left\{\frac{\int_0^1 \frac{t}{\mu^*}\,\mathrm{d}t}{\int_0^1 \frac{1}{\mu^*}\,\mathrm{d}t}\int_0^t \frac{1}{\mu^*}\,\mathrm{d}t - \int_0^t \frac{t}{\mu^*}\,\mathrm{d}t\right\}, \tag{33}$$

$$S = \int_0^1 (G/\theta)\,\mathrm{d}t \tag{34}$$

where $\theta = T/T_1$, $\theta_2 = T_2/T_1$ and $t = x/H$. The maximum value of V_z is in the points with the nondimensional transverse coordinate $t = t_m$ in which the derivative

$$\left.\frac{\mathrm{d}G}{\mathrm{d}t}\right|_{t=t_m} = 0. \tag{35}$$

Solving the equation (35) yields

$$t_m = \int_0^1 (t/\mu^*)\,\mathrm{d}t \bigg/ \int_0^1 (1/\mu^*)\,\mathrm{d}t. \tag{36}$$

From eq. (32) it follows:

$$t = \int_1^\theta \kappa^*\,\mathrm{d}\theta \bigg/ \int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta, \quad \mathrm{d}t = \left(\kappa^* \bigg/ \int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta\right)\mathrm{d}\theta. \tag{37}$$

After substituting eqs (37) into eqs (33), (34) and (36), we obtain the following equations for the function G and the coefficient S and the coordinate t_m:

$$G = \frac{2}{\left[\int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta\right]^2}\left\{\frac{\int_1^{\theta_2}\left[(\kappa^*/\mu^*)\int_1^\theta \kappa^*\,\mathrm{d}\theta\right]\mathrm{d}\theta}{\int_1^{\theta_2}(\kappa^*/\mu^*)\,\mathrm{d}\theta} \times \int_1^\theta (\kappa^*/\mu^*)\,\mathrm{d}\theta - \int_1^\theta\left[(\kappa^*/\mu^*)\int_1^\theta \kappa^*\,\mathrm{d}\theta\right]\mathrm{d}\theta\right\}, \tag{38}$$

$$S = \int_1^{\theta_2}[(G\kappa^*)/\theta]\,\mathrm{d}\theta \bigg/ \int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta, \tag{39}$$

$$t_m = \int_1^{\theta_2}\left[(\kappa^*/\mu^*)\int_1^\theta \kappa^*\,\mathrm{d}\theta\right]\mathrm{d}\theta \bigg/ \int_1^{\theta_2}(\kappa^*/\mu^*)\,\mathrm{d}\theta\int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta. \tag{40}$$

In eqs (38)–(40), the nondimensional temperature θ is the new independent variable of integration. When

$$\kappa^* = \theta^\alpha, \quad \mu^* = \theta^\beta \tag{41}$$

the eqs (38)–(40) are:

$$\theta = [1 + t(\theta_2^{1+\alpha} - 1)]^{1/(1+\alpha)}, \tag{42}$$

$$G = \frac{2(1+\alpha)}{(2+2\alpha-\beta)(1-\theta_2^{1+\alpha})^2} \times \left\{(1-\theta^{2+2\alpha-\beta}) - \frac{(1-\theta_2^{2+2\alpha-\beta})}{(1-\theta_2^{1+\alpha-\beta})}(1-\theta^{1+\alpha-\beta})\right\}, \tag{43}$$

$$S = \frac{2(1+\alpha)^2}{(2+2\alpha-\beta)(1-\theta_2^{1+\alpha})^3}\,\varphi, \tag{44}$$

$$\varphi = \left\{\frac{1}{(1+2\alpha-\beta)}\frac{(1-\theta_2^{2+2\alpha-\beta})}{(1-\theta_2^{1+\alpha-\beta})}(1-\theta_2^{1+2\alpha-\beta}) - \frac{1}{(2+3\alpha-\beta)}(1-\theta_2^{2+3\alpha-\beta}) - \frac{1}{\alpha}\,\theta_2^{1+\alpha-\beta}\frac{(1-\theta_2^{1+\alpha})}{(1-\theta_2^{1+\alpha-\beta})}(1-\theta_2^{\alpha})\right\}, \tag{45}$$

$$t_m = \left\{(1-\theta_2^{1+\alpha-\beta}) - \frac{(1+\alpha-\beta)}{(2+2\alpha-\beta)} \times (1-\theta_2^{2+2\alpha-\beta})\right\}\Big/(1-\theta_2^{1+\alpha-\beta})(1-\theta_2^{1+\alpha}). \tag{46}$$

The formulae obtained enable us to directly estimate the T, V_z and p distributions when the values of T_1, T_2 and p_0 are given. As seen from eq. (46), the upper plate temperature increase causes a shift of the points with the maximum value of V_z to the surface of the cooler lower plate. When the equations for coefficient κ^* and μ^* are equal to (41) the value of nondimensional coordinate t_m does not exceed

$$t_m^* = (1+\alpha)/(2+2\alpha-\beta). \tag{47}$$

The analysis showed that, when T_2, $n^{(2)}$, p_0, Q, H are constant and the temperature T_1 rises, the value of V_z at each point of the gas flow and the longitudinal drops p monotonously increase. This is clearly evident from Fig. 2 and Fig. 3. The curves in Fig. 2 show the dependence of the function $B(t)$ on the nondimensional coordinate t when $T_1/T_2 = 1$ (curve 1), $T_1/T_2 = 3$ (curve 2), $T_1/T_2 = 6$ (curve 3). The curve 2 and curve 3 first are given. The function $B(t)$ is equal to:

$$B(t) = V_z(n^{(2)}bH/Q) = (n^{(2)}G/n^{(1)}S), \tag{48}$$

where $n^{(2)} = n|_{t=1} = p_0/kT_2$; $n^{(2)}bH/Q = constant$. The function $B(t)$ depends on the nondimensional coordinate t and the ratio T_1/T_2. The function $B(t)$ reaches a maximal value when $t_m = 0.5$ (curve 1), $t_m = 0.5595$ (curve 2) and $t_m = 0.5870$ (curve 3). The curve in Fig. 3 shows the dependence of the ratio γ upon T_1/T_2

$$\gamma = (p_0 - p)/[(p_0 - p)|_{T_1/T_2=1}] = a/(a|_{T_1/T_2=1}). \tag{49}$$

The ratio γ depends only upon T_1/T_2. The curves in Fig. 2 and Fig. 3 have been drawn on the assumption that

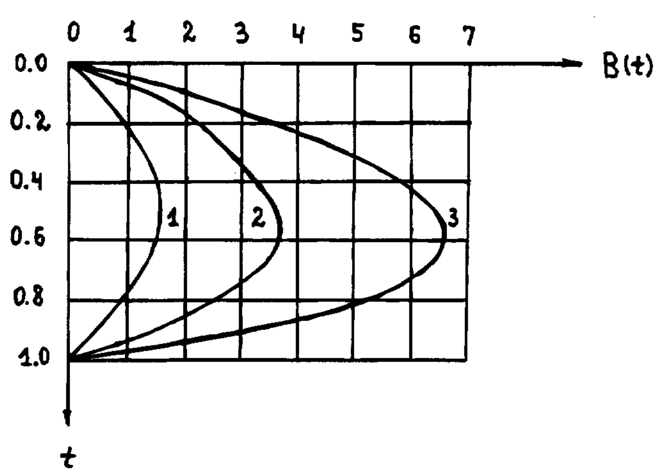

Fig. 2. The dependence of the function $B(t)$ on t.

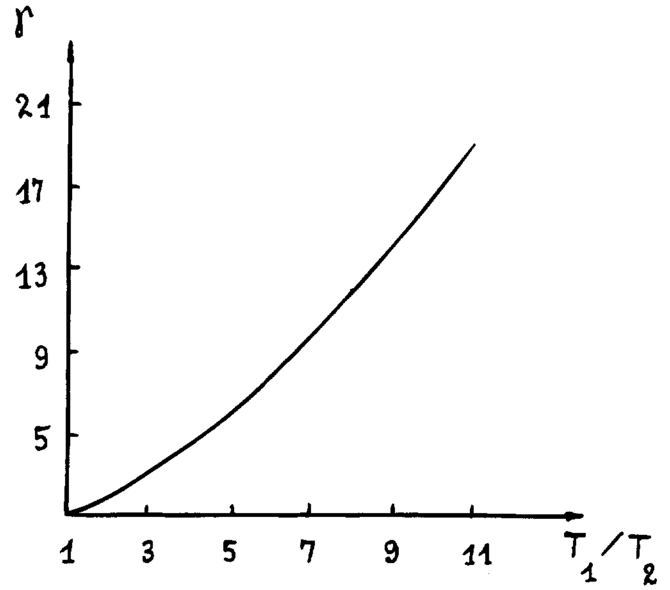

Fig. 3. Relationship between γ and T_1/T_2.

$\alpha = \beta = 0.7$. Comparing the curves on Fig. 2 and Fig. 3 it follows that, with the increase of T_1/T_2, the pressure drops will grow faster than the value of the component V_z. This is due to the simultaneous increase of the value of the coefficient μ and the component V_z when the temperature T_1 rises.

4. Peculiarities of thermophoretic deposition

In our case aerosol particles move to the lower plate along well-defined trajectories. The trajectories of the particles are described by the following differential equation:

$$\mathrm{d}x/U_x = \mathrm{d}z/U_z, \tag{50}$$

where $U_x = U_{Tx} + U_{gx}$ and $U_z = V_z$ are the transversal and longitudinal components of the partical velocity. The values of the component V_z can be determined from the eq. (26). From eq. (20) it follows that the derivative

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{1}{\kappa H}\int_{T_1}^{T_2}\kappa\,\mathrm{d}T, \tag{51}$$

or

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{1}{\kappa^* H}\,T_1\int_1^{\theta_2}\kappa^*\,\mathrm{d}\theta. \tag{52}$$

After substituting eqs (51), (52) into eq. (7), the following equations for the component U_{Tx}, when shape of the particle is near spherical, are obtained:

$$U_{Tx} = f_T\,\nu\,\frac{1}{\kappa HT}\int_{T_2}^{T_1}\kappa\,\mathrm{d}T, \tag{53}$$

$$U_{Tx} = f_T\,\nu^{(1)}\,\frac{\mu^*}{\kappa^* H}\int_{\theta_2}^{1}\kappa^*\,\mathrm{d}\theta, \tag{54}$$

where $\nu^{(1)} = \mu^{(1)}/mn^{(1)} = \nu|_{t=0}$ is the coefficient of kinematic viscosity in the points with $t = 0$. The components U_x and U_z depend only upon x. Considering this fact, we can integrate eq. (50) and get the following expression for the trajectory coordinates of the particle motion:

$$z = \int_{x(0)}^{x}(U_z/U_x)\,\mathrm{d}x, \tag{55}$$

where $x^{(0)} = x|_{z=0}$, x, z are the coordinates of the trajectory. The length l_c for complete particle deposition for the given values of T_1, T_2, p_0 and Q can be obtained from:

$$l_c = \int_0^H (U_z/U_x)\, \mathrm{d}x. \tag{56}$$

Consider now the peculiarities of the pure thermophoretic (when $U_{Tx} \gg U_{gx}$) deposition of spherical particles. It will be easier to evaluate l_c if we proceed in the integral of eq. (56) from the independent variable x to a new independent variable θ, taking eqs (37) into account. Thus, eq. (56) may be written as

$$l_c = l_c^{(0)}\Psi, \tag{57}$$

where $v^{(2)} = v|_{t=1}$, $n^{(2)} = n|_{t=1}$, $\mu^{*(2)} = \mu^*|_{\theta=\theta_2}$, $(n^{(2)}v^{(2)}/n^{(1)}v^{(1)}) = (\mu^{*(2)}/\mu^{*(1)})$,

$$l_c^{(0)} = \frac{QH}{bn^{(2)}v^{(2)}}, \tag{58}$$

$$\Psi = \left(\frac{n^{(2)}v^{(2)}}{n^{(1)}v^{(1)}}\right)\frac{1}{S}\frac{1}{\left[\int_{\theta_2}^1 \kappa^*\, \mathrm{d}\theta\right]^2}\int_{\theta_2}^1 (G\kappa^{*2}/f_T\mu^*)\, \mathrm{d}\theta. \tag{59}$$

In obtaining eq. (57), we considered eqs (26), (37) and (54). The value of the coefficients S and G can be estimated by eqs (38), (39), (43) and (44). With the constant value of f_T (as, for instance, for small particles) and the degree dependence, of κ^* and μ^* upon θ, the expression for Ψ is written as

$$\Psi = \frac{(1+\alpha)}{f_T(1-\theta_2^{1+\alpha})\varphi}\theta_2^{\beta} \times \left\{\frac{1}{(2+3\alpha-2\beta)}\frac{(1-\theta_2^{2+2\alpha-\beta})}{(1-\theta_2^{1+\alpha-\beta})}(1-\theta_2^{2+3\alpha-2\beta}) - \frac{1}{(3+4\alpha-2\beta)}(1-\theta_2^{3+4\alpha-2\beta}) - \frac{1}{(1+2\alpha-\beta)} \times \theta_2^{1+\alpha-\beta}\frac{(1-\theta_2^{1-\alpha})}{(1-\theta_2^{1+\alpha-\beta})}(1-\theta_2^{1+2\alpha-\beta})\right\}. \tag{60}$$

If deposition occurs at small temperature drops, when $(1-\theta_2) \ll 1$, we obtain the following equation for the trajectory coordinates and the length of complete particle deposition:

$$z = l_c^{(0)}\frac{1}{f_T(1-\theta_2)}[3(t^2 - t^{(0)2}) - 2(t^3 - t_0^{(0)3})],$$

$$l_c = l_c^{(0)}\frac{1}{f_T(1-\theta_2)}, \quad l_c^{(0)} = \frac{QH}{bnv},$$

where n, v and f_T are the mean values of n, v and f_T in the cross-sections; $t^{(0)} = x^{(0)}/H$. When T_1 rises, both the thermophoretic velocity of the aerosol particles and the longitudinal component of the gas velocity increase. Hence it is interesting to define the dependence of the length l_c upon T_1. When the value of Q, T_2, p_0, b and H are constant, the dependence of l_c upon Q_1 is defined by the function Ψ. The function Ψ depends only upon the ratio T_1/T_2. Numerical evaluation by means of eqs (59) and (60) show that, when T_1 increases, l_c decreases monotonously. This is illustrated by the curves in Fig. 4, demonstrating the dependence of the function Ψ upon the ratio T_1/T_2 when the degree of dependence of κ^* and μ^* upon θ are given by eqs (41) with

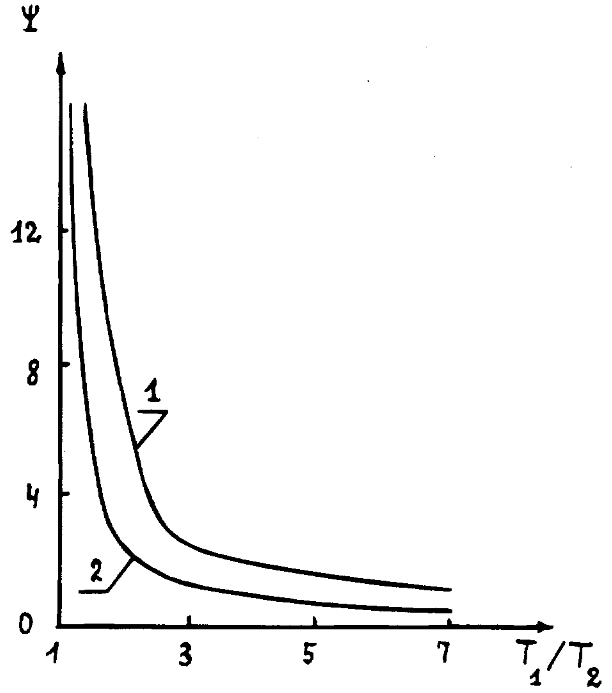

Fig. 4. Relationship between the function Ψ and T_1/T_2 when: $\lambda^{(2)}/R = 0.05$ (curve 1) and $\lambda^{(2)}/R = 5$ (curve 2).

$\alpha = \beta = 0.7$. The curves in Fig. 4 show the results of calculations for iron particles with $\lambda^{(2)}/R = 0.05$ (curve 1), $\lambda^{(2)}/R = 5$ (curve 2) suspended in an air stream with $p_0 = 1$ atm, $T_2 = 293$ K. Here, R is the radius of the particle and $\lambda^{(2)}$ is the gas mean free path when $p_0 = 1$ atm, $T_2 = 293$ K. The values of the coefficient f_T were estimated by eqs (3) and (4) with $q_\tau = 1$ and $q_E = 1$.

In the process of thermophoretic deposition the heat transfers from the upper to the lower plate. Equation (61) can be used in estimating the heat flux Q_T:

$$Q_T = -bl_c\kappa\frac{\mathrm{d}T}{\mathrm{d}x}. \tag{61}$$

After substituting eq. (52) into eq. (61), we obtain:

$$Q_T = T_1\kappa^{(1)}bl_c\frac{1}{H}\int_{\theta_2}^1 \kappa^*\, \mathrm{d}\theta. \tag{62}$$

With account of eq. (56), the expression for Q_T develops into

$$Q_T = Q_T^{(0)}F, \quad F = \Psi\left(\frac{T_1\kappa^{(1)}}{T_2\kappa^{(2)}}\right)\int_{\theta_2}^1 \kappa^*\, \mathrm{d}\theta, \tag{63}$$

where $Q_T^{(0)} = (\kappa^{(2)}T_2Q/n^{(2)}v^{(2)})$, $\kappa^{(2)} = \kappa|_{T=T_2}$. The function F depends only upon the ratio T_1/T_2. From the equation for Q_T (63) it follows that the heat flux Q_T does not depend on the distance H between the plates and the width b of the plates. The numerical evaluation has shown that, when T_1 increases (with the fixed value of Q, T_2, p_0), Q_T also increases monotonically. This is well demonstrated in Fig. 5, showing the curves of the dependence of the function F upon the ratio T_1/T_2 for iron particles with $\lambda_2/R = 0.05$ (curve 1) and $\lambda^{(2)}/R = 5$ (curve 2). The evaluation has been done under the assumption that the particles are in an air stream with $p_0 = 1$ atm and $T_2 = 293$ K.

Experimental studies of the thermophoretic deposition of aerosol particles in plane-parallel channels with considerable transversal temperature drops were not yet done. Fulford *et al.* [10] experimentally examined the effect of the

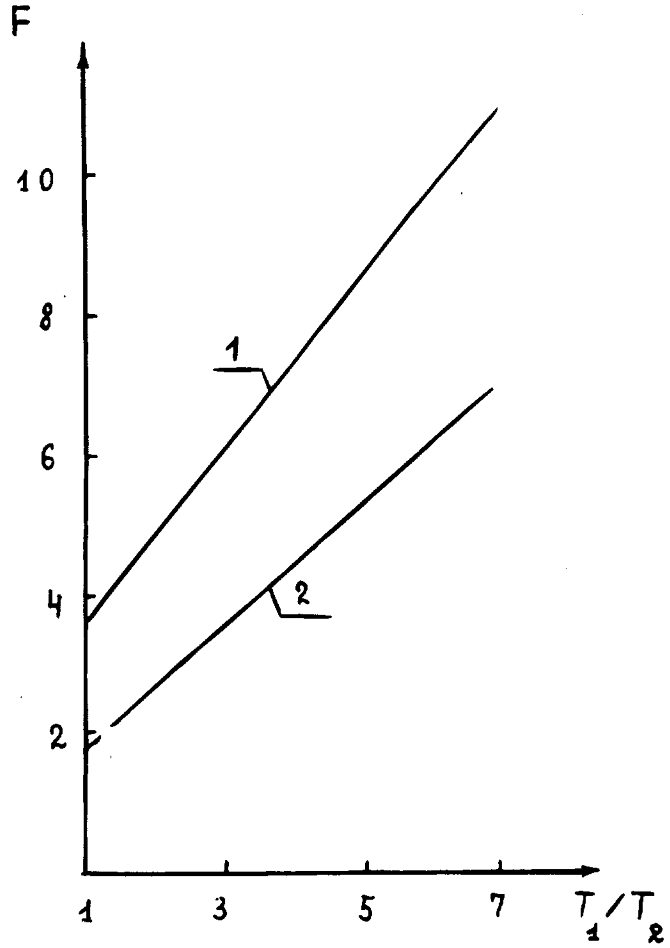

Fig. 5. Relationship between the function F and T_1/T_2 when: $\lambda^{(2)}/R = 0.05$ (curve 1) and $\lambda^{(2)}/R = 5$ (curve 2).

thermophoretic force on deposition of particles in the plane-parallel channel with small transversal temperature drops. They showed that appreciable effects can be obtained even with small temperature drops for particles up to about 30 μm in diameter (which are small for conventional dust removal techniques) in a horizontal laminar air stream. The equipment used in this experimental study consisted of a long horizontal rectangular channel (of cross-section 10.1 cm by 0.63 cm) constructed of two brass plates at the top and bottom with transparent "Lucite" acrylic sheets forming the sides. A sheet metal chamber of size 30 × 25 × 8 cm was attached at the inlet end of the channel to serve as a calming section for the inlet air. A channel entrance length of 91 cm was provided in front of the working section to ensure a fully developed velocity profile before entry to the working section (61 cm long). The brass plates were held at different temperatures by circulating water through external baffled jackets attached to them. Temperatures were measured by means of thermocouples embedded in the plate surfaces and a suitable potentiometer. The plates were carefully earthed to prevent accumulation of electric charges. Particles were dispensed isokinetically into the main air stream in a thin stream down the centre of the channel through a small nozzle immediately upstream of the working section. The collector described after modification may be used for the experimental study of the thermophoretic deposition of particles in channels with considerable transversal temperature drop.

5. Conclusions

Formulae have been obtained which allow evaluation of thermophoretic deposition of aerosol particles in plane-parallel channels with considerable transversal temperature drops. These formulae show that an increase of the upper plate temperature causes a decrease of the complete thermophoretic capture of particles. The resultant data may ultimately be used in practical applications; for example, in devices for the purification of gas streams from aerosol particles of micron and sub-micron size. These particles present difficulties for conventional dry air cleaning devices such as fibrous filters, cyclones, and electrostatic precipitator [17].

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