

A Film Model of Sound Propagation in Gas–Liquid Foams: 1. The Sound Velocity

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Abstract—A new (film) model is proposed for the propagation of sound vibrations in gas–liquid foams. The model explains the effect of sound retardation in foams and qualitatively agrees with the experimental data on the velocity of sound propagation in foams of various structures.

In the 1970s, scientists at the Novosibirsk Institute of Thermal Physics (Siberian Division, USSR Academy of Sciences) intensely studied the process of the propagation of weak perturbations in gas– and vapor–liquid mixtures. The results of those studies were summarized in a monograph [1]. The propagation of sound vibrations in such media was shown to be satisfactorily described by a homogeneous model, which represents a gas–liquid mixture as a continuous medium whose density and compressibility are determined by the values averaged over the whole system volume. In particular, the sound velocity c in a gas–liquid medium is determined by the relationship $c^2 = \frac{\partial P}{\partial \rho}$, known from the acoustics of homogeneous media. In the parameters characterizing the mixture, this equation is represented in the form

$$c^{-2} = \rho \left(\frac{\varphi}{\rho_2 c_2^2} + \frac{1-\varphi}{\rho_1 c_1^2} \right). \quad (1)$$

Here, ρ and φ are the density of the mixture and the volume gas content in it, respectively; ρ_1 and ρ_2 are the densities of the liquid and gas, respectively; c_1 and c_2 are the sound velocities in these media; and P is the pressure.¹

The $c(\varphi)$ dependence is plotted in Fig. 1. Experiments show that the measured velocity values at low gas contents display a good agreement with the left (descending) branch of this curve. The right (ascending) branch lies in the region of the gas contents $\varphi > 0.5$, which did not interest the authors of [1].

Gas–liquid mixtures with the gas content $\varphi > 0.75$ are foams, that is, structured gas–liquid systems where gas bubbles lose their individual mobilities. The interest to the acoustic properties of foams was evoked by their ability to effectively dampen shock (blast) waves. This is why the first acoustic studies of foams con-

cerned the propagation of shock waves in them, with a destruction of the foam structure (for example, see [2]). The propagation of weak perturbations in foams has received far less attention until recently.

In the “foam language,” dependence (1) looks as follows (the $\rho_2 c_2^2$ product is neglected in comparison with $\rho_1 c_1^2$):

$$c^2 = \frac{c_2^2 \rho_2 K^2}{(K-1)[\rho_2 K + (\rho_1 - \rho_2)]}, \quad (2)$$

where K is the foam expansion, that is, the value determining the volume fraction of the liquid in the foam and related to the gas content as $K = 1/(1 - \varphi)$.

Using the known relationship for gases

$$c_2^2 \rho_2 = \gamma P \quad (3)$$

and neglecting the gas density ρ_2 in the denominator in comparison with the liquid density ρ_1 , one can rewrite Eq. (2) in the form

$$c_2^2 = \frac{\gamma P K^2}{(\rho_1 + K \rho_2)(K-1)}, \quad (4)$$

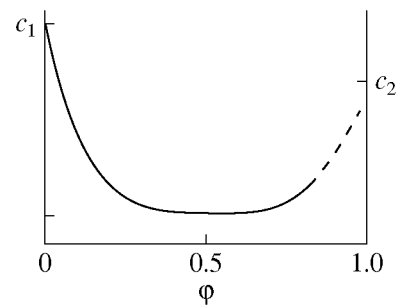


Fig. 1. The dependence of the sound velocity in gas–liquid emulsions on the gas content.

¹ Hereafter, the subscripts 1 and 2 denote parameters characterizing the liquid and the gas, respectively.

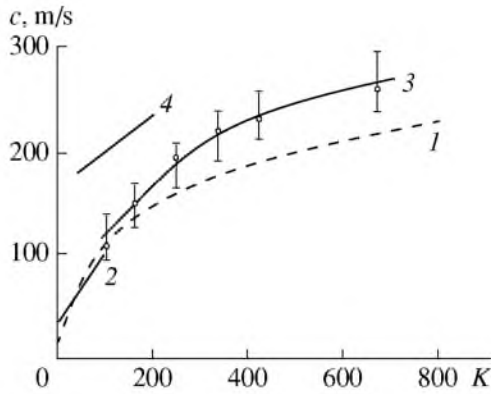


Fig. 2. The dependences of the sound velocity in foams on the expansion: (1) the homogeneous model [1], and (2–4) the experimental data of [3–5], respectively. The points on curve 3 are the values calculated according to the formulas of the film model.

where $\gamma = 1.4$ is the adiabatic constant for diatomic gases.

The dependence of the sound velocity in a foam on the expansion plotted in accordance with Eq. (4) is shown in Fig. 2 (curve 1). The same figure presents three experimental dependences obtained in [3–5] (curves 2, 3, and 4, respectively). It is seen that the measured values of the sound velocity in foams are very far from those predicted by the homogeneous model (1).

The authors of [6] tried to explain the significant difference between the experimental results of [5] and the theory while remaining within the framework of the homogeneous model. They assumed that the sound propagation process in foams is significantly affected by the motion of the liquid along the foam channels. Varying the “free” (in fact, fairly definite!) parameter in the formula for the water permeability of foams [7], the authors of [6] obtained another dependence, which passed through the experimental points. According to this model, the sound velocity in a foam must be characterized by a strong frequency dispersion.

However, comparing other experimental data in Fig. 2 with the theoretical curve 1, one has to suppose that the sound velocity in foams does not agree with the homogeneous model. The results closest to this dependence (but not coinciding with it!) are the ones we have obtained for low-expansion fine foams (curve 2). Assuming that the propagation of sound in foams is not an adiabatic process, we managed to make our measurements fit the theoretical curve 1. Even the polytropic constant was estimated ($n \approx 1.23$). Later, we understood that such an approximation is not justified, because (i) Eq. (2) was transformed into Eq. (4) using the rigorous relationship (3) for a diatomic gas (air), which does not allow a substitution of γ by n , and (ii) the propagation of sound in gases is a predominantly adiabatic process because of their low thermal

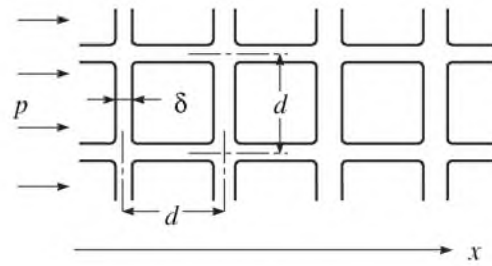


Fig. 3. The model of a “cubic” foam.

conductivity (only helium displays significant deviations from the adiabaticity).

Analyzing these (and other) experimental facts, one has to suppose that sound is propagated in foams otherwise than in gas–liquid emulsions: the very mechanism of the sound propagation is different.

In homogeneous media, the acoustic perturbation is transferred from one molecule to another, and the sound wave moves according to the molecular mechanism. In quasi-homogeneous gas–liquid mixtures, the presence of small noninteracting gas bubbles at large distances from each other makes the sound pulse split in two: the weak but fast “precursor” moves with the velocity of sound in the liquid, and the main energy flux is retarded by the pulsing gas bubbles [1].

In high-expansion foams, the main “structural element” is represented by two-sided liquid films. The sound wave in a foam is transferred from one film to another, that is, according to the film mechanism. Let us consider this mechanism using a simplified model of foam schematically shown in Fig. 3. Gas bubbles with a linear size d are separated by plane-parallel liquid films of thickness δ . Let a plane sound wave of pressure $p = p_0 \exp[-i(\omega t - k_2 x)]$ with the cyclic frequency ω and wavenumber $k_2 = 2\pi/\lambda_2$ advance on the foam along the X axis. The velocity of the gas particles in this wave is determined by the relationship

$$v = \frac{p}{\rho_2 c_2} = \frac{p}{\rho_2 c_2} e^{-i(\omega t - k_2 x)}, \quad (5)$$

where $\rho_2 c_2$ is the wave resistance of the medium (air) [8]. The thickness δ of the foam films is much smaller than the length λ_2 of the sound waves. Therefore, for waves in the acoustic range, a foam film is an obstacle in the form of a “localized mass” with the surface density $\mu = \rho_1 \delta$, loaded on the sound resistance $z_2 = \rho_2 c_2$ of the air behind the film. The sound resistance of the localized mass is $z_1 = -i\omega\mu$ [8], and the total sound resistance is

$$z = z_2 + z_1 = \rho_2 c_2 - i\omega\mu = r e^{-i\varepsilon},$$

where $r = \sqrt{\rho_2^2 c_2^2 + \omega^2 \mu^2}$, and $\varepsilon = \arctan \frac{\omega\mu}{\rho_2 c_2} \approx \frac{\omega\mu}{\rho_2 c_2}$.

The last of these equalities follows from the fact that

$\omega\mu \ll \rho_2 c_2$ for water–air foams in the acoustic frequency range. Therefore, one may assume $r \approx \rho_2 c_2$.

The velocity of the gas particles in the wave transmitted through the film is determined by the relationship

$$\begin{aligned} v_f &= \frac{p}{z} = \frac{p_0 e^{-i(\omega t - k_2 x)}}{r e^{-i\varepsilon}} = \frac{p_0}{r} e^{-i(\omega t - k_2 x - \varepsilon)} \\ &\approx \frac{p_0}{\rho_2 c_2} e^{-i(\omega t - k_2 x - \varepsilon)}. \end{aligned} \quad (6)$$

Comparing Eqs. (6) and (5), one can note

$$v_f = v e^{-i\varepsilon}, \quad (7)$$

that is, the wave transmitted through the foam is delayed by phase from comparison with the incident wave by the value of $\varepsilon \approx \omega\mu/(\rho_2 c_2)$. At a certain distance x , the sound wave will encounter x/d films, and the phase lag will reach $\varepsilon x/d$. Thus, the pressure wave in the foam may be represented as $p = p_0 e^{-i(\omega t - k_2 x - \varepsilon x/d)}$. On the other hand, neglecting the absorption, one can specify the same wave as $p = p_0 e^{-i(\omega t - k_f x)}$, where $k_f = 2\pi/\lambda_f$ is the wavenumber, and λ_f is the wavelength in the foam. Therefore, $k_f x = k_2 x + \varepsilon x/d$, or $k_f = k_2 + \varepsilon/d = 2\pi/\lambda_f$. Using this expression, we obtain

$$\lambda_f = \frac{2\pi/k_2}{1 + \varepsilon/(k_2 d)} = \frac{\lambda_2}{1 + \varepsilon/(k_2 d)},$$

and the sound velocity in the foam, which is equal to $c_f = \omega/k_f$, turns out to be related to the sound velocity in the air c_2 as

$$c_f = \frac{c_2}{1 + \varepsilon/(k_2 d)} \approx \frac{c_2}{1 + \mu/(\rho_2 d)} = \frac{c_2}{1 + \rho_1 \delta/(\rho_2 d)}. \quad (8)$$

This is just the expression for the sound velocity in the film model. Thus, the lower velocity of sound in a foam is mostly due to the retardation of the sound wave on the foam films because of their inertia rather than to the compressibility of the medium; this effect is virtually independent of the sound wave frequency. That is, according to the film model, the propagation of sound vibrations in foams takes place far from the frequency dispersion range. Note that the molecular mechanism is characterized by a significant frequency dispersion of the sound [6]. This fact represents a fundamental difference between the film and molecular mechanisms of sound propagation in foams.

The absence of a sound dispersion in the 100–1000 Hz frequency range was mentioned by the authors of [4]. Neither did we observe any frequency dependence of the sound velocity in a foam for pulse actions on the foam [3] or in experiments with monochromatic waves [9]. Thus, the experiments provide qualitative evidence in favor of the film mechanism of sound propagation in foams.

Now let us consider quantitative estimates. Dependence (8) shows that the sound velocity in a foam with a constant dispersity ($d = \text{const}$) is wholly determined only by the surface density μ of the films, or, to put it more precisely, only by their thickness δ . The problem is to find how the thickness of the foam films depends on the generalized structural characteristics of the foam: its expansion K and dispersity (average bubble size) d .

To analyze various processes in a foam, a polyhedral model of the foam structure is often used. According to this model, foam bubbles are pentagonal dodecahedra separated by plane-parallel liquid films. At three-film junctions (along the edges of the dodecahedra), the model suggests the existence of cylindrical channels with triangular cross sections; the concave walls of these channels have a radius of curvature r determined by the rarefaction in the liquid. This model yields the following equation for the foam expansion as a function of the structural microparameters (film thickness δ and radius r):

$$K = \frac{1.3d^2}{4.2d\delta + 1.6r^2}, \quad (9)$$

where d is the diameter of a sphere whose volume is equivalent to that of a foam bubble.

However, the polyhedral model does not enable one to estimate the film thickness. Therefore, the formula used in the calculations is just a modification of Eq. (9) where the fraction of the film-confined liquid is neglected (assuming $\delta = 0$).

Another model of the foam structure is the cellular model [10]. It makes allowance for the fact that a significant part of the liquid in low- and medium-expansion foams is contained in the structure nodes—at the junctions of the foam channels (in the polyhedral model, the volume of the nodes is regarded as negligible [11]). This model is closer to the structure of real foams, but it has the same drawback as the polyhedral one: it does not consider the volume of the liquid contained in the films, which are regarded as infinitely thin.

To estimate the fraction of the film-confined liquid, the authors of one of the most recent works on the foam structure [12] regarded the foam films as plane-parallel (as in the polyhedral model) and assumed their thickness to be equal to the experimental values obtained for free films [13].

Even Hooke and Newton were already interested in two-sided liquid films (soap bubbles). The first monograph on soap films was written at the end of the XIX century. As a rule, the objects of those studies were single (“free”) films, the factors of their stability, and the processes of their thinning and rupture. A wealth of both theoretical and experimental data on liquid films, including their thicknesses, has been accumulated by now. However, even those works do not answer the

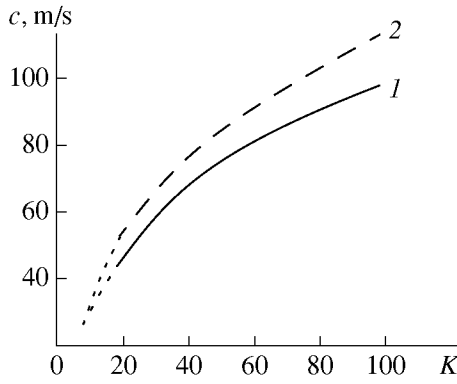


Fig. 4. (1) The expansion dependence of the sound velocity in a foam found in [3] in comparison with the (2) dependence calculated according to the homogeneous model.

question about the actual thickness of foam films. This is due to several reasons; the main two are as follows.

(1) Films in foams are not free; they represent only one of the elements of the complex foam architecture. Therefore, the models of foam films used in the studies are very far from their prototypes.

(2) The states of foam films are far from the conditions that are regarded as “equilibrium” in physical chemistry. In the process of their internal destruction (syneresis, growth of bubbles due to diffusion, etc.), foam films continuously change their thickness and area.

There have been virtually no studies of films that are incorporated into the foam structure. Thus, presently there are no clear ideas concerning the thickness (and shape) of films in foams of various structures. It is therefore impossible to obtain sufficiently rigorous theoretical dependences for comparing the relationships of the film model with the experiment. Nevertheless, measurements conducted by the authors of [4] enable us to evaluate the applicability of the film model for describing the propagation of sound in foam. The experiments in [4] were conducted for coarse ($d \sim 2$ mm) high-expansion ($K \leq 700$) foams, whose structure was close enough to polyhedral. According to our estimates, the area of films in such foams reaches 86% of the total interfacial area. Therefore, to estimate the film thicknesses in these experiments, one can use relationships from the polyhedral model of the foam structure as the first approximation. Assuming that all the liquid is confined to the foam films (that is, neglecting the liquid in the channels and thus assuming $r = 0$), formula (9) yields

$$K \approx \frac{1.3d}{4.2\delta} \text{ or } \delta \approx \frac{0.3d}{K}, \quad (10)$$

this relationship may be regarded as the upper limit of the film thickness in the polyhedral model of the foam structure.

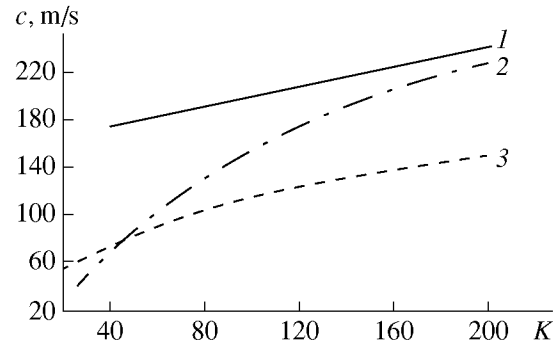


Fig. 5. The dependences of the sound velocity in a foam on the expansion: (1) experiment [5], (2) film model, and (3) homogeneous model.

Real foam is not a regular monodisperse structure; therefore, let us write the functional relationship (10) in the form

$$\delta = \frac{ad}{K}, \quad (11)$$

where a is an empirical coefficient whose value can be experimentally determined. Substituting Eq. (11) into formula (8) and comparing this dependence with the measurement results in [4], we obtained a value of $a = 0.23 \pm 0.02$. In Fig. 2, points obtained according to Eq. (8) with the δ values calculated according to Eq. (11) are superimposed on the experimental curve 3. They are not beyond the ranges of the measurement errors given by the authors of [4].

The foams studied in our work [3] lie in another range of structural parameters. Our measurements of the sound velocity were conducted for fine ($d \approx 0.22$ mm) low-expansion ($K = 18\text{--}100$) foams. Even if the foams were monodisperse, the area of the films in them barely reached 50% of the total interfacial area. According to our estimates, the polydispersity of our foams amounted to $K_{\min} \approx 12$ (see [7]). In foams with such a polydispersity and low expansion, films are just beginning to form, and the foam structure is still fairly close to spherical. The sound velocity in such foams cannot be very different from the values calculated according to the homogeneous model. This is just what our measurements show: the experimental curve 1 in Fig. 4 is fairly close to the one calculated according to the homogeneous model (4). They virtually converge at low expansions and rapidly diverge as the films are formed (with increasing expansion): the molecular mechanism is gradually substituted by the film mechanism.²

²The fact that our curve runs below and curve 3 in Fig. 2 (plotted according to the data of [4]) runs above the “homogeneous” curve is explained by the different dispersities of the foams. Velocities close to the sound velocity in gas are unattainable for fine foams because of a limitation on the thickness of liquid films—see [14] and formula (8). Therefore, any experimental curve will sooner or later cross the homogeneous model dependence.

The most difficult problem is to interpret the experimental results obtained by Moxon *et al.* in Australia [5] (curve 1 in Fig. 5). The structure of the foams used in that work is intermediate between the samples used in the previous two cases. Those foams consisted of fairly large bubbles ($d \sim 1.5$ mm) but had a comparatively low expansion ($K = 40\text{--}200$). Films in such foams are already distinct enough, but a significant amount of the liquid is still contained in nodes and channels of the foam structure. According to our estimates, the area of the films in these experiments amounted to 43–65% of the whole interfacial area; i.e., this is just the case where the foam structure is already far from spherical but still not polyhedral enough. The techniques used above (for the “extreme” structures) are inapplicable for such foams. For example, if we distribute all the liquid among the films, they become so thick that the sound velocity in the foam turns out to be even lower than according to the homogeneous model. With increasing expansion, the amount of the liquid in nodes decreases, and the fraction of the channel-confined liquid, which may be “excluded” from foams (according to relationships of the cellular model), increases. The dependence of the sound velocity on the expansion calculated for the experimental conditions of [5] is plotted in Fig. 5 (curve 2). The film thickness δ was found from the complete polyhedral relationship (10) by substituting the actual radii of curvature r calculated according to equations of the cellular model of the foam structure. Apparently, the sound velocity in a foam at an expansion of 40 is significantly lower than the experimental value and close to the velocity predicted by the homogeneous model (curve 3). However, the calculated film thicknesses approach real values with increasing expansion, and the calculated curve tends to the experimental one.

Thus, using the concepts of the film model, one can explain the experimental dependences of the sound velocity in foams of various structures from a unified standpoint. A quantitative verification of the film model is difficult at present because of the lack of theoretical approaches and experimental data on the thicknesses of foam films. An additional complication in this problem is that real foam films are not plane-parallel. The interfaces bounding the films have a fairly high curvature (which also varies over their area!) in low- and

medium-expansion foams. Therefore, formula (8) contains some efficient film thickness, which is difficult to estimate.

Arguments in favor of the proposed model of sound propagation in foams are not limited to the considerations provided in this paper. Using the film model, one can also explain the main specific features of sound absorption in foams. This topic is covered in the second part of this work.

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