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On the problem of stable profiles of deluvial slopes

by

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Zusammenfassung. Dieser Aufsatz beschreibt mathematische Modelle der Hangspülung. Der Fall einer unveränderlichen Untergrenze des Hangs wird ebenfalls in Betracht gezogen.

Summary. This article describes mathematical models of the deluvial process with and without basal recession.

Résumé. L'article suivant décrit des modèles mathématiques du processus deluvial. On prend aussi en considération la fixation de la base du profil.

The general solution

In modelling the deluvial process, one commences with the continuity equation for unconsolidated material first obtained by EXNER (1922).

$$\frac{\partial q}{\partial X} = -\frac{\partial y}{\partial t}, \qquad (1)$$

where q is the river sediment discharge.

The model of deluvial slope development is analogous to that for steam profile development in a river (VELIKANOV, 1958).

In the case of a river M. A. VELIKANOV has obtained the following value for a river sediment discharge:

$$q = a\left(\frac{\partial y}{\partial X}\right)^{4/3} \qquad (2)$$

After equation (2) has been linearized, then the basic equation of hydrodynamics is obtained from equation (1) above:

$$\frac{\partial y}{\partial t} = K\frac{\partial^2 y}{\partial X^2} \qquad (3)$$

From the model for the slow wash of viscous material along the surface of a slope the following sediment transport law is obtained:

$$q = -\frac{gh^3 \cdot \sin a}{3\,\eta} \cdot \varrho \qquad (4)$$

where $\frac{gh^3 \sin a}{3\eta}$ is the water discharge which is considered to be constant; and ϱ is the sediment concentration.

Some physical theory similar to M. A. VELOKANOV's is necessary for the definition of the ϱ value depending on current flow rate along the slope or from it. As an example of one approximation we may write ϱ in the form:

$$\varrho = (AX^2 + BX + C)\frac{\partial y}{\partial X} \qquad (5)$$

for convex-concave slopes.

The solution of equation (1) under the conditions (4), (5) may be obtained using Legendre polynomials (V. M. MOSKOVKIN, A. M. TROFIMOV, YU. V. BABANOV, 1975):

$$y(Z, t) = \sum_{n=0}^{\infty} C_n P_n(Z) \exp\,[A(n+1)n\,t] \qquad (6)$$

$$C_n = \left(n + \frac{1}{2}\right) \int_{1-}^{1+} \varphi(Z)P_n(Z)dZ \quad (n = 0, 1, 2 \ldots), \qquad (7)$$

where the variable X is connected with the variable Z by the relation:

$$X = P + (q-P)\frac{(1-z)}{2} \qquad (8)$$

where P, q are the roots the quadratic $AX^2 + BX + C = 0$.

A boundary condition function is required for $y(X, t)$, and the function $\varphi(X)$ sets the initial profile of the slope. An important property of solutions to equation (3) of parabolic type is that they 'forget' the initial conditions as time passes, i.e. their solutions with t increasing depend much less on details of initial conditions. We may consider the process in the opposite time sense for a brief period, using equation (3) as an illustration.

$$\frac{\partial y}{\partial t} = -K\frac{\partial^2 y}{\partial X^2}. \qquad (9)$$

Taking this into account the period of time couldn't be longer than that for the current in which the deluvial process took the place described by equation (3).

The more it is desired to know about previous conditions, the more precisely we must know present ones. Small flunctuations of present profile of relief lead to great changes of previous state.

Thus, considering the problem in the opposite direction for a long period of time makes no sense.

Constant basal recession

Let us now consider a particular solution obtained from equation (3) under conditions of steady slope-base recession. This may be done with the help of the specific boundary condition:

(10) $y(Bt, t) = 0.$

It shows that the rate of slope recession goes at a constant rate B. In general, we may write the boundary condition as:

$$y(f(t), t) = 0.$$

But in case (10) the exact analytical solution for the slope profile can be obtained as follows:

To make the solution of equation (3) easier for the boundary condition of a movable base (equation (10)) we make the transformations:

(11) $X' = X - Bt$
$t' = t$

Equation (3) then becomes:

(12) $$\frac{\partial y}{\partial t} = K \frac{\partial^2 y}{\partial X^2} + B \frac{\partial y}{\partial X}.$$

To make the writing easier we omit primes for the transformed t and X. The same equation had been discovered by M. HIRANO (1971, 1972). The boundary condition (10) for movable boundary changes into the simpler one for a fixed boundary:

(13) $y(0, t') = 0.$

The initial condition is: $y(X, 0) = \varphi(X)$; which is unchanged by the transformation (11) because:

(14) $y(X', 0) = \varphi(x')$

The solution to equation (12) may be obtained by the method of separation of variables for the half plane $X' \geqslant 0$.
The parameter of separation we took as

$$-\lambda^2 - \frac{B^2}{4K^2}.$$

The final solution of (12), (13) and (14), taking account of the transformation (11) leads to the solution in the form:

$$y(X,t) = \frac{1}{2\sqrt{\pi K t}} \int_0^\infty \varphi(Z) e^{\frac{B}{2K} Z + \frac{B^2}{4K} t - \frac{B}{2K} X} \times \left[e^{\frac{-(Z - X + Bt)^2}{4Kt}} - e^{\frac{-(Z + X - Bt)^2}{4Kt}} \right] dZ.$$

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