Z. Geomorph. N. F.	28	1	77–94	Berlin · Stuttgart	März 1984

The dynamic models of geomorphological systems (the qualitative theory of dynamic systems' application)

by

A. M. TROFIMOV and V. M. MOSKOVKIN, Kazan, USSR

with 1 figure

Zusammenfassung. Dieser Aufsatz befaßt sich mit einem breiten Band von Modellen – neu für die Geomorphologie –, die auf der qualitativen Theorie dynamischer Systeme beruhen. Folgende Modelle werden diskutiert: zum ersten dynamischen Modelle von Küsten-Hangsystemen – ein Modell der Interaktion zwischen Hangbereichen der Spüldenudation und -akkumulation mittels des dazwischenligenden Transportbereichs sowie dynamischen Modelle der Hangunterschneidung; zum zweiten ein dynamisches Modell eines Schutthangs, ein dynamisches Modell von Rinnenspülungsprozessen und dynamische Modelle, die auf dem Gleichgewichtskonzept von F. AHNERT beruhen.

Die behandelten Modelle sind anwendbar auf die Analyse der Stabilität und Kontrolle von Hangsystemen und auf die Vorhersage ihrer Reaktionen auf Veränderungen von Systemkomponenten, einschließlich der Abschätzung von Reaktionsverzögerungszeiten (lag effects) und der Zeitspannen bis zur völligen Anpassung des jeweiligen Systems an diese Veränderungen (relaxation times).

Summary. This paper deals with a wide range of models, new for geomorphology, that are based on the qualitative theory of dynamic systems. The following models are discussed: first, dynamic models of shore slope systems – a model of the interaction between wash denudation and accumulation zones by way of the intervening transit zone and dynamic models of undercut slopes; second, dynamic models of slope systems – a dynamic model of a talus slope, a dynamic model of gully processes and dynamic models based on the equilibrium concept of F. AHNERT.

The models discussed can be applied to the analysis of stability, to the control of slope systems and to the forecasting of their reactions to changes of system components, including assessment of the resulting lag effects and relaxation times.

Résumé. Cet article traite d'un large éventail de modèles – nouveaux en géomorphologie – qui sont basés sur la théorie qualitative des systèmes dynamiques. Les modèles suivants sont discutés: (1) des modèles dynamiques de systèmes de pentes de plages – un modèle de l'interaction entre l'érosion par l'eau et les zones d'accumulation par l'intermédiaire de la zone de dérive et des modèles dynamiques de versants sapés à la base. (2) des modèles dynamiques de systèmes de pentes – un modèle dynamique des pentes d'éboulis, un modèle dynamique des processus de ravinement et des modèles dynamiques basés sur le concept d'équilibre de F. AHNERT.

Les modèles discutés peuvent être appliqués à l'analyse de stabilité, au contrôle des systèmes de pentes et à la prévision de leurs réactions aux changements des composantes du système incluant l'établissement des effets de retard et des temps de relaxation.

0372-8854/84/0028-0077 $ 4.50

The first mathematical models of geomorphological processes, based on the qualitative theory of dynamic systems, were formulated in biochemical and biological kinetics and ecology (LOTKA 1925, VOLTERRA 1931); at present they are intensively being developed, especially in the field of ecology (ALEKSEYEV 1976, SMITH et al. 1976). This theoretical work may find application in the modelling of geomorphological processes as well, for instance, in the construction of models of geographical interaction for the forecasting of landform evolution (SIMONOV et al. 1976). We suggest, new for geography and geomorphology, a wide scope of models, based on the qualitative theory of dynamic systems. It is not advisable to divide these models rigidly into groups. We will view them rather as a single group of dynamic models of natural (slope) systems.

All the models under consideration in this paper are reduced to the dynamic systems, which are a maximum of the second order (the system of the two ordinary autonomous differential equations of first order). In a mathematical sense there are some differences in the structure of the suggested models. First, there are the dynamic models (systeme) that consist of the system of the ordinary differential equations themselves of the first order. Second, there are the dynamic models that consist of one ordinary differential equation of the first order, while the differential equations which enclose it are transformed into algebraic ones. The latter simplified models are known, for instance, in chemical kinetics when in some of the equations of a dynamic system the velocities' change (dC_i/dt) of the (C_i) reacting substances' concentrations are proportional to each other and these differential equations are transformed into algebraic ones reducing them to one ordinary differential equation of the first order.

The dynamic models of geomorphological (slope) systems are applicable to analysing stability, to forecast and control, to the accounting of time lags and to the determination of relaxation time in the systems. The first means a time lapse of any external effect on the system until the latter reacts to this effects. The second means the time lapse after a change of external conditions until the system is completely adapted to the new conditions (achievement of dynamic equilibrium). Due to negative feedbacks the system has a tendency towards some stable state (dynamic equilibrium) when the external factors remain stable. In this connection one of the problem of an optimum consists of determining these states and transforming the system into them as soon as possible. This is equivalent to a time decrease in the system. The models mentioned above constitute a particularly adequate technique for developing the theory of the dynamic equilibrium relief.

The aim of this paper is restricted to the construction of dynamic models, to the analytical solution of dynamic systems and to the analysis of their stable states.

1 *The dynamic models of slope systems*

1.1 *The model of interaction between denudation area and accumulation area through the area of a slope's transit*

A slope's development can be modelled using the dynamic system of second order which describe the interaction of a weathered waste of the denudation area with the accumulation area by way of the transit area (TROFIMOV & MOSKOVKIN 1978)

(1.1–1) $$dy_1/dt = f_1(y_1, y_2) - f_2(y_1, y_2)$$
$$dy_2/dt = f_2(y_1, y_2) + f_3(y_2) - f_4(y_2)$$

where y_1, y_2 are the weathered waste's amounts in the denudation and accumulation regions: $f_1(y_1, y_2)$, $f_3(y_2)$ are the rates of the weathered waste's development in the denudation and accumulation regions; $f_2(y_1, y_2)$ is the rate of the weathered waste's inflow into the accumulation area from the denudation area; $f_4(y_2)$ is the removal rate of the weathered waste from the accumulation area by transport. At the initial moment one can set $y_1(O) = a$, $y_2(O) = v$. Generally, the thickness of a weathered waste is noticeable in the accumulation region. Thus it may be considered that in most cases $f_3(y_2) = O$. The system (1.1–1) can be tested for a stability of the special point's solution, which corresponds to the dynamic regime $\frac{dy_1}{dt} = \frac{dy_2}{dt} = O$, which indicates that the amount of waste developing in the denudation region is equal to that of the amount coming into the accumulation region. The latter, in its turn, is equal to the waste's amount being removed from the accumulation area ($f_3(y_2) = O$).

In the absence of undercutting ($f_4(y_2) = O$) a substantial waste accumulation occurs in the accumulation area. This produces a feedback in the first system's equation (1.1–1). With the waste increase in the accumulation area, there begins a decrease of waste flow to this area from the denudation area through the transit area, due do the decrease in the steepness of the transit area. In the course of time the slope development stabilizes. The transit area is stable when there is no decrease of the denudation area by enroachment of the accumulation area, i. e. when $f_1(y_1 \cdot y_2) = f_1(y_1)$. However, in nature, the denudation region's reduction occurs more often, which causes a stabilization in a slope's development as well. In this case one can design the following model: the weathered waste's development rate in the denudation area is represented in the form

(1.1–2) $$f_1(y_1, y_2) = \alpha (h_o - h)\ 1$$

i. e. when the weathered waste's layer thickness equals zero ($h = o$), its development rate is the greatest; when weathering does not effect the bedrock there exists the limit thickness. In the formula (1.1–2), 1 is the length of the denudation area. If the profile shapes of the denudation and accumulation areas are sufficiently rectilinear, the denudation area's length depends upon the accumulation area in the following way:

(1.1–3) $$1 = k\,(y_{2max} - y_2)^{1/2}$$

When $y_2 = y_{2max}$ the denudation region disappears, i. e. the accumulation region overlaps it completely. The thickness h in the formula (1.1–2) represents the ratio of the weathered waste's amount (y_1) to the length of the denudation area (the plain problem is being considered

(1.1–4) $$h = y_1/l = y_1/k\,(y_{2max} - y_2)^{1/2}$$

Consider the incoming waste's velocity into the accumulation area to be proportional to the waste's amount in the denudation area, i.e. $f_2 = \beta y_1$, the waste's removal from the accumulation region being constant (λ = const). Taking into account the above mentioned (when $f_3 = O$), one comes to the following set of expressions

(1.1–5) $d\,y_1/dt = -(\alpha + \beta)\,y_1 + \alpha\,h_o\,k\,(y_{2max} - y_2)^{1/2}$
$d\,y_2/dt = \beta\,y_1 - \lambda$

The special system's point is as follows

(1.1–6) $\bar{y}_1 = \lambda/\beta,\ \bar{y}_2 = y_{2max} - [(\alpha+\beta)^2\,\lambda^2/\beta^2\,\lambda^2\,h^2\,k^2]$

When $\bar{y}_2 < O$ a bed slope cut will occur (the accumulation area disappears). This will occur when $\lambda \gtrless (y_{2max})^{1/2}\,\beta\alpha\,h_o\,k/(\alpha+\beta)$.

To define the special point's stability character (1.1–6), we linearize the system

(1.1–5) at the special point
(1.1–7) $y_1 = \xi + \bar{y}_1,\ y_2 = \eta + \bar{y}_2$

Putting (1.1–7) into the system (1.1–5) and neglecting the members with order higher than the first one, we obtain

(1.1–8) $d\,\xi/dt = -(\alpha + \beta)\,\xi - \dfrac{\alpha^2\,h_o^2\,k^2\beta}{2\,\lambda\,(\alpha+\beta)}\,\eta$

$d\,\eta/dt = \beta\,\xi$

The solution of the linear system (1.1–8) in the form

(1.1–9) $\xi = C_{11}\exp(\lambda_1 t) + C_{12}\exp(\lambda_2 t);\ \eta = C_{21}\exp(\lambda_1 t) + C_{22}\exp(\lambda_2 t)$

gives the following meanings for

(1.1–10) $\lambda_{12} = \dfrac{(\alpha+\beta)}{2} \pm \left(\dfrac{(\alpha+\beta)^2}{4} - \dfrac{(\alpha\,h_o\,k\,\beta)^2}{2\,\lambda\,(\alpha+\beta)}\right)^{1/2}$

From the last ratio the following conclusions on the special point's stability character can be drawn:

When $\dfrac{(\alpha+\beta)^2}{4} \geqslant \dfrac{(\alpha\,h_o\,k\,\beta)^2}{2\,\lambda\,(\alpha+\beta)}$

we have a stable bound, which means the aperiodic waning tendency to the special point.

When $\dfrac{(\alpha+\beta)^2}{4} < \dfrac{(\alpha\,h_o\,k\,\beta)^2}{2\,\lambda\,(\alpha+\beta)}$

we have the stability focus, which means a periodic waning tendency to the special point.

The critical parameter value (waste removal rate from the accumulation area), which divides the two stability regions, will equal

(1.1–11) λ critical $= 2\,(\alpha\,h_o\,k\,\beta)^2/(\alpha+\beta)^3$

When λ is larger or equal to this critical value, one obtains a stable bound. When $\lambda = O$ (absence of wash) one fails to construct a linearized system, but the special point's character $\bar{y}_1 = O,\ \bar{y}_2 \equiv y_{2max}$ is clear from the physical point of view, namely, the waste amount brought into the accumulation area increases slowly to the maximum value $\lim_{t\to\infty} y_2 = y_{2max}$. Thus, the special point is the stable bound. The trajectories on the phase plane can be qualitatively drawn by the isoclinal lines' method (fig. 1). On the curve $y_1 = \dfrac{\alpha\,h_o\,k}{\alpha+\beta}\,(y_{2max}^2 - y_2^2)^{1/2}$ the tangents to the trajectory are of the vertical direction, while on the straight line $y_1 = O$ they are of the horizontal one. These two typical curves $y_1 = \dfrac{\alpha\,h_o\,k}{\alpha+\beta}\,(y_{2max} - y_2)^{1/2}$ and $y_1 = O$ are called the isoclinal lines. The first one separates the regions where the derivatives dy_2/dy_1 have different signs. When constructing the models (1.1–5) the wash area was assumed to increase at the same time as the accumulation area dininishes. With large talus wash this condition is not observed, a bench being formed at the initial talus.

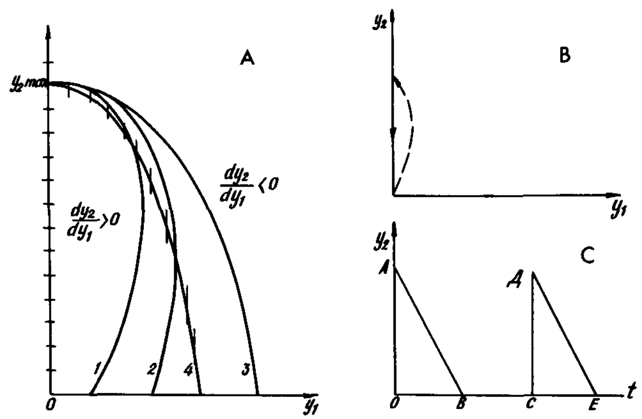

Fig. 1. A: Qualitative character of phase trajectories' behaviour when wash at the basement of a slope is absent.
B: Closed broken phase trajectory characterizing a cyclic development of abraision-landslide or a landfall-abrasion slope.
C: Periodioc in time dynamics process of accumulative abrasion-landslide region or landfall-abrasion slope.

1.2 *The dynamic models of abrasion slopes (undercut slopes)*

Considering the development of the steep slope which is being undercut by wash at its base, we start from the following obvious circumstances. First, the stronger the wash the more quickly increases the slope's steepness. Second, the steeper the slope, the more waste is removed from the upper parts of the slope and the less is the base being cut. Third, the greater the slope's steepness, the more quickly it dinishes. Fourth, the water flow's energy cuts the slope base and reworks the wasted accumulating there. The slope's development is also assumed to be going on continuously without any large waste falls. As a result, we come to the following dynamic system (Trofimov & Moskovkin 1978):

(1.2–1)
$$\frac{d\alpha}{dt} = c_1 v_{cut} - c_2 \alpha$$
$$\frac{dm}{dt} = v_{cut} + v_{up} - v_{rew}$$
$$v_{cut}\, d = V(1 - [m/m_o]),\ d > 1,\ v_{rew} + v_{cut}\, d = \text{const},\ v_{up} = k\,\alpha$$

where α is the average slope angle; v_{cut} is the undercutting intensity on the bedrock; v_{rew} is intensity of reworking of the accumulated waste at the slope base (m); v_{up} is the intensity of waste inflow from the upper parts of the slope; c_1, c_2, k, d are positive constants. The second equation of the system is the balance ratio, the third describes a cut; i. e. when m = o, the intensity of a cut is maximum and there is some value $M = m_o$ when the waste cover prevents any cutting of the bedrock (v_{cut} = O). The constantz α indicates that the influence of a wave flow upon the slope base is not equivalent (does not etqually act upon the bedrock and the accumulating debris). Last, but not an equation of the system, is the condition of a continuous intensity of influence on the slope base by a wave flow. The equations system (1.2–1) reduces to the following dynamic system of second order:

(1.2–2)
$$dm/dt = v_{up.} + \frac{1 + d}{d} V (1 - [m/m_o]) - V,$$
$$d\, v_{up.}/dt = \frac{c_1 k}{d} V (1 - [m/m_o]) - c_2 v_{up.}$$

The special point of the initial equation system (1.2–1) is as follows:

(1.2–3)
$$\bar{m} = m_o (c_1 k + c_2) / (c_1 k + c_2 (d + 1)),$$
$$\bar{v}_{up.} = V c_1 k / (c_1 k + c_2 (d + 1)),$$
$$\bar{v}_{cut.} = V c_2 / (c_1 k + c_2 (d + 1)),$$
$$\bar{v}_{rew.} = V (c_1 k + c_2) / (c_1 k + c_2 (d + 1)),$$
$$\bar{\alpha} = V c_1 / (c_1 k + c_2 (d + 1)).$$

While analysing the equation system (1.2–2) we shall need the first two coordinates of the special point. The linearization of the system (1.2–4) gives

(1.2–4)
$$d\,\xi / dt = -\frac{c_1 k}{m_o d} \eta - c_2 \xi,$$
$$d\,\eta / dt = \xi - \frac{(1 + d) V}{d\, m_o} \eta$$

where $m = \bar{m} + \xi$, $v_{up} = \bar{v}_{up} + \eta$

The roots of the characterizing equation of thisd system are as follows

(1.2–5)
$$\lambda_{1,2} = -\frac{d\, c_2 m_o + (d + 1) V}{2 m_o d} \pm \frac{\sqrt{[c_2 m_o d - V(1 + d)]^2 - 4 c_1 k\, m_o V d}}{2 m_o d}$$

When $\Delta = [c_2 m_o d - V(1 + d)]^2 - 4 c_1 k m_o V d \geq O$ there is the stable bound and when there is the stable focus. Thus, in some time certain definite relationships are established between v_{cut}, v_{up}, v_{rew}, α, m. We see that when the influencing factor V is constant there are no unstable solutions at any parameter values of the system. Now we introduce a lag effect into the system (1.2–1). The fact is that the slope's cut does not lead to an increase of steepness at once but in some time T: the slope's steepness increase does not lead to an increase of waste removal from the upper parts of the slope at once either, but only in time τ. In this case we come from the system (1.2–1) to the system with the lag arguments:

(1.2–6)
$$d\,\alpha (t)/dt = c_1 v_{cut} (t - T) - c_2 \alpha (t)$$
$$d\, m(t)/dt = v_{cut} (t) + v_{up} (t + \tau + T) - v_{rew}(t),$$
$$v_{rew} (t) + v_{cut} (t)\, d = V,$$
$$v_{up} (t + \tau) = k\, \alpha (t),$$
$$v_{cut} (t)\, d = V (1 - [m(t)/M_o])$$

Applying the Laplace transformations to the system (1.2–6) $\alpha \to A$, $m \to M$, $v_{cut} \to U_{cut}$, $v_{up} \to U_{up}$, $v_{rew} \to U_{rew}$ where, for instance, $A = \int_0 \alpha(t) \exp(-st)\, dt$, we come to the following algebraic system of equations:

(1.2–7)
$$- \hat{\alpha}_o + As = c_1 \exp(-sT)\, U_{cut} - c_2 A,$$
$$- \hat{m}_o + Ms = U_{cut} - U_{rew} + U_{up} \exp[s(\tau + T)],$$
$$U_{rew} + U_{cut}\, d = V/S,$$
$$U_{up} \exp(s\tau) = k A,$$
$$U_{cut}\, d = (V/S) - (V/m_o) M,$$

where $\hat{\alpha}_o = \alpha(O)$, $\hat{m}_o = m(O)$. Solve this system relatively to M

(1.2–8)
$$M = \frac{[(c_2 + s) + \hat{m}_o s (c_2 + s) d + k c_1 V + k \hat{\alpha}_o S \exp(s T)]}{s[d m_o s^2 + (m_o c_2 d + (1 + d) V) S + (1 + d) V c_2 + k c_1 V]}$$

From this m(t) will be expressed as the inverse transformation of Laplace

(1.2–9)
$$m(t) = \frac{1}{2 \pi i} \int_{c-i\infty}^{c+i\infty} \frac{m_o \exp(st) [(c_2 + s) V + m_o s (c_2 + s) d + k c_1 V + k \hat{\alpha}_o S \exp(s T)]\, ds}{s [d m_o s^2 + (m_o c_2 \alpha + (1 + \alpha) V) S + (1 + d) V c_2 + k c_1 V]}$$

The dominator of the subintegral expression has the following zeroes:

$$(1.2\text{–}10)\quad S_1 = O;\; S_{2,3} = -\left(\frac{c_2}{2} + \frac{(1+d)\,V}{2\,m_o\,d}\right) \pm \left[\left(\frac{c_2}{2} - \frac{(1+d)\,V}{2\,m_o\,d}\right)^2 - \frac{k\,c_1\,V}{m_o\,d}\right]^{1/2}$$

When $\frac{c_2}{2} - \frac{(1+d)\,V}{2\,m_o\,d} > \left(\frac{k\,c_1\,V}{m_o\,d}\right)^{1/2}$, the roots $S_{2,3}$ are valid negative numbers. Otherwise we have complex roots with valid negative parts. Thus, unstable solutions arise. The integral (1.2–9) is derived by the deductions' sum at the point $S_{1,2,3}$. Here calculate only one deduction at the point $S_1 = O$, which gives the stable limit value for m(t).

$$(1.2\text{–}11)\quad \text{Deduction}_{\,s\,=\,o}\;\varphi(t) = \lim_{t\to\infty} m(t) = \frac{m_o\,(c_2 + k\,c_1)}{(1+d)\,c_2 + k\,c_1}$$

where φ(t) is the subintegral expression in the formula (1.2–9). We see that the expression (1.2–11) coincides with the stable value for m(t) of the previous model, in which the lag factor was not being taking into account. The stability character of the special points did not change either. Thus, when the influencing factor is constant (V = const) no unstable solutions arise. Actually, in nature the instability of the wave-cut slopes might happen if the influencing factor's intensity (V) increases sharply. We assumed that the slope development would take place according to an uninterrupted model process. But if the slope angle (α) exceeds some critical value ($\alpha > \alpha_c$), there occur large landslides and this model does not apply any more. One can describe slope development but the cyclic process of coastal abrasion and landslides (HUTCHINSON 1975) or by abrasion and earthfall in the bounds of the model (1.1–1). Now it is possible to show what such slopes will look like both on the phase plane and during a time (fig. 1B, C). Let at the moment of time t = O an earthfall or a landslide occur from the wave-cut slope. On the phase plane (fig. 1B) this corresponds to the leap from the point O to the point A. Further, there occurs the removal of the accumulated debris until it disappears completely. After the accumulated mass has been removed, there occurs the cut of the bedrock slope, the point on the phase plane being at O, i. e. during this period $y_2 = O$. If the slope's stability is suddenly interrupted, an earthfall or earthslide occurs, i. e. there occurs the leap again from the point O to the point A (in fig. 1B) (this corresponds to the HUTCHINSON sheme (1975), when the landslides of the same thickness recur periodically). On the plane (t, y_2) (fig. 1B) the direct line AB corresponds to the Process of removing the accumulated landslide debris, the direct line BC corresponds to the cutting back of the bedrock slope, and the direct line CD corresponds to an earthfall or a landslide. Then the process is periodical.

Let's analyse now the simplified system (1.2–1) without taking into account the change of the average inclination (α). Consider the slope to be rectilinear, the wash's intensity being constant and equaling $V_s = v\,\ell\,\varrho$, where v is the denudation velocity along the normal to the slope, ℓ is the slope's lewngth, ϱ is the volumetric rock

weight. The simplified system without the equation comprising α, will be written as follows:

$$(1.2\text{–}12)\quad \begin{aligned} dm/dt &= v_{cut} + v_{up} - v_{rew},\\ v_{cut}\,d &= V(1 - [m/m_o]),\\ v_{cut}\,d + v_{rew} &= V = \text{const} \end{aligned}$$

The system (1.2–12) is reduced to the one ordinary differential equation

$$(1.2\text{–}13)\quad \frac{dm}{dt} = v_{up} + \frac{V}{d}\left(1 - \frac{m}{m_o}\right) - \frac{Vm}{m_o}$$

the solution of which at the initial condition m(O) = M has the form (TROFIMOV & MOSKOVKIN 1980)

(1.2–14)

$$m(t) = \left[\left(v_{up} + \frac{V}{d}\right)\left\{1 - \exp\left[-\frac{V}{m_o}\left(\frac{d+1}{d}\right)t\right]\right\} + \frac{V}{m_o}\left(\frac{d+1}{d}\right)\right.$$
$$\left. M \exp\left[-\frac{V}{m_o}\left(\frac{d+1}{d}\right)t\right]\right]\left(\frac{V}{m_o}\left(\frac{d+1}{d}\right)\right)^{-1}$$

From the expression (1.2–14) it is seen that in the course of time there occurs some definite amount of waste at the slope foot

$$(1.2\text{–}15)\quad \lim_{t\to\infty} m(t) = m_o\,(v_{up}d + V)/V\,(1+d)$$

When $v_{up} = V$, $\lim_{t\to\infty} m(t) = m_o$, the intensity of the bedrock cutting is equal to zero ($v_{cut} = O$) and the intensity of the waste removal rate is equal V ($v_{rew} = V$). Thus, the waste's amount which is removed from the slope in the same as that which is transported to the slope foot from upslope (the first case by HUTCHINSON 1975).

When $v_{up} < V$, $\lim_{t\to\infty} m(t) < m_o$, there occurs undercutting of the bedrock slope (the second case b HUTCHINSON).

When $v_{up} > V$, $\lim_{t\to\infty} m(t) > m_o$, waste is accumulated at the slope foot (the third case by HUTCHINSON).

The development and concretization of the models considered here, on the basis of the change of the influencing factor V and of hypotheses derived from practical experience, permits rational measures against abrasional cutting at the slope foot. The critical value of the waste amount (m_o) should be determined from theoretical, natural and experimental studies. According to the limit expression (1.2–15) the relaxation time equals infinity; this applies to the analysis of random dynamic systems. In fact, however, due to the rapid exponential tendency towards a stable regime some finite time may be taken as the relaxation time, at which the value m (t) is sufficiently close to m_o. It may be derived by the solution of equation (1.2–15) relative to a time. By changing the model's parameters it is possible to vary the relaxation time, i. e. the time of the system's transformation into the stable state.

While considering the space problem one should write a more general balance equation of the waste instead of the second equation of the system (1.2–1).

(1.2–16) $dm/dt = v_{up} + v_{cut.\ V.} + v_{acc.\ V.} - v_{rew.\ V.} + v_{cut.\ V.f.} + v_{acc.\ V.f.} - v_{rew.\ V.f.}$

where $v_{cut.\ V.}$, $v_{cut.\ V.f.}$ are the intensities of the waste subbly due to abrasional undercutting of the bedrock slope by waves and longshore currents; $v_{rew.\ V.}$, $v_{rew.\ V.f.}$ are the intensities of the waste removal being reworked by the waves and longshore currents.

The problem is to enclose the equation (1.2–16) by the ratios which are analogous to the model's hypotheses (1.2–1). This is a theoretical aspect of the application of equation (1.2–16). A practical predicting aspect of its applicastion may be as follows. Let all the components of the equation (1.2–16) have been determined for the given time moment (t = O) by experiment and let their algebraic sum be equal $\bar{v}$. Then, with the initial waste amount m(O) = M according to the equation (1.2–16), find a waste amount for a small period ahead during which it may be considered that $\bar{v}$ = const (SHIROKOV, MOSKOVKIN & TROFIMOV 1979).

(1.2–17) $m(t) = M + \bar{v}\, t$

When $\bar{v} > O$ there occurs a waste (m) increase, when $\bar{v} < O$ a dedrease is taking place.

2 *Dynamic models of slope systems*

2.1 *The dynamic model of a talus slope*

A well-known concepts of LEHMANN (1933) is fruitful from the point of constructing a dynamic model of the talus waste accumulation at the base of a bedrock slope. Actually, with the suggestions of this concept one may write down the following dynamic system:

(2.1–1)
$$dw(t)/dt = k\, v\, l(t)$$
$$dl(t)/dt = \alpha\, (w(t) - w_o)$$

where l(t) is the lenght of a bare bedrock slope that produces waste by weathering; w(t) is the talus waste volume at the random moment of time t; w is the maximum talus waste volume, corresponding to the cover of the entire bedrock slope by the talus (l = O); v is the bedrock slope's recession velocity in a direction normal to ik; k is the loosening coefficient which characterizes the waste's volume increase in comparison with the same waste's mass: α = const. The first equation of the system (2.1–1) is the balance ratio, postulasting that the amount of the waste removed from the bare bedrock slope equals the amount that is accumulated on the talus. The second equation states that the rate of shortening in the length of the bare bedrock

slope decreases with the talus volume's increase ($w_o > w$). It fulfills a feedback in the system bedrock slope-talus.

Having divided the first equation into the second one solved the obtained ordinary differential equations, we will have

(2.1–2) $w = w_o - (vk/\alpha)^{1/2}$

While solving it was assumed that $w = w_o$ when l = O. The coefficient α we find from the condition w = O when $l = l_o$ (l_o is the initial slope length): $\alpha = \frac{vkl_o^2}{w_o}$.

Putting the expression (2.1–2) into one of the system's equations (2.1–1), wefinally obtain

(2.1–3)
$$w(t) = w_o\left[1 - \exp\left(-\frac{k\, v\, l_o}{w_o}\, t\right)\right]$$

Thus, the talus waste's accumulation grows to Wo by the exponential law. From the ratios (2.1–3) it is possible to determine the time of a full cycle of a talus slope's development and the age of it.

2.2 *The dynamic model of gulling*

Assuming a slope to be developing while covered by a talus type (south arid zone) we obtain the waste's prime balance equation at a random point of a symmetrical gully in the form of

(2.2–1) $dw(t)/dt = 2k_p\, v\, l(t) - \lambda$,

where w(t) is the volume of waste, at the bottom of a gully per unit of its bed length, m^2; kp is the loosening coefficient; v is the normal rate of the gully slope's recession, m/yr; is the length of the retreating segment of the gully slope, m; λ is the rate of gully cleaning by a bed process, m^2/yr.

This equation is close to the first equation of the system (1.2–1). To enclose the equation (1.2–1) and construct the dynamic system of second order one should know the way of the dependence $\frac{dl}{dt} = f(w)$ when λ = const. Consider the geomorphological aspect of the equation (2.2–1).

It should be noted that a fall process goes more gradually with respect to time than a bed one. At the initial stage of a gully growth when it has a triangular cross profile form and a small length of a denudation segment l(t), bed processes proceed with time to clean a gully from the removed upslope's waste ($\lambda > 2\, k_p\, v\, l(t)$). There is a critical waste volume, which cannot be removed completely by one or several intensive (for the given conditions) floods $W_{crit} = 2k_p l_{crit} v$ hence $l_{crit} = \frac{W crit}{2k_p v}$.

Afterwards the gully has a tendency of ceasing its growth in depth; a process of progressive waste accumulation and widening of the bottom begins. The growing taluses at the gully's slopes reduce their correlative denudation segments, which deminishes the rate of waste supply to the gully; finally the taluses usually cover the steep slopes of the gullies completely. This is rather favoured by the cleaning velocity (λ) in the course of time due to a decrease of the catchment region at the top of the gully.

The processes of accumulation and waste removal into gullies of the arid zone are of a cyclic character. The process of more or less continuous falling and accumulation of talus waste into the gully bed is stopped by rapid cleaning when a micromudflow occurs. As a result the two components of the balance equation (2.2–1) have a different time lapse and are mutually exclusive. In fact, when the process of a contiuous falling and accumulation of waste is going on in the bed, the velocity of its cleaning may be considered to be zero (λ = O); when there occurs an intensive and momentary flood ($\lambda \neq$ O) the first number of the equation (2.2–1) may be neglected.

The estimate of the slope component is simpler in the equation (2.2–1). For this the data for average rates of denudation of gullied talus slopes are needed. With a concrete estimate of the slope component of the equation (2.2–1) appply it to the whole length of the gully (L), then the waste accumulation rate will be expressed by the value $2\ k_p\ vl\ L$, m^3/yr.

New gullies that are developed in the upper cretaceous marls of the right slope of the Gipsy gully (near the town of Bakhchesarai) and can be cleaned, have the following parameters (KLYUKIN & MOSKOVKIN 1979): Coef kp = 1,3; v = 5,4 mm/yr; l = 1–3 m, L = 50–200m; they therefore have the following rate interval of waste accumulation: 0,7–8,4m^3/yr. With the average frequency of micromudflows in this region, once per five years (KLYUKIN, RUDNITZKY, TOLSTUKH, KHARLAMOVA 1979), one obtains a waste accumulation volume at the interval 3–40m^3. This volume of waste accumulation in gullies corresponds to the volume of a possible single transport event by micromudflow for the considered region.

The above considered peculiarities of gully development are typical for the gullies developed in stable rocks, which corresponds to the third type of the slope process, i. e. falling, in the gullies of the southern arid zone according to the works of KOSOV YE. & LYUBIMOV B. P. (1979).

2.3 *Dynamic models based on the equilibrium concept of* F. AHNERT

Let us turn to the construction of dynamic slope models, namely, to the AHNERT equilibrium model as the basis (1967) and his comprehensive model of slope development (1973). This leads us to the dynamic model of slope development for the weathered waste's equilibrium at a definite point of the slope. According to AHNERT's equlibrium concept the change of the weathered waste at a point is fulfilled due to the waste which comes from the uplying points, that which is removed from this point and that which comes by weathering at this point. Further, the balance equations of the waste at the definite point led to the construction of the comprehensive model of the slope's development (AHNERT 1973). To realize this the program COSLOP-2 in the FORTRAN language was constructed. It consists of the main program, which serves as the subprograms' selection and references to them, recalculation of the slope's parameters, data print-out and 17 subprograms, which are divided into the following groups: three subroutines to set the initial slope profile; one for determining of weathering with an account of geological structure (two subroutines). The calculation of weathering can be carried out by quasimechanical, chemical and combined models; six programs for calculating the slope's baselevel change (stream incision) at aconstant, slowing and increasing rate; three subroutines of the slope processes: viscous-plastic waste flow according to SOUCHEZ's formula (1964) $R = v\ (\sin \alpha - \frac{a}{h})$, a simple wash by the formula $R = q \sin \alpha$ and compound wash by the ZINGG formula (1940) $R = k \sin (1 + d^m)$, where, as distinct from a simple one, the wash's intensity increase is taken into account down the slope (d); two subroutines, modelling a landslide mechanism of the left and right part of the slope respectively. In the above formulas h is the thickness of the waste's layer; v, d, q, k, m = const, m < 1. Modelling by a computer of the Univac-1108 type has shown that the development of the initially rectilinear slope under the influence of (1) viscousplastic flow leads to a convex profile; of (2) compound wash leads to a concave profile; of (3) the combination of the first two processes – to a convex-concave profile; of (4) a viscous plastic flow, with the account of a constant velocity of the slope's denudation basis lowering leads to a convex stable profile; of (5) a compound wash, with the account of a constant velocity of the slope's denudation – to a concave stable profile. The further development of the comprehensive model has led to the construction of its space analogue (AHNERT 1976) and the analysis of the slope's profile sensitiveness to slope parameter changes (MOON 1975). Our investigations are carried out in the way of the construction and solution of the closed equations describing the waste balance at the definite point of a slope (TROFIMOV & MOSKOVKIN 1979). The waste balance equation at this point is assumed as the basis

(2.3–1) $y_1 - y_2 + y_3 = h - h_o$

where y_1 is the waste amount coming from the higher-lying points; h_o is the initial thickness of the waeathered waste, y_1 (O) being equal y_2 (O) = y_1 (O) = O, h (O) = h. To describe the process of weathering take the G. CHANG hypothesis (1958) on the exponential decrease of the weathering velocity with the depth

(2.3–2) $dy_3/dt = B\exp(-\beta h)$ (AHNERT 1976)

For the summarized wash's rate take the ratio which is analogous to the R. SOUCHEZ formula (1964)

(2.3–3) $d(y_2 - y_1)/dt = A\sin\alpha = Ai$

where A = const, $\sin\alpha = i$ is the slope's indication in the considered point of the slope. Thus, one has the closed equations' system (2.3–1 ÷ 2.3–3). By differentiating the equation (2.3–1) in time with the account of (2.3–3) one obtains

(2.3–4) $-A_i + (dy_3/dt) = dh/dt$

By differentiating the equation (2.3–2) in time, having expressed dh/dt from it and putting it into the equation (2.3–4) we come to the following differential equation:

(2.3–5) $\frac{d^2y_3}{dt^2} + \beta\left(\frac{dy_3}{dt}\right)^2 - A_i\beta\frac{dy_3}{dt} = 0$

Having solved it relatively to dy_3/dt, we obtain

(2.3–6) $dy_3/dt = A_iC\exp(\beta A_it)/[1 + C\exp(\beta A_it)]$

From the initial condition $dy_3/dt_1|_{t=0} = \beta\exp(-\beta h_o)$, we find a constant of integration

(2.3–7) $C = B\exp(-\beta h_o)/[A_i - B\exp(-\beta h_o)]$

Putting (2.3–6) into (2.3–2) we obtain h

(2.3–8) $h = -\frac{1}{\beta}\ln\left[\frac{A_i C\exp(\beta A_i t)}{B(1 + C\exp(\beta A_i t))}\right]$

The maximum values for dy_3/dt and h are equal to:

(2.3–9) $\lim_{t\to\infty}(dy_3/dt) = A_i$

$\lim_{t\to\infty} h = -\frac{1}{\beta}\ln\left(\frac{A_i}{B}\right) = \bar{h} \geq 0$, (when $A_i \leq B$)

Thus, when the summarized wash (Ai) is less or equal to the maximum weathering intensity ($B = \max\{B\exp(-\beta h)\}$) the development goes in the way of reaching the dynamic equilibrium $y_1 - y_2 + y_3 = \bar{h} - h_o = const.$ In the other case ($A_i > B$) the weathered layer ($h < o$) does not exist, which corresponds to a bare bedrock slope. F. AHNERT (1967) and A. V. PEZDNYAKOV (1976) have pointed out the dynamic equilibrium to be stablized in the process of a slope's development between weathering and denudation.

If instead of the hypothesis (2.3–2) it is assumed that the rate of decreases linearly with the depth

(2.3–10) $dy_3/dt = k(h_{max} - h)$

then solving the equation system (2.3–1; 2.3–3; 2.3–10), we obtain

(2.3–11) $dy_3/dt = A_i - [A_i + k(h_o - h_{max})]\exp(-kt)$

from where $\lim(dy_3/dt) = A_i$; bearing in mind (2.3–10) we obtain

(2.3–12) $\lim_{t\to\infty} h = h_{max} - \frac{A_i}{k} = \bar{h}.$

From the expression (2.3–12) follows that the less the slope's inclination the more thickness of the waste layer is established (this is true for the previous model as well, see formula (2.3–9)).

When $A_i > kh_{max}$ a weathered layer does not exist.

When $A_i < kh_{max}$ the dynamic equilibrium is established $y_1 - y_2 + y_3 = \bar{h} - h_o =$ const. as well as in the previous model.

Considering the previous models we can draw the following conclusion: if the summarized wash does not exceed the maximum rate of weathering (which is obtained when $h = o$) the slope development goes in the way of obtaining the dynamic equlibrium at every point of the wash's region, in the transit region the ratio $h = const$ being always fulfilled. At the accumulation region the thickness h increases all the time. When no shift of the waste occurs, then the maximum equilibrium will be obtained ($\lim_{t\to\infty} h = const$).

Now analyse the case when the third system's equation (2.3–1–2.3–3) is written for the viscous waste flow (AHNERT 1977)

(2.3–13) $d(y_2 - y_1)/dt = R = Ah^r\sin\alpha$

where r = const. In this case the equations' system is reduced to one: –

(2.3–14) $Ah^r\sin\alpha + B\exp(-\beta h) = dh/dt$

For the stationary system's state, the solution of the process equation (when $dh/dt = 0$) is:

(2.3–15) $-Ah^r\sin\alpha + B\exp(-\beta h) = 0$

When $r > 0$ the equation (2.3–15) has a single solution $h = \bar{h}$. Let us show this solution to be stable. Let r = 1, then putting h in the form of $h = \bar{h} + h'$, where h' is the small value and putting it into the equation (2.3–14), ignoring the terms of which are higher than the infinitesimals of second order, we obtain

(2.3–16) $h' = C\exp[-(A\sin\alpha + \beta B\exp(-\beta\bar{h}))t]$

Because $A\sin\alpha + \beta B\exp(-\beta\bar{h}) > 0$, then $\lim_{t\to\infty} h' = 0$, which was to be proved.

If the initial thickness value ho is close to $\bar{h}$, we get the following solution for h:

(2.3–17) $h = (h_o - \bar{h})\exp[-(A\sin\alpha + \beta B\exp(-\beta\bar{h}))t] + \bar{h}$

When r = 3 (the analogue of a viscous flow by SOUCHEZ's formula) we obtain by the analogous way

$$(2.3\text{–}18)\quad h = (h_o - \bar{h}) \exp\left[-(3\bar{h}^2 A \sin\alpha + \beta B \exp(-\beta\bar{h}))t\right] + \bar{h}$$

The analogous result (stable solution) one gets as well with a random $r > O$. Thus, when $r > O$ the dynamic equilibrium is obtained at every point, i.e. the weathered thickness's value tends to the constant value $\bar{h}$.

We obtain analogous results when R (wash removal rate) is set according with the viscous-plastic flow model of AHNERT (1977): $R = \varkappa (h \sin\alpha - \varkappa_1)$ and when the weathering rate is set by the combined scheme (physical and chemical weathering), where the chemical rate is set by a continuous piece-like smooth function (AHNERT 1977).

$$(2.3\text{–}19)\quad \frac{dy_3}{dt} = w = w_o \left(1 + k_o \frac{h}{h_c} - \frac{h^2}{h_c^2}\right),\ h \leqslant h_c$$

$$\frac{dy_3}{dt} = w = w_o\, k_o \exp\left[-(h - h_c)\right],\ h > h_c$$

Note, unlike AHNERT's formula, we introduced the coefficient into the formula for physical weathering rate (2.3–2). This coefficient β characterizes the weathering rate's decrease with depth.

In the case, when the weathering rate is set by the formula (2.3–19) or by the combined scheme with the maximum function w different from w_o, $w > w_o$, there exist two possible stable states. For instance, stability will take place when $r = O$ and $w_o < A\sin\alpha < w_o k_o$, $k_o > 1$, for the straight line $R = A\sin\alpha$ in this case will cross the schem's function (2.3–19) at two points $\bar{h}_1$, $\bar{h}_2$, which will be the stationary states of the equation $-R + w = dh/dt$. The tendency to this or the other stable state will depend on the initial condition h_o. As $O < \bar{h}_1 < h_c$ and $\bar{h}_2 > h_c$, then when $O < h_o < h_c$, $\lim_{t\to\infty} h = h_1$, while $h_o > h_c$, $\lim_{t\to\infty} h = \bar{h}_2$.

At present mathematical methods for the stability analysis of dynamic systems with a small random perturbation have been worked out (VENTSELE & FRAIDLIN 1970). Here several trajectories that come out from a stable state beyond the region's boundary are estimated, and the character of this comes out as well. An application of this method to the analysis of the stability of ecosystems is beimg worked out by V. A. SVETLOSANOV (1976). It is applicable to the analysis of slope systems as well (TROFIMOV & MOSKOVKIN 1980b). Introduce into the right part of the equation (2.3–14) the small random perturbations $\varepsilon\, w_t$, where ε is the small parameter, w_t is the random VINEROV's process. Estimate the time of the system's change from the stable state $h = \bar{h}$ to the boundary state $h = o$, which corresponds to the degradation time of the initial state and its transition into an other system. For instance, a gentle slope, which has the weathered waste's layer ($h \neq o$) under the action of the perturbations' series is likely to transfer into an other slope type – the steep bedrock slope ($h = o$), the functioning of which will go in an other way. This is derived by (VENTSELE & FRAIDLIN 1970; SVETLOSANOV 1976)

$$(2.3\text{–}20)\quad M_x\, \tau_1^{\varepsilon} \sim \exp\left[4\, U(O)/2\varepsilon^2\right]$$

where U (h) is the potential of the right part of the equation (2.3–14), which is derived with the help of the integration of it by h. The conformity with our case we obtain:

$$(2.3\text{–}21)\quad M_x \tau_1^{\varepsilon} \sim \exp\left\{\left[\left(\frac{A\bar{h}^r}{B}\sin\alpha - 1\right)\left[-\frac{2B\bar{h}}{\ln(A\bar{h}^r \sin\alpha/B)}\right] + \frac{2A}{r+1}\bar{h}^{r+1}\sin\alpha\right] / \varepsilon^2\right\}$$

We estimate the numerator of this expression when $\bar{h} \to o$. After a number of simplifications, we finally obtain the following limit $\lim_{\bar{h}\to O} \frac{2B}{r}\bar{h} = O$, i.e. when the slope's system is in the stable state with a minimum depth of the weathered layer, a small lapse of time is needed to transform it in the state $h = o$ (system's degradation state). When $\bar{h} \to \infty$ the numerator of the expression (2.3–21) goes to infinity as well, the system with the great $\bar{h}$ being stable to the small random perturbations. In fact, with a large thickness of the weathered waste's layer it is unlikely to disappear quickly (the transform into the state $h = o$).

The other analysis's aspect is likely to be bound up with introducing the controlling factors into the dynamic systems. In this case there appears a possibility to solve the problems of optimum control by slope's systems (TROFIMOV & MOSKOVKIN 1978, 1980). Different anti-erosion, anti-abrasion and other measures – factores of man's activity – can serve as the control factors.

In conclusion, it should be said that the dynamic models allow to investigate the stationary-dynamic regimes of the slope's system for stability, to solve optimum control problems, to take into account stochastic factors and lagging factors and to determine the relaxation time in the slope's systems as well. The analysis of different concrete systems has shown the systems' development goes in the direction of establishing the dynamic equilibrium due to the presence of negative feedbacks, the influencing factor's intensity being constant.

Acknowledgements

The authors are indebted to Professor Dr. FRANK AHNERT (Aachen) for his critical reading of the manuscript.

References

AHNERT, F. (1967): The role of the equilibrium concept in the interpretation of landform of fluvial erosion and deposition. – L'evolution des Versants. Macar P. (Editor). Liege: 23–41.
– (1971): COSLOP-2. A comprehensive model program for simulating slope profile development. – Geocom. Programs, n. 8: 99–122.
– (1976): Trechmernaya teoreticheskaya model razvitija formi reljefa. – Mezchdungarodnaya geografiya. I. Geomorphologiya i paleogeographiya". M.: 233–236. [in Russian]
– (1977): Some comment on the quantitative formulation of geomorphological processes in a theoretical model. – Forth Surf. Processes, **2**, (2–3): 191–201.
ALEKSEYEV, V. V. (1976): Dinamicheskie modeli vodnikch biogeotzenosov. – Tchelovek i biosfera, **I**, M.: 3–137. [in Russian]
CHANG, JEN-HU (1958): Ground temperature Blue Hill. – Meteor. Labor. (Harvard Univ.). Milton Mass.: 1–107.
HUTCHINSON, J. N. (1975): The response of London clay cliffs to differing rates of toe erosion. – Build. Res. Establ. Car. Pap. 27: 15.
KLYUKIN, A. A. & MOSKOVKIN, V. M. (1979): Opredelenie absolutnogo vozrasta ovragov Predgornogo Krima po srenei scorosti otstupaniya krutikch sklonov. – Gemorphologiya, 3: 66–72 [in Russian]
KLYUKIN, A. A.; RUDNITSKII, G. V.,; TOLSTIKCH, E. A. & KCHARLAMOVA, J. B. (1979): Vivetrivanie mergelei v selevikch baseinakch Krima. – Vopposi inzchenernoi geodinamiki. **5**, Tashkent: 48–55 [in Russian]
KOSOV, B. F. & LYUBIMOV, B. P. (1979): Otsenka deformatsii ovrazchnich sklonov gravitatsionimi progtsesamimi dlya prognozirovaniya rosta ovragov. – Erosiya potchv ruslovimi protsesami, **7**, MGU; 90–100. [in Russian]
LEHMANN, O. (1933): Morphologische Theorie der Verwitterung von Steinschlagwänden. – Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich, **78**, 3–4: 88–122.
LOTKA, A. J. (1925): Elements of physical biology. – Williams and Wilkins, Baltimore: 460.
MOON, P. (1975): The sensitivity of AHNERT's slope development model to variation in slope forming parametres – S. Afr. Geogr. J., **57**, 2: 111–117.
POZDNYAKOV, A. A. (1976): Razvitie sklonov i nekotorie zakanomernosti formirovaniya reliefa. – Nauka, M., 112. [in Russian]
SHIROKOV, V. M.; MOSKOVKIN, V. M. & TROFIMOV, A. M. (1979): Teoreticheskie aspektie formirovanija beregov vodokchranilitch. – Prognozi pererabotki beregov vodohranich. **5**, Cherrasi: 94–96. [in Russian]
SIMONOV, YU. G. (1976): Modeli geografitcheskogo vzaimodeistvija dlja prognozirovanija evolutsii okruzchayntchei sredi. – Vestnik M. G. U. Ser. geogr., N 4: 3–9. [in Russian]
SMITH, J. (1976): Modeli v ekologii. – Mir, M.: 184. [in Russian]
SOUCHEZ, R. (1964): Viscosite, plasticite et rupture dans. – L'Evolution des versants. Ciel et Terre, **80**, 11–12: 389–410.
SVETLOSANOV, V. A. (1976): O stabilinosti ekosistem. – Vestnik MGU, Sergeogr., 4: 89–94. [in Russian]
TROFIMOV, A. M. & MOSKOVKIN, V. M. (1978): Klimatitcheskaja i obschegeografitcheskaja obuslovenost dinamicheskih modelei razvitija skonov. – Klimat, relief i dejatelnost chelveka, **1**, Kazan: 119–121. [in Russian]
– – (1979): K voprosu isledovanija matematicheskoi modeli balanza vivetrelogo material na sklone. – Matematiko-geograficheskoe modelirovanie, Kaznan: 39–41. [in Russian]
– – (1980): Theoretical preconditions of the control over the exogenic processes. – 24 IGC. Main session, **1**, Tokyo: 30–31.
– – (1980b): O dinamicheskom sostojanii prirodnih sistem. – Modelirovanie geograficheskih sistem Kazan: 23–27. [in Russian]
VENTSEL, A. D. & FRAINDLIN, M. J. (1970): O malih slutchainih vosmuschenijah dinamicheskih sistem. – UMN, t. 25, **1** (151): 3–55. [in Russian]
VOLTERRA, V (1917): Vanation and fluctuations of the number of individuals in animal species living together. – Animal Ecology. Mc. Graw. – Hill. New York: 109–448.
ZINGG, A. W. (1940): Degree and length of land slope as it affects soil in runoff. – Agr. Eng., **21** (40): 56–64.

Address of the authors: Dept. of Geography. Kazan State University. Lenin's street, 18. Kazan, USSR.