Suppression of density effect in the polarization bremsstrahlung for relativistic charged particles crossing a thin target

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Abstract

The peculiarities of polarization bremsstrahlung emitted when a relativistic charged particle penetrates a thin layer of an amorphous medium are considered. In particular, suppression of the density effect, which otherwise limits the emission, is predicted. The physical nature of this suppression is analogous to that taking place in ionization energy losses of a relativistic particle crossing a thin target.

Keywords: Relativistic electron; Density effect; Polarization bremsstrahlung

1. Introduction

When a fast, charged particle moves through a medium, polarization bremsstrahlung occurs due to the excitation of electrons in the medium by the electromagnetic field of the fast particle [1]. In the case of a condensed medium, the projectile field is screened due to polarization of the medium. This effect (the density effect) causes moderation of relativistic particle ionization energy losses (Fermi effect [2]). An analogous effect has been predicted for the polarization bremsstrahlung of relativistic particles moving through an unbounded medium [3,4].

It is well known that the Fermi effect is suppressed in the case of a relativistic particle crossing a sufficiently thin target [5]. The physical reason for such suppression is very simple. The particle's field in vacuum in front of the target is transformed to the screened field inside the target over the emission formation length $l_{coh} \approx \gamma^2/\omega$ [6] ($\gamma = (1 - v^2)^{-1/2}$, v is the particle's velocity, $c = 1$). Therefore, the structure of the particle field does not change essentially in the frequency range where the inequality $l_{coh} \gg L$ is valid (L is the thickness of the target).

One can expect suppression of the density effect to occur as well in polarization bremsstrahlung of a relativistic particle crossing a thin layer of medium. This work is devoted to the detailed analysis

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of this phenomenon. It is shown that the emission is formed in the case under consideration by the scattering of both virtual photons associated with the fast particle and real transition radiation photons emitted from the *in*-surface of the target. Detailed analysis shows that the contribution of transition radiation completely compensates the suppression of ordinary polarization bremsstrahlung caused by the density effect.

It should be noted that the role of the density effect in atomic K-shell excitation by relativistic electrons crossing a thin layer of condensed medium has been studied in detail in [7]. Since the nature of the effects considered in [7] and our paper is the same and consists in the preservation of the structure of a fast particle's vacuum field in a sufficiently thin target, the results obtained in these papers are similar to each other.

2. General expressions

Let us consider the structure of the electromagnetic field emitted by a relativistic particle moving with the constant velocity v along the axis $\boldsymbol{e}_x$, which is normal to a plate of amorphous medium with thickness L. To find the Fourier transform of the electric field

$$\boldsymbol{E}_{\omega\boldsymbol{k}} = (2\pi)^{-4} \int \mathrm{d}t\,\mathrm{d}^3r\,\boldsymbol{E}(\boldsymbol{r}, t)\exp(\mathrm{i}\omega t - \mathrm{i}\boldsymbol{k}\boldsymbol{r})$$

by means of the Maxwell equations, it is necessary to determine the induced current density for the medium electrons. We use the following simple expression in this paper:

$$\boldsymbol{j} = -\frac{\mathrm{e}^2}{m}\boldsymbol{A}(\boldsymbol{r}, t)\hat{n}, \qquad \hat{n} = \sum_{\alpha}\sum_{\beta=1}^{Z}\delta(\boldsymbol{r} - \boldsymbol{r}_\alpha - \boldsymbol{r}_{\alpha\beta}), \tag{1}$$

which is generally accepted within the framework of X-ray scattering theory [8]. The formula (1) is valid in the frequency range $I \ll \omega \ll m$ (I is the mean ionization potential of an atom, and m is the electron mass). These relations allow us to consider atomic electrons as free ones during the emission process and to neglect the Compton shift of the frequency of emitted photons. It is very important that the electron coordinates $\boldsymbol{r}_{\alpha\beta}$ are approximately constant during the process of relativistic particle interaction with an atom. In formula (1), $\boldsymbol{A}$ is the electromagnetic vector potential and α is an index for individual atoms, while β is the index for electrons in a given atom of atomic number Z.

Substitution of expressions (1) into the Maxwell equations permits us to obtain the following equation for the transverse component of the electrical field $\boldsymbol{E}^{\mathrm{tr}}_{\omega\boldsymbol{k}} = \sum_{\lambda=1}^{2}\boldsymbol{e}_{\lambda\boldsymbol{k}}E_{\lambda\boldsymbol{k}}$:

$$\begin{aligned}(k^2 - \omega^2)E_{\lambda\boldsymbol{k}} &+ \int \mathrm{d}^3k' \sum_{\lambda'=1}^{2} \boldsymbol{e}_{\lambda\boldsymbol{k}}\boldsymbol{e}_{\lambda'\boldsymbol{k}'}G(\boldsymbol{k}' - \boldsymbol{k})E_{\lambda'\boldsymbol{k}'} \\ &= \frac{\mathrm{i}\omega\mathrm{e}}{2\pi^2}\boldsymbol{e}_x\boldsymbol{e}_{\lambda\boldsymbol{k}}\delta\left(k_x - \frac{\omega}{v}\right), \\ G &= \frac{\mathrm{e}^2}{2\pi^2 m}\sum_{\alpha}\sum_{\beta=1}^{Z}\exp[\mathrm{i}(\boldsymbol{k}' - \boldsymbol{k})(\boldsymbol{r}_\alpha + \boldsymbol{r}_{\alpha\beta})],\end{aligned} \tag{2}$$

where $\boldsymbol{e}_{\lambda\boldsymbol{k}}$ is the polarization vector, $\boldsymbol{k}\boldsymbol{e}_{\lambda\boldsymbol{k}} = 0$.

The function $G(\boldsymbol{k}' - \boldsymbol{k})$ describes both reflecting and scattering properties of the target. Considering the polarization bremsstrahlung as being due to the scattering of the total electromagnetic field associated with a penetrating fast particle consisting of the primary projectile field and the generated transition radiation field on the fluctuations of the target electron density, one may separate the average and random quantities in Eq. (2),

$$\begin{aligned}E_{\lambda\boldsymbol{k}} &\equiv \bar{E}_{\lambda\boldsymbol{k}} + \tilde{E}_{\lambda\boldsymbol{k}}, \\ G(\boldsymbol{k}' - \boldsymbol{k}) &\equiv \bar{G}(\boldsymbol{k}' - \boldsymbol{k}) + \tilde{G}(\boldsymbol{k}' - \boldsymbol{k}), \\ \bar{G} = \langle G\rangle &= \frac{4\mathrm{e}^2 n_0}{m}F(\boldsymbol{k}' - \boldsymbol{k})\delta(\boldsymbol{k}'_\parallel - \boldsymbol{k}_\parallel) \\ &\quad\times \frac{\sin(k'_x - k_x)L/2}{k'_x - k_x}.\end{aligned} \tag{3}$$

Here, the brackets $\langle\cdot\rangle$ mean averaging over the coordinates $\boldsymbol{r}_\alpha$ and $\boldsymbol{r}_{\alpha\beta}$, n_0 is the atomic density of the medium , $F(\boldsymbol{k}' - \boldsymbol{k})$ is the atomic formfactor, and $\boldsymbol{k}_\parallel$ is the $\boldsymbol{k}$ component parallel to the target surface. The equations for $\bar{E}_{\lambda\boldsymbol{k}}$ and $\tilde{E}_{\lambda\boldsymbol{k}}$ are

$$(k^2-\omega^2)\bar{E}_{\lambda k}+\int \mathrm{d}^3k'\sum_{\lambda'=1}^{2}\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{\lambda' k'}(\bar{G}(\boldsymbol{k}'-\boldsymbol{k})\bar{E}_{\lambda' k'} + \langle\tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\tilde{E}_{\lambda' k'}\rangle)=\frac{\mathrm{i}\omega\,\mathrm{e}}{2\pi^2}\boldsymbol{e}_x\boldsymbol{e}_{\lambda k}\delta\Big(k_x-\frac{\omega}{v}\Big),$$

$$(k^2-\omega^2)\tilde{E}_{\lambda k}+\int \mathrm{d}^3k'\sum_{\lambda'=1}^{2}\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{\lambda' k'}(\bar{G}(\boldsymbol{k}'-\boldsymbol{k})\tilde{E}_{\lambda' k'} + \tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\bar{E}_{\lambda' k'}+\tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\tilde{E}_{\lambda' k'} - \langle\tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\tilde{E}_{\lambda' k'}\rangle)=0. \tag{4}$$

X-ray scattering in an amorphous medium is a weak process ($\tilde{G}\ll G$). Therefore, the random component of the field $\tilde{E}_{\lambda k}$ is smaller than the average component in the case under consideration, when the thickness of the target is smaller than X-ray extinction length. This circumstance allows us to neglect the "nonlinear" terms in (4) proportional to $\tilde{G}\tilde{E}_{\lambda k}$.

Considering Eqs. (4) separately inside and outside the target, one should take into account the inequality $k_xL\approx\omega L\gg 1$ that is practically valid in the X-ray region. This permits simplification of the coefficient $(\sin(k'_x-k_x)L/2)/(k'_x-k_x)$ to $\pi\delta(k'_x-k_x)$ in formula (3). The final equation for $\bar{E}_{\lambda k}$ inside the target is

$$(k^2-\omega^2\varepsilon(\omega))\bar{E}_{\lambda k}=\frac{\mathrm{i}\omega\,\mathrm{e}}{2\pi^2}\boldsymbol{e}_x\boldsymbol{e}_{\lambda k}\delta(k_x-\omega/v), \tag{5}$$

where $\varepsilon(\omega)=1-\omega_0^2/\omega^2$ and $\omega_0=(4\pi Zn_0\mathrm{e}^2/m)^{1/2}$ is the plasma frequency of the medium.

It is easy to see that Eq. (5) and the analogous equation for the field $\bar{E}_{\lambda k}$ outside the target following from (5) in the limit $\omega_0\to 0$ coincide with the ordinary equations of X-ray transition radiation theory [6]. The well-known solution for $\bar{E}_{\lambda k}$ inside the target takes the form

$$\bar{E}_{\lambda k}=\frac{\mathrm{i}\omega\,\mathrm{e}}{2\pi^2}\boldsymbol{e}_x\boldsymbol{e}_{\lambda k}\Bigg[\frac{1}{k^2-\omega^2\varepsilon(\omega)}\delta(k_x-\omega/v) + \left(\frac{1}{k^2-\omega^2}-\frac{1}{k^2-\omega^2\varepsilon(\omega)}\right)\times\exp\left(\mathrm{i}\frac{L}{2}\left(\sqrt{\omega^2\varepsilon(\omega)-k_\parallel^2}-\frac{\omega}{v}\right)\right)\delta\Big(k_x-\sqrt{\omega^2\varepsilon(\omega)-k_\parallel^2}\Big)\Bigg], \tag{6}$$

where $\boldsymbol{k}=\boldsymbol{k}_\parallel+\boldsymbol{e}_x\omega/v$, with $\boldsymbol{e}_x\boldsymbol{k}_\parallel=0$.

The equation for the polarization bremsstrahlung field $\tilde{E}_{\lambda k}$ following from (4) is given by

$$(k^2-\omega^2\varepsilon(\omega))\tilde{E}_{\lambda k}=-\int \mathrm{d}^3k'\sum_{\lambda'=1}^{2}\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{\lambda' k'}\tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\bar{E}_{\lambda' k'}, \tag{7}$$

inside the target, and

$$(k^2-\omega^2)\tilde{E}_{\lambda k}=0, \tag{8}$$

outside the target.

The expression (6) for $\bar{E}_{\lambda k}$ is used in formula (7). It is important to note that the exact expression for $\tilde{G}\equiv G-\bar{G}$ must be used in (7) in order to avoid singularities in final results.

3. Spectral–angular distribution of the polarization bremsstrahlung

Eqs. (6)–(8) allow the calculation of all characteristics of the polarization bremsstrahlung in the wave zone behind the target. Using the ordinary solutions of Eqs. (7) and (8) and the well known boundary conditions for electromagnetic field on the *out*-surface of the target, one can obtain the following expression for $\tilde{E}_{\lambda k}$ in vacuum behind the target:

$$\tilde{E}_{\lambda k}=\frac{\mathrm{i}\omega\,\mathrm{e}}{2\pi^2}a_{\lambda k_\parallel}\delta\Big(k_x-\sqrt{\omega^2-k_\parallel^2}\Big),$$

$$a_{\lambda k_\parallel}=-\exp\left(-\mathrm{i}\sqrt{\omega^2-k_\parallel^2}\frac{L}{2}\right)\int\frac{\mathrm{d}k_x}{k^2-\omega^2\varepsilon(\omega)}\times\exp\left(\mathrm{i}k_x\frac{L}{2}\right)\int \mathrm{d}^3k'(\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{2k'})(\boldsymbol{e}_x\boldsymbol{e}_{\lambda k'})\times\tilde{G}(\boldsymbol{k}'-\boldsymbol{k})\Bigg[\frac{1}{k'^2-\omega^2\varepsilon(\omega)}\delta(k'_x-\omega/v) + \left(\frac{1}{k'^2-\omega^2}-\frac{1}{k'^2-\omega^2\varepsilon(\omega)}\right)\times\exp\left(\mathrm{i}\left(\sqrt{\omega^2\varepsilon(\omega)-k'_\parallel 2}-\frac{\omega}{v}\right)\frac{L}{2}\right)\times\delta\Big(k'_x-\sqrt{\omega^2\varepsilon(\omega)-k'_\parallel 2}\Big)\Bigg]. \tag{9}$$

Here, $e_{1k} = k_{\parallel} + e_x/|k_{\parallel} + e_x|$ and $e_{2k} = k + e_{1k}/|k + e_{1k}|$. To obtain the emission amplitude A_n, it is necessary to calculate the Fourier integral

$$E_\lambda^{\mathrm{Rad}} = \int \mathrm{d}^3k\, \tilde{E}_{\lambda k}\, \mathrm{e}^{\mathrm{i}knr}, \tag{10}$$

where n is the unit vector along the direction of the emitted photon. The integral (10) for the field E_λ^{Rad} in the wave zone can be calculated by the stationary phase method. The result of the integration is as follows:

$$E_\lambda^{\mathrm{Rad}} = A_\lambda \frac{\mathrm{e}^{\mathrm{i}\omega r}}{r}, \quad A_\lambda = -2\pi \mathrm{i}\omega n_x a_{\lambda\omega n_{\parallel}}, \tag{11}$$

where $n = n_{\parallel} + e_x n_x$ with $e_x n_{\parallel} = 0$.

Using expressions (9) and (11) for the emission amplitude A_n, one can obtain the following final formula for the polarization bremsstrahlung spectral–angular distribution:

$$\begin{aligned} \omega \frac{\mathrm{d}N_\lambda}{\mathrm{d}\omega\,\mathrm{d}\Omega} &= \langle |A_\lambda|^2 \rangle \\ &= \mathrm{e}^2\omega^2 \left\langle \left| \int \mathrm{d}^3 g \frac{|k_{\parallel} + g_{\parallel}|}{|k + g|} e_{\lambda k} e_{2k+g} \tilde{G}(g) \right. \right. \\ &\quad \times \left[A\delta\left(g_x - \omega/v + \omega\sqrt{n_x^2 - \omega_0^2/\omega^2} \right) \right. \\ &\quad + B \cdot \delta\left(g_x - \sqrt{\omega^2\varepsilon(\omega) - (k_{\parallel} + g_{\parallel})^2} \right. \\ &\quad \left. \left. \left. \left. + \omega\sqrt{n_x^2 - \omega_0^2/\omega^2} \right) \right] \right|^2 \right\rangle, \end{aligned}$$

$$A = \frac{1}{\omega^2/v^2\gamma^2 + \omega_0^2 + (k_{\parallel} + g_{\parallel})^2},$$

$$\begin{aligned} B = &\left(\frac{1}{\omega^2/v^2\gamma^2 + (k_{\parallel} + g_{\parallel})^2} \right. \\ &\left. - \frac{1}{\omega^2/v^2\gamma^2 + \omega_0^2 + (k_{\parallel} + g_{\parallel})^2} \right) \\ &\times \exp\left(\mathrm{i}\left(\sqrt{\omega^2\varepsilon(\omega) - (k_{\parallel} + g_{\parallel})^2} - \omega/v \right) L/2 \right), \end{aligned} \tag{12}$$

where

$$k = \omega\left(n_{\parallel} + e_x\sqrt{n_x^2 - \omega_0^2/\omega^2} \right).$$

This formula shows that the polarization bremsstrahlung yield is formed by the scattering of two different fields. The term in square brackets in (12), proportional to the coefficient A, describes the amplitude of the ordinary polarization bremsstrahlung arising due to the scattering of the equilibrium field of the primary particle moving in the medium with dielectric permeability $\varepsilon(\omega) = 1 - \omega_0^2/\omega^2$. The second term, proportional to the coefficient B, corresponds to the contribution of the scattering of transition radiation field emitted from the *in*-surface of the target. The aim of the further analysis is to study the separate contributions of these emission mechanisms to the total polarization bremsstrahlung yield as well as the interference between them.

4. Discussion

To study the general result (12), let us rewrite it in the form

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \omega \frac{\mathrm{d}N^{\mathrm{PB}}}{\mathrm{d}\omega\,\mathrm{d}\Omega} + \omega \frac{\mathrm{d}N^{\mathrm{STR}}}{\mathrm{d}\omega\,\mathrm{d}\Omega} + \omega \frac{\mathrm{d}N^{\mathrm{INT}}}{\mathrm{d}\omega\,\mathrm{d}\Omega}, \tag{13}$$

where the first term corresponds to the ordinary polarization bremsstrahlung, the second describes the scattered transition radiation and the last term contains interference.

Taking into account the anisotropic character of the polarization bremsstrahlung spectral–angular distribution [4], let us define a two-dimensional angular variable Θ in accordance with the formula

$$n = e_x\left(1 - \frac{1}{2}\Theta^2\right) + \Theta, \qquad e_x\Theta = 0, \quad \Theta \ll 1. \tag{14}$$

Substituting (14) into (12) and (13) and writing $g_{\parallel} = \omega x$, one can obtain the following expression for the ordinary polarization bremsstrahlung distribution:

$$\omega \frac{dN_\lambda^{PB}}{d\omega d^2\Omega} = e^2\omega^2 \int d^2x d^2x' |\boldsymbol{\Theta}+\boldsymbol{x}||\boldsymbol{\Theta}+\boldsymbol{x}'| \times (\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{\lambda k+g})(\boldsymbol{e}_{\lambda k}\boldsymbol{e}_{2k+g'}) \times \frac{\langle \tilde{G}(\boldsymbol{g})\tilde{G}^*(\boldsymbol{g}')\rangle}{(\gamma^{-2}+\omega_0^2/\omega^2+(\boldsymbol{\Theta}+\boldsymbol{x})^2)(\gamma^{-2}+\omega_0^2/\omega^2+(\boldsymbol{\Theta}+\boldsymbol{x}')^2)}, \tag{15}$$

where

$$\boldsymbol{g} = \omega\left(\boldsymbol{x}+\boldsymbol{e}_x\frac{1}{2}\left(\gamma^{-2}+\frac{\omega_0^2}{\omega^2}+\Theta^2\right)\right),$$
$$\boldsymbol{k} = \omega\left(\boldsymbol{\Theta}+\boldsymbol{e}_x\left(1-\frac{1}{2}\left(\frac{\omega_0^2}{\omega^2}+\Theta^2\right)\right)\right).$$

After averaging over atomic coordinates $\boldsymbol{r}_\alpha$ (they are assumed to be independent; see the formulae (2) and (3)) and electronic coordinates $\boldsymbol{r}_{\alpha\beta}$ (we use the Fermi–Thomas atom model with exponential screening), the expression for the correlator $\langle \tilde{G}(\boldsymbol{g})\tilde{G}^*(\boldsymbol{g}')\rangle$ is given by

$$\langle \tilde{G}(\boldsymbol{g})\tilde{G}^*(\boldsymbol{g}')\rangle = \frac{Z^2e^4n_0}{\pi^2m^2}\frac{1}{(1+g^2R^2)(1+g'^2R^2)} \times \frac{2\sin(g'_x-g_x)L/2}{g'_x-g_x}\delta(\boldsymbol{g}'_\parallel-\boldsymbol{g}_\parallel), \tag{16}$$

where n_0 is the density of atoms, R the screening radius, and m is the electron mass. The incoherent contribution of atomic electrons to the polarization bremsstrahlung is neglected in (16) (see [4]).

Substitution of expression (16) into formula (15) and summation over emitted photon polarizations allow us to obtain the very simple formula

$$\omega\frac{dN^{PB}}{d\omega d^2\Omega} = \frac{Z^2e^6n_0L}{\pi m^2}\left[\ln\left(\frac{1}{\omega_0^2R^2(1+\omega^2/\gamma^2\omega_0^2)}\right)-2\right], \tag{17}$$

which is valid within the range of emitted photon frequencies ω and observation angles Θ satisfying the requirement

$$\omega^2R^2\Theta^2 \ll 1. \tag{18}$$

The result (17) coincides with that obtained in the previous work [4] (on condition (18)) devoted to the study of a relativistic particle polarization bremsstrahlung in an unbounded medium.

In accordance with (17), the emission yield saturates in the frequency range $\omega \ll \gamma\omega_0$ because of the polarization of medium electrons.

We should remember that the observation angle Θ must be much larger than the characteristic emission angle for relativistic particles $\Theta_{em} \approx \sqrt{\gamma^{-2}+\omega_0^2/\omega^2}$ in order to separate the polarization bremsstrahlung from the more intensive transition radiation and ordinary bremsstrahlung. On the other hand, within the experimentally most interesting frequency range $\omega \leqslant \gamma\omega_0$, where the discussed density effect takes place, the condition (18) reduces to the limitation $\Theta^2 \ll 1/\gamma^2\omega_0^2R^2$. Therefore, the range of permissible observation angles is determined by the inequalities

$$\gamma^{-2}+\frac{\omega_0^2}{\omega^2} \ll \Theta^2 \ll \frac{1}{\gamma^2\omega_0^2R^2} \ll 1. \tag{19}$$

The last inequality shows that an experiment requires electron beams with sufficiently high particle energy, of the order of a hundred MeV.

The influence of density effect on the polarization bremsstrahlung spectrum is illustrated in Fig. 1.

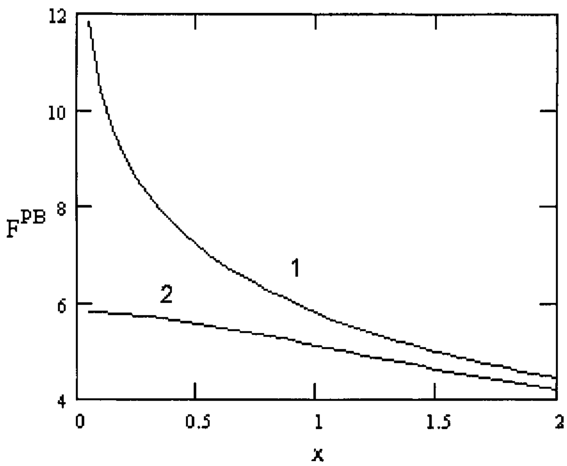

Fig. 1. The influence of density effect on ordinary polarization bremsstrahlung in an infinite medium ($x=\omega/\gamma\omega_0$). $\omega dN^{PB}/d\omega d^2\Theta = N_0F^{PB}$, $N_0 = Z^2e^6n_0L/\pi m^2$, $\omega_0R = 5\times10^{-3}$. 1: without taking the density effect into account; 2: taking into account the density effect.

Let us consider the second term in the formula (13) describing the scattered transition radiation. Calculations analogous to those done above allow us to obtain, on the same conditions, the following expression for an emission spectral–angular density:

$$\omega \frac{\mathrm{d}N^{\mathrm{STR}}}{\mathrm{d}\omega\,\mathrm{d}^2\Omega} = \frac{Z^2 \mathrm{e}^6 n_0 L}{\pi m^2}\left[\left(1+2\frac{\omega^2}{\gamma^2\omega_0^2}\right)\ln\left(1+\frac{\gamma^2\omega_0^2}{\omega^2}\right)-2\right]. \tag{20}$$

In contrast to the wide spectrum (17) formed by the scattering of the spectrally wide projectile field, the spectrum (20) is concentrated within the narrow frequency range $\omega \leqslant \gamma\omega_0$ corresponding to the transition radiation spectrum. The shape of the spectrum (20) is shown in Fig. 2.

In accordance with (20), the scattered transition radiation yield is proportional to the target thickness L. This result is the consequence of the applied perturbative approach to the solution of the general equation (4). Therefore, the value of L must be nominally much smaller than the distance over which the transition radiation wave changes substantially due to scattering.

It should be pointed out that, in the frequency range $\omega \ll \gamma\omega_0$, where the ordinary polarization bremsstrahlung saturates due to the density effect, the sum of the distributions (17) and (20) is given by the formula

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega\,\mathrm{d}^2\Omega} \approx \frac{Z^2 \mathrm{e}^6 n_0 L}{\pi m^2}\ln\left(\frac{\gamma^2}{\omega^2 R^2}\right), \tag{21}$$

which coincides with the distribution (17) for the ordinary polarization bremsstrahlung without taking into account the density effect.

In accordance with the result (21) (the main result of this work), the density effect does not appear in the process of polarization bremsstrahlung caused by a relativistic charged particle penetrating a thin target. To explain this result, it should be noted that, in the frequency range $\omega \ll \gamma\omega_0$, the screened field of the penetrating particle is suppressed (see coefficient A in the general formula (12)). On the other hand, the transition radiation field becomes similar to the vacuum field (see coefficient B in formula (12)). Therefore, the main contribution to total emission yield determined by the scattered transition radiation becomes comparable to that due to the scattering of the vacuum field of the fast particle.

The formula (21) is valid in the case where the transition radiation field does not change essentially on the target thickness L. Since scattering in an amorphous medium is a weak process, the main process changing the transition radiation wave is a photoabsorption. Therefore, the considered phenomenon may be observed under the condition

$$L \ll l_{\mathrm{ab}}, \tag{22}$$

where l_{ab} is the photoabsorption length.

Let us consider the last term in the general formula (13) describing the interference effect. After doing calculations similar to those done above, we come to

$$\omega \frac{\mathrm{d}N^{\mathrm{INT}}}{\mathrm{d}\omega\,\mathrm{d}^2\Omega} = \frac{Z^2 \mathrm{e}^6 n_0 L}{\pi m^2}\left[\left(1+2\frac{\omega^2}{\gamma^2\omega_0^2}\right)\frac{\sin\sigma(1+\omega_0^2\gamma^2/\omega^2)}{\sigma(1+\omega_0^2\gamma^2/\omega^2)} + \cos\sigma(1+\omega_0^2\gamma^2/\omega^2) + \left(\sigma(1+\omega_0^2\gamma^2/\omega^2) - \frac{2}{\sigma}\frac{\omega^4}{\gamma^4\omega_0^4}\right) si(\sigma(1+\omega_0^2\gamma^2/\omega^2))\right.$$

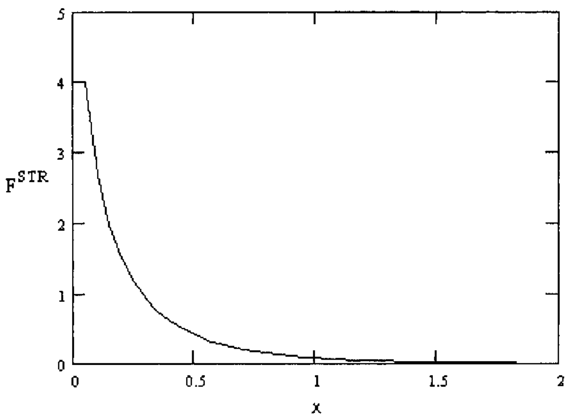

Fig. 2. The spectral density of scattered transition radiation ($x = \omega/\gamma\omega_0$). $\omega\,\mathrm{d}N^{\mathrm{STR}}/\mathrm{d}\omega\,\mathrm{d}^2\Omega = N_0 F^{\mathrm{STR}}$.

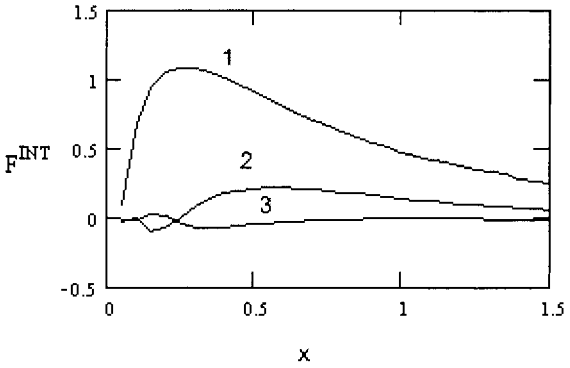

Fig. 3. The contribution of the interference item to polarization bremsstrahlung spectral density ($x = \omega/\gamma\omega_0$). $\omega \mathrm{d}N^{\mathrm{INT}}/\mathrm{d}\omega \mathrm{d}^2\Omega = N_0 F^{\mathrm{INT}}$. 1: $y = \omega_0 L/2\gamma = 0.1$; 2: $y = 0.5$; 3: $y = 1$.

$$
- 2\frac{\omega^2}{\gamma^2\omega_0^2} ci(\sigma(1 + \omega_0^2\gamma^2/\omega^2))
+ \frac{2}{\sigma}\frac{\omega^4}{\gamma^4\omega_0^4}(\cos\sigma\omega_0^2\gamma^2/\omega^2 si(\sigma)
+ \sin\sigma\omega_0^2\gamma^2/\omega^2 ci(\sigma))\Bigg], \quad (23)
$$

where $\sigma = \omega L/2\gamma^2$. The phase deviation and the difference between phase velocities of virtual photons of the particle field and real photons of the transition radiation stipulate the oscillations in the spectrum (23) depending on the target thickness L. The interference effect contribution is illustrated in Fig. 3. It is easy to see that the relative contribution of this effect is not large.

In accordance with Fig. 3, the influence of the interference effect decreases when increasing the thickness of the target because of increase of the phase shift between the different components of the total emission amplitude (see (12)), that is, the contribution of the transition radiation field and the particle's field to the formation of the polarization bremsstrahlung yield. That is why the compensation of the reduction of the polarization bremsstrahlung yield due to the density effect decreases when increasing the thickness L, as follows from Fig. 4, where the spectrum of the total emission calculated by (13) is compared with the spectrum of ordinary polarization bremsstrahlung without taking into account the density effect.

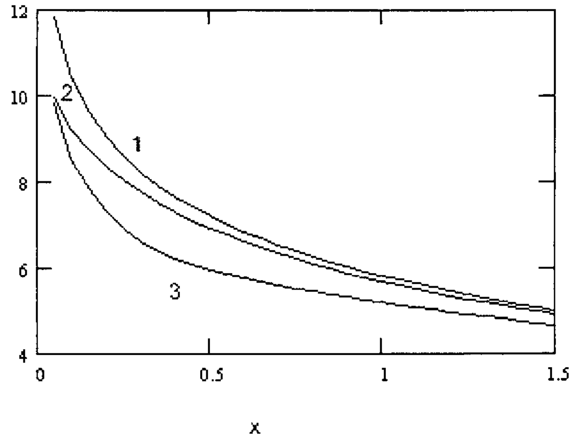

Fig. 4. The spectrum of a total polarization bremsstrahlung versus the target thickness L. 1: F^{PB} without taking into account the density effect; 2: $F^{\mathrm{PB}} + F^{\mathrm{STR}} + F^{\mathrm{INT}}$ for $y = \omega_0 L/2\gamma = 0.1$; 3: $F^{\mathrm{PB}} + F^{\mathrm{STR}} + F^{\mathrm{INT}}$ for $y = 1$.

5. Conclusion

Thus, the analysis carried out shows that the density effect does not take place in polarization bremsstrahlung of relativistic electrons crossing a thin layer of medium. The physical nature of the predicted phenomenon is connected with the contribution of transition radiation scattering to the total polarization bremsstrahlung yield, completely compensating the decrease of ordinary polarization bremsstrahlung (arising due to the scattering of the field of the fast particle).

The thickness of the target must be smaller than a photoabsorption length in order to observe the predicted effect experimentally.

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