Relativistic-Electron Scattering on an Atomic String of a Crystal at Ultrasmall Angles of Particle Incidence on a String

N. F. Shul'ga[1), *] and V. I. Truten'

State Research Center Kharkov Institute for Physics and Technology, ul. Akademicheskaya 1, Kharkov, 310108 Ukraine

Abstract—The classical and quantum scattering of fast electrons on an atomic string of a crystal is considered at angles of particle incidence on the string that are much smaller than the critical angle of axial channeling. The investigation was performed within the simplest approximation of the continuous atomic-string potential in the form of a cutoff Coulomb potential. For this case, the azimuthal scattering of particles at an angle exceeding 180° in the plane orthogonal to the string axis is shown to be possible for all impact-parameter values. It is demonstrated that, in particle scattering on a string, an effect can occur that is similar to the Ramsauer–Townsend effect, which consists in a considerable reduction of the total cross section for slow-electron scattering on atoms.

1. INTRODUCTION

When a fast charged particle is incident on an atomic string of a crystal at a small angle ψ with respect to its axis, correlations between successive collisions of the particle with string atoms are of importance. Because of these correlations, the particle is scattered predominantly at the azimuthal angle φ in the plane orthogonal to the string axis [1]. At angles of particle incidence that exceed the critical angle of axial channeling, ψ_c, typical values of the azimuthal scattering angle are small. However, these typical values of the azimuthal scattering angle increase with decreasing angle ψ, achieving 180° at $\psi \sim \psi_c$. Of particular interest is scattering of negatively charged particles on an atomic string because, at all possible values of the angle ψ, such particles can approach closely the string axis, undergoing intense interactions with the string field.

In the present study, we analyze special features of the scattering of a negatively charged particle on an atomic string of a crystal at ultrasmall angles of particle incidence on the string. The study is performed within the simplest approximation relying on a continuous potential of the atomic string in the form of a cutoff Coulomb potential. It is shown that, in the region $\psi \ll \psi_c$, the character of particle scattering on a string changes drastically with decreasing ψ. In particular, it turns out that, at some value of $\psi = \psi_g$ ($\psi_g \ll \psi_c$), the classical angle of particle scattering is 180° for all impact-parameter values. This effect is similar to the effect of giant glory in slow-electron scattering on an atom [2]. For $\psi < \psi_g$, the angle of particle scattering on the string is shown to exceed 180° for all impact-parameter values resulting in scattering.

We have considered quantum aspects of particle scattering on an atomic string. For $\psi < \psi_g$, the quantum-scattering cross section can oscillate considerably about the classical value over the entire range of azimuthal scattering angles. This effect is similar to the generalized Ramsauer–Townsend effect for slow-electron scattering on atoms at a large angle [3, 4].

We also investigate the scattering cross section as a function of the angle φ. At small angles ψ, the total scattering cross section is shown to have a rather complicated structure. In particular, this cross section appears to have a deep minimum at a certain value of ψ. This result is similar to the Ramsauer–Townsend effect, which consists in a considerable reduction of the total cross section for slow-electron scattering on an atom (see [3]).

2. QUANTUM- AND CLASSICAL-SCATTERING CROSS SECTIONS

The motion of a fast charged particle incident on an atomic string of a crystal at a small angle ψ with respect to its axis (z axis) is determined primarily by a continuous string potential—that is, by the potential of string atoms that is averaged along the z axis [5, 6]. The z component of the momentum is conserved in this field, so that the particles undergo scattering only at the azimuthal angle φ in the (x, y) plane, which is orthogonal to the string axis. As a result, we arrive at the two-dimensional problem of particle scattering on a potential $U(\rho)$ in the (x, y) plane, where $\rho = \sqrt{x^2 + y^2}$.

This formulation of the problem is valid both for the quantum and for the classical treatment of the process of particle scattering on an atomic string. In classical

[1)] National Research Center Kharkov Institute for Physics and Technology, ul. Akademicheskaya 1, Kharkov, 310108 Ukraine and Belgorod State University, Studencheskaya ul. 12, Belgorod, 308007 Russia.

* e-mail: shulga@kipt.kharkov.ua

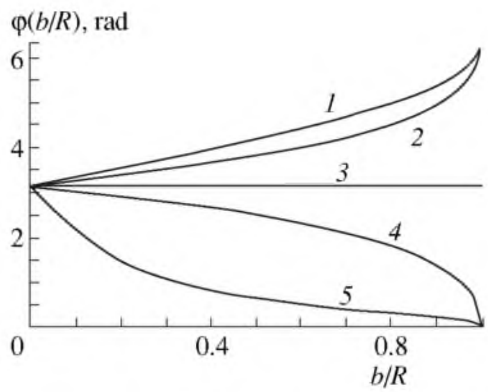

Fig. 1. Deflection function $\varphi = \varphi(b/R)$ (R is the cutoff radius of the string potential) for electron scattering by an atomic string of a silicon crystal at ψ = (*1*) $0.05\psi_c$, (*2*) $0.2\psi_c$, (*3*) ψ_g, (*4*) $0.4\psi_c$, and (*5*) $0.8\psi_c$.

mechanics, the angle of azimuthal particle scattering in the field $U(\rho)$ is given by [1]

$$\varphi(b) = \pi - 2b\int_{\rho_0}^{\infty}\frac{d\rho}{\rho^2}\left(1 - \frac{U(\rho)}{\varepsilon_\perp} - \frac{b^2}{\rho^2}\right)^{-1/2}, \qquad (1)$$

where b is the string impact parameter, $\varepsilon_\perp = \varepsilon\psi^2/2$ is the transverse particle energy, ε is the particle energy, and ρ_0 is the minimum distance between the particle and the string axis.

If the function $\varphi(b)$ is known, we can find the cross section for classical particle scattering on the string, $\frac{d\sigma_c}{d\varphi}$, at angles in the interval $(\varphi, \varphi + d\varphi)$. The result is

$$\frac{d\sigma_c}{d\varphi} = L\psi\left|\frac{d\varphi}{db}\right|^{-1}, \qquad (2)$$

where L is the string length.

The cross section for quantum particle scattering on the atomic-string field has the form [1]

$$\frac{d\sigma}{d\varphi} = \frac{L\psi}{2\pi p_\perp}\left|\sum_{l=-\infty}^{\infty} e^{il\varphi}(e^{2i\eta_l} - 1)\right|^2, \qquad (3)$$

where η_l are phase shifts for scattering on the field $U(\rho)$ and $p_\perp = \varepsilon\psi$. Summation is performed over the integral values of the angular momentum in the transverse plane.

In the semiclassical approximation, the phase shifts η_l are given by

$$\eta_l = \int_{\rho_0}^{\infty} d\rho\sqrt{p_\perp^2 - 2\varepsilon U(\rho) - l^2\rho^{-2}} - \int_{\rho_0}^{\infty} d\rho\sqrt{p_\perp^2 - l^2\rho^{-2}}, \qquad (4)$$

where ρ_0 is the root of the radicand (at $l = 0$, we have $\rho_0 = 0$).

According to (3), the total cross section for particle scattering on the string is

$$\sigma_{tot} = \frac{4L\psi}{p_\perp}\sum_{l=-\infty}^{\infty}\sin^2\eta_l. \qquad (5)$$

In order to find the scattering cross section, it is necessary to know the form of the function $U(\rho)$. The continuous potential of the atomic string has a rather complex form; therefore, simpler forms of the potential energy $U(\rho)$ are often used to approximate the potential of the particle interaction with the string.

For the function $U(\rho)$, we use here the form (see [1, 5, 7])

$$U(\rho) = \begin{cases} -\alpha\left(\frac{1}{\rho} - \frac{1}{R}\right), & \rho \le R \\ 0, & \rho > R, \end{cases} \qquad (6)$$

where $\alpha = U_0 a$, a being the Thomas–Fermi radius of the screening of the atomic potential $U_0 = \pi Z e^2/2d$ (here, $Z|e|$ and e are the charges of the atomic nucleus and the electron, respectively, and d is the spacing between the string atoms), while R is the cutoff radius of the string potential.

Functions of the form $U(\rho) = \alpha/\rho$ are often used in the theory of particle channeling in crystals [5, 7]. In this approximation, the potential $U(\rho)$ is cut off at the distance equal to half the spacing between the nearest atomic strings of the crystal.

The problem of particle scattering on the potential (6) was also considered in studying slow-electron scattering on atoms [2, 3, 8].

3. RESULTS AND DISCUSSION

Formulas (1)–(5) allow us to study particle scattering on an atomic string within the quantum and within the classical framework.

The calculated deflection function $\varphi = \varphi(b)$ for a negatively charged particle scattered by an atomic string is shown in Fig. 1 at various values of the parameter ψ/ψ_c, where $\psi_c = \sqrt{2U_0/\varepsilon}$ is the critical angle of axial channeling.

Our results reveal that the character of particle scattering on an atomic string changes significantly at $\psi \sim \psi_g$, where $\psi_g = \sqrt{\alpha/\varepsilon R}$.

For $\psi > \psi_g$, $\varphi(b)$ is a monotonically decreasing function of the impact parameter, the particles being scattered here at angles that do not exceed 180°.

At $\psi = \psi_g$, the particles are scattered at $\varphi(b) = 180°$ for all impact-parameter values. This effect is similar to the effect of giant glory in slow-electron scattering on atoms [2].

For $\psi < \psi_g$, $\varphi(b)$ is a monotonically increasing function of the impact parameter, the particles being scattered at angles in excess of 180° at all impact-parameter values.

Figure 2 shows the differential cross sections for the scattering of 10-MeV electrons incident on an atomic string of a silicon crystal at angles of ψ = (*a*) $0.8\psi_c$ and (*b*) $0.2\psi_c$ with respect to the ⟨100⟩ axis. These values of ψ correspond to the angles of particle incidence such that $\psi > \psi_g$ in the former case (*a*) and $\psi < \psi_g$ in the latter case (*b*). The values of $F = (La\psi)^{-1}\frac{d\sigma}{d\varphi}$ are plotted along the ordinate. The thin and the thick curve correspond to the calculations by formulas (2) (classical scattering) and (3) (quantum scattering), respectively. The cutoff radius of the potential was taken to be R = 0.96 Å, which corresponds to half the mean spacing between the atomic strings in the plane orthogonal to the ⟨100⟩ axis of a silicon crystal.

These results show that, at $\psi = 0.8\psi_c$, the quantum and classical cross sections coincide over a considerable region of scattering angles; modest oscillations of the quantum-scattering cross section about the values of the classical-scattering cross section in the region of small scattering angles are caused by diffraction effects in particle scattering on the localized string potential.

At $\psi = 0.2\psi_c$, the classical cross section increases monotonically with increasing scattering angle. Here, the quantum cross section develops sizable oscillations about the values of the classical cross section over the whole region of scattering angles. The oscillations are caused by a significant contribution to the scattering cross section (3) from terms associated with small values of the quantum number l. These oscillations are similar to those that occur in the generalized Ramsauer–Townsend effect for slow-electron scattering on atoms at large angles [3, 4].

Figure 3 displays the total cross section for the scattering of 10-MeV electrons on an atomic string of the silicon crystal [see Eq. (5)] as a function of the angle of particle incidence on the string. Here, the quantity $G = (2RL\psi)^{-1}\sigma_{tot}$ is plotted along the ordinate. Our results demonstrate that the total cross section has a rather complicated structure in the region of small angles ψ. In the region of large values of ψ ($\psi \geq \psi_c$), the total cross section decreases smoothly with increasing ψ. In the region of small ψ, the cross section exhibits sharp variations. The reason for this is that the contributions of some terms drop out from the sum over l (5) when ψ decreases.

The total cross section is close to zero at $\psi \sim 0.04\psi_c$. This effect is analogous to the Ramsauer–Townsend effect, which consists in a significant decrease in the total cross section for slow-electron scattering on atoms in a certain region of particle energies (see [3] and references therein).

In conclusion, we note that the above special features of electron scattering on the string field are peculiar not only to the approximation of the string field in

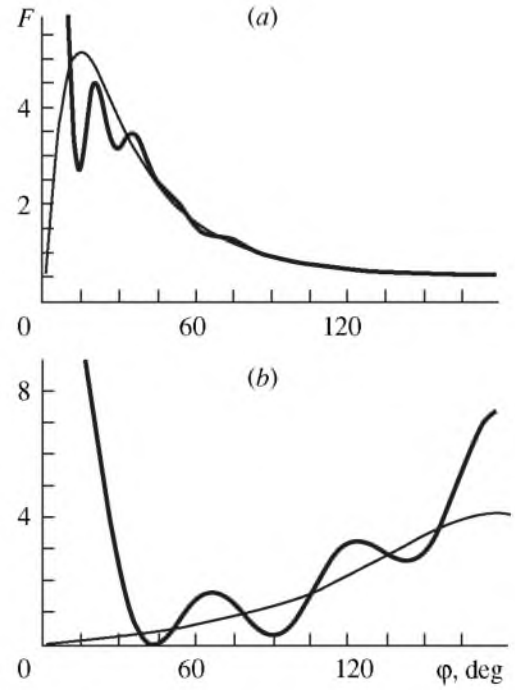

Fig. 2. Differential cross sections for the scattering of 10-MeV electrons incident on an atomic string of a silicon crystal at angles of ψ = (*a*) $0.8\psi_c$ and (*b*) $0.2\psi_c$ with respect to the ⟨100⟩ axis. The values of $F = (La\psi)^{-1}\frac{d\sigma}{d\varphi}$ are plotted along the ordinate. The thin and the thick curve represent the results of the calculations by formulas (2) (classical scattering) and (3) (quantum scattering), respectively.

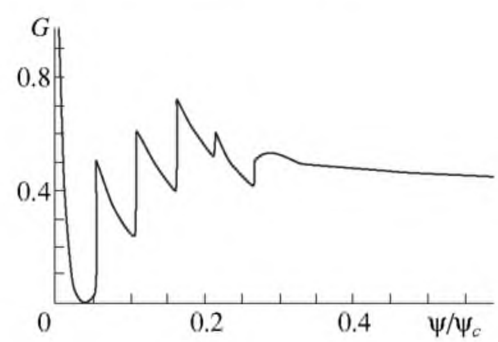

Fig. 3. Total cross section for the scattering of 10-MeV electrons on an atomic string of a silicon crystal as a function of the angle of incidence. The values of $G = (2RL\psi)^{-1}\sigma_{tot}$ and of ψ/ψ_c are plotted along the ordinate and the abscissa, respectively.

the form (6) but also to other approximations that fall off fast with increasing distance.

ACKNOWLEDGMENTS

This work was supported in part by the Ukrainian Foundation for Basic Research, by the International

Science and Educational Program of Ukraine (project Landau–Pomeranchuk–Migdal effect and QSU 082231), and by the Russian Foundation for Basic Research (project no. 98-02-16160).

REFERENCES

1. A. I. Akhiezer and N. F. Shul'ga, *High-Energy Electrodynamics in Matter* (Nauka, Moscow, 1993).
2. Yu. N. Demkov and J. Los, Phys. Lett. A **46**, 13 (1974).
3. N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon, Oxford, 1965; Mir, Moscow, 1969).
4. William F. Egelhoff, Jr., Phys. Rev. Lett. **71**, 2883 (1993).
5. J. Lindhard, Usp. Fiz. Nauk **99**, 249 (1969).
6. D. S. Gemmell, Rev. Mod. Phys. **46**, 129 (1974).
7. I. A. Golovchenko, Phys. Rev. B **13**, 4672 (1976).
8. F. M. Morse, Rev. Mod. Phys. **4**, 577 (1932).