Orientation effects in angle distribution of the relativistic electron radiation in single crystals

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Abstract

The spectral–angular distribution of the radiation of relativistic electrons in a crystal is very sensitive to the radiation mechanism. The results of an experimental research of the orientation effects in ultrarelativistic electron radiation in Si single crystals are presented in this work. The orientation dependences of the radiation angular distributions measured in the conditions of the axis effect manifestation are interpreted. The contribution of abovebarrier electrons in the radiation angle distribution is shown. The effect of "capture" of the radiation beam by crystal axis observed in the experiment has been explained by features of abovebarrier electron movement.

Keywords: Relativistic electron; Photons; Angle distribution; Single crystal; Orientation effects

1. Introduction

The question of the relation of channeled [1] and abovebarrier [2] relativistic electron radiation contributions is still open. The spectral–angular distribution of relativistic electron radiation in crystal is an object sensitive to radiation mechanism; therefore it can be used for testing the validity of theoretical models. The angular distributions of soft gamma-radiation of the 900 MeV electron in thick diamond crystal were measured for different orientations of ⟨1 0 0⟩ axis in [3,4]. In that experiment a maximum in angular distribution of radiation was observed. The maximum position in the distribution was shifted in proportion with crystallographic axis orientation angle in relation to projectile particles direction with a factor of proportionality 0.8. The authors of [3,4] connect the obtained result to the manifestation of scattering of the electrons on atom chain potential that is not conclusively because under such crystal thickness (10 mm of diamond) the main effect in scattering is caused by incoherent scattering on separate atoms. In [5] the results of [3,4] is interpreted as confirmatory evidence for the main contribution of channeled electrons to the radiation in thick crystals. Nevertheless the fact that moving maximum is not exactly aligned with the crystal direction (ignored by the authors of [5]) makes this interpretation incorrect.

2. Experiment

To clear up the radiation mechanism we have carried out a series of measurements, in which the angular distribution ($I_\Phi(\theta)$) of soft ($\omega \geqslant 50$ eV) gamma-radiation of 300 and 1200 MeV electrons in 70, 350, 1500, 3000 and 15,000 μm thick Si crystals was measured by means of a small (0.5×0.5 mm^2) scanning detector and the angular distribution dependence on orientation angle Φ (angle between the $\langle 1\,1\,1\rangle$ axis and electron beam direction) was traced. These experiments were carried out on Kharkov linac LUE – 2 GeV. The beam of accelerated electrons with the transverse dimensions $\leqslant$3 mm and divergence of $\leqslant 2\times10^{-4}$ rad was formed on the crystal, mounted into a goniometer [6], which permits to align the crystal with the accuracy of 5×10^{-5} rad. The electrons passed through the crystal were deflected by magnet and directed at the second electron monitor, by which the beam current was controlled. The bremsstrahlung gamma-quanta passing through the vacuum canal of 8 m long and then through the output foil (aluminum of 1 mm thickness) was registered by detector.

In these experiment the angular resolution was defined by divergence of the electron beam formed on single crystal target and also by the solid angle of radiation registration. The scanning detector allowed registering the transverse distribution of scattered electrons (or radiation) in horizontal plane. There was opportunity in experiment to register a family of curves of angular distribution of the radiation in all transverse area by changing the scanning detector position in vertical plane. Of special interest is the angular distribution of scattered electrons in a plane given by direction of projectile and the crystal axis. The geometry of measurement of the relativistic electron radiation by the deminutive scanning detector is shown in Fig. 1 with the appropriate identification. The angle ($\vec{V}_c$, $\langle 1\,1\,1\rangle$) in this experiment was situated always in horizontal plane. There were considered only positive angles Φ because of symmetry.

The angular distributions of the radiation of 300 MeV electrons shown in Fig. 2 demonstrate the effect of the radiation intensity maximum

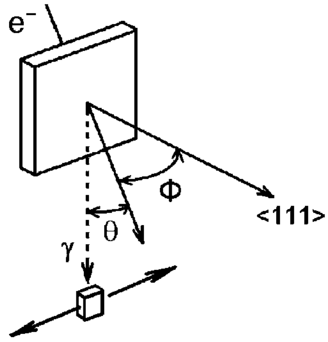

Fig. 1. The geometry of the experiment on measurement of angular distribution of radiation generated in single crystal by relativistic electrons: θ is the angle of observation of radiation, Φ is the angle between $\langle 1\,1\,1\rangle$ crystal axis and direction of projectile electron movement.

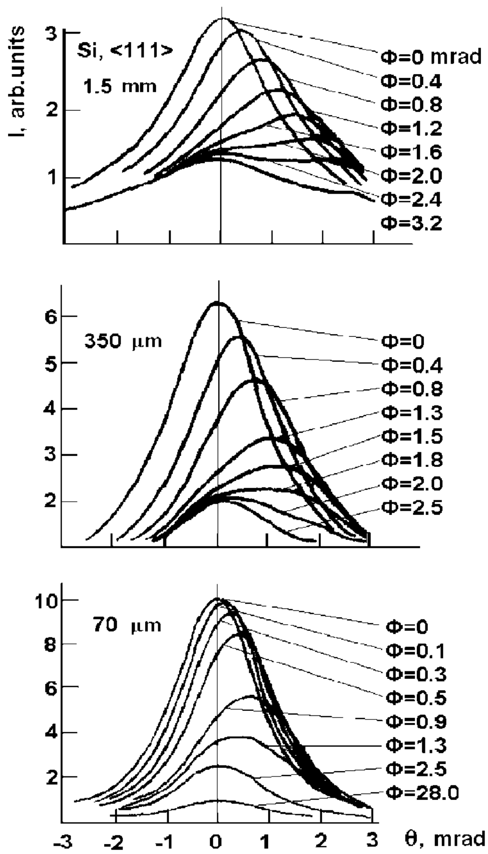

Fig. 2. The flux density angular distribution of the gamma-quanta radiated by 300 MeV electrons in Si crystal of different thickness: θ is radiation angle in plane formed by axis of projectile electron beam and crystal axis $\langle 1\,1\,1\rangle$. The angle θ is counted from projectile particle beam axis.

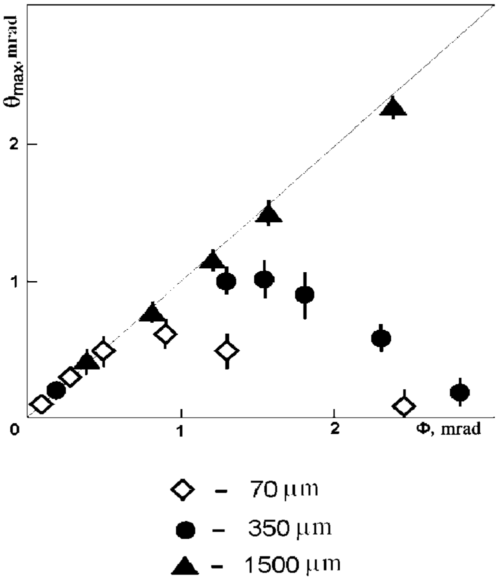

Fig. 3. The orientation dependence of the position of maximum in the angular distribution of radiation generated in crystals of different thickness by 300 MeV electrons: ◇ – 70 μm, ▲ – 350 μm, ● – 1500 μm.

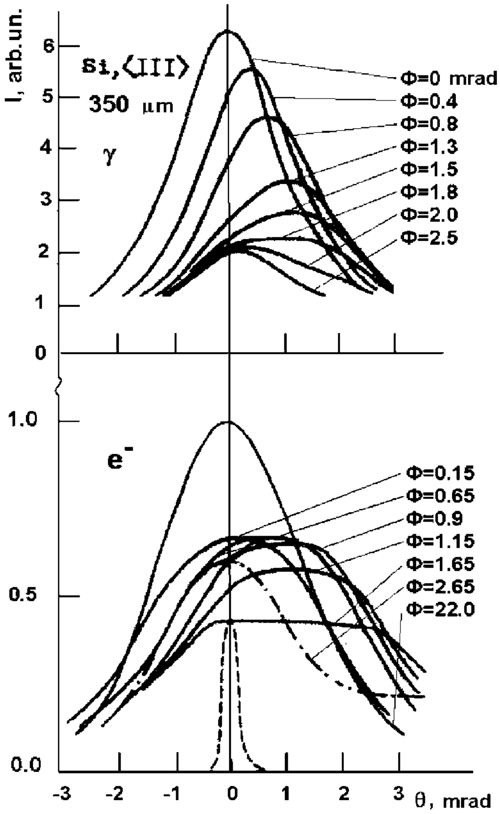

Fig. 4. The angular distribution of gamma-radiation and scattering of 300 MeV electrons measured in Si crystal of 350 μm thickness for different values of the ⟨1 1 1⟩-axis orientation angle Φ.

displacement relative to the electron beam direction (the angle θ is measured from projectile beam direction). Fig. 3 demonstrates the dependence of maximum displacement value θ_{max} on the crystal axis orientation (angle Φ). One can see that displacement of the maximum is depended on the crystal thickness too. With the proviso that $\Phi > 1/\gamma$ maximum in angle distribution of the radiation breaks down into two maxima, one of which corresponds to initial direction of electron beam movement and the other corresponds approximately to direction of the crystal axis.

The radiation angular distribution character is changed with the crystal thickness. In case of a thin crystal the maximum displacement is observed without its bifurcation. In this case the formation of radiation angular distribution is defined to a great extent by coherent multiple scattering of electrons from chains of atoms in crystal. Fig. 4 demonstrates the correlation between the angle distributions of scattered electrons and of the radiation in crystal of 350 μm thickness.

Fig. 5 presents the results of measurement of angular distributions of radiation generated in crystal Si of 70 μm thickness by 1200 MeV electrons. Angular dependences of gamma-quantum flux density in horizontal plane $I(\theta_H)$ were obtained for four values of orientation angle of ⟨1 1 1⟩ axis of crystal under fixed values of observation angle in vertical plane ($\theta_V = 0, 0.3, 0.6, 0.9$ mrad). In the character of the angular distributions shown in this figure one can trace a significant effect of the electron scattering processes in crystal. In particular, the curve family of the angular distribution of radiation for orientation angle $\Phi = 2\theta_c$ demonstrates crescent distribution of radiation, by which appropriate distribution of azimuthal electron scattering on the atom chains in crystal can be matched.

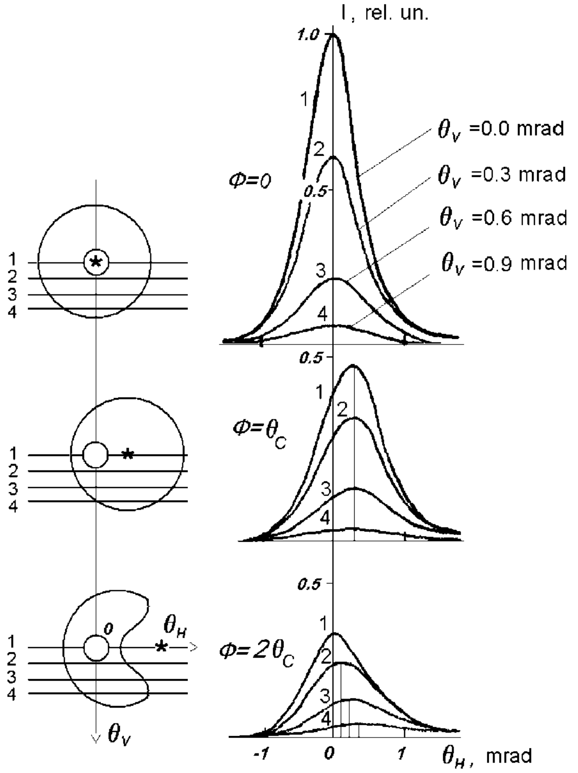

Fig. 5. The two-dimensional distribution of 1200 MeV electron radiation generated in Si crystal of 70 μm thickness: the scanning was realized in horizontal (angle θ_H) plane under fixed values of angle in vertical plane θ_V; θ_c is Lindhard angle; Φ is angle of $\langle 1\ 1\ 1\rangle$ axis orientation; ○ – incident electron beam; ★ inlinefig – direction of $\langle 1\ 1\ 1\rangle$ crystal axis.

3. Discussion

The data obtained were used for comparison with theoretical calculations of the abovebarrier electron angular distribution. The calculation were carried out for case of a thin crystal, when the main effect in scattering of the relativistic electrons is coherent azimuthal scattering [7] on the chains of atoms, and also in case of a thick crystal, when the incoherent scattering of particles is the main. In both cases the coherent contribution of atoms in chain was considered in the radiation. The initial expression for spectral–angular density of the relativistic particle radiation [7], taken for consideration is

$$\frac{dE_{n\omega}}{d\omega\cdot d\Omega} = \left(\frac{e^2}{8\cdot\pi^2}\cdot\frac{E^2+E1^2}{E1^2}\cdot\omega^2\right)\left\langle\left[\left|\left(\vec{n}\cdot\int\vec{v}\cdot e^{i\omega 1(t-\vec{n}\cdot\vec{r})}dt\right)\right|\right]^2 + \frac{m^2\cdot\omega^2}{E^2\cdot(E^2+E1^2)}\cdot\left(\left|\int e^{i\omega 1(t-\vec{n}\vec{r})}dt\right|\right)^2\right\rangle,$$

where $\omega 1 = \omega\cdot\frac{E}{E1}$, $E1 = E-\omega$, $\vec{n}$ is unit vector in the radiation direction, $\vec{v} = d\vec{r}/dt$, $\vec{r}(t)$ is classical trajectory. Trajectory is fixed by the angle Φ to atomic chain and the impact parameter b. The brackets $\langle\cdots\rangle$ mean the averaging over the initial conditions on the particle trajectory. Then the angular distribution of abovebarrier particle radiation can be obtained as

$$\frac{dN}{d\Omega} = \int dz\,d\vec{\Phi}F(z,\vec{\Phi})n_{tr}\Phi\int db\int\frac{d\omega}{\omega}\frac{dE_{\vec{n}\omega}}{d\omega\cdot d\Omega}f(\omega),$$

where $f(\omega)$ is the detector affectivity function, $F(z,\vec{\Phi})$ is angular distribution function of scattering electrons, z is the particle scattering length in the crystal target. The function $F(z,\vec{\Phi})$ must be used in a different form for a thin and thick target.

Let us interpret the experimental results related to case of thin crystal in the beginning. In frame of the considered approach a shifting of maximum in angular distribution of radiation is interpreted by multiple azimuthal scattering of electrons by the chains of atoms. This scattering shifts the centroid of radiating particles flow aside of crystal axis [7,8]. In this case the radiation angular distribution calculations were carried by [9]

$$\frac{dN}{d\Omega} = \frac{Z^2\cdot e^6 n_{\text{norm}}\cdot R\cdot\gamma^3\cdot\exp\left(\frac{u^2}{R^2}\right)\cdot L}{\pi\cdot a^2\cdot m^2\cdot\beta}\cdot N_1(\alpha,\beta,\phi), \tag{1}$$

$$N_1(\alpha,\beta,\phi) = \int_{-\pi}^{\pi}F(\chi,\alpha,\beta,\phi)\int_0^1\frac{f(y)}{y\cdot\sqrt{1+v^2}}\cdot(A_1\cdot B_1+A_2\cdot B_2)\,dy\,d\chi,$$

$$F(\chi,\alpha,\beta,\phi) = \frac{1+\frac{2}{\kappa}\cdot\left(\frac{\pi^2}{6}+\frac{\pi\cdot|\chi|}{2}+\frac{\chi^2}{4}-\sum_k\exp\left(\frac{-u^2\cdot\kappa^2}{k^2}\cdot\cos(k\cdot\chi)\right)\right)}{(1+\alpha^2+\beta^2-2\cdot\alpha\cdot\beta\cdot\cos(\chi-\phi))^2},$$

$$A_1 = \left[1 + 2 \cdot \frac{u^2}{R^2} \cdot (1 + v^2)\right] \cdot \left(1 - \Phi\Phi\left(\frac{u}{R} \cdot \sqrt{1 + v^2}\right)\right) - \frac{2}{\sqrt{\pi}} \cdot \frac{u}{R} \cdot \sqrt{1 + v^2} \cdot \exp\left[\frac{-u^2}{R^2} \cdot (1 + v^2)\right],$$

$$A_2 = \left[\left[1 - 2 \cdot \frac{u^2}{R^2} \cdot (1 + v^2)\right] \cdot \left(1 - \Phi\Phi\left(\frac{u}{R} \cdot \sqrt{1 + v^2}\right)\right) - \frac{2}{\sqrt{\pi}} \cdot \frac{u}{R} \cdot \sqrt{1 + v^2} \cdot \exp\left[\frac{-u^2}{R^2} \cdot (1 + v^2)\right]\right] \cdot \frac{v^2}{1 + v^2},$$

$$B_1 = 1 + (1 - y)^2 - 8 \cdot (1 - y) \cdot \frac{\alpha^2 \cdot \sin(\chi - \phi)^2}{(1 + \alpha^2 + \beta^2 - 2 \cdot \alpha \cdot \beta \cdot \cos(\chi - \phi))^2},$$

$$B_2 = 1 + (1 - y)^2 - 8 \cdot (1 - y) \cdot \frac{(\beta - \alpha \cdot \cos(\chi - \phi))^2}{(1 + \alpha^2 + \beta^2 - 2 \cdot \alpha \cdot \beta \cdot \cos(\chi - \phi))^2},$$

where $\alpha = \gamma \cdot \theta$, $\beta = \gamma \cdot \phi$, θ and ϕ are polar and azimuthal angles of vector $\vec{n}$ with respect to crystal axis, u is mean-square amplitude of thermal oscillations of atoms in crystal, a is a distance between atoms in chain, $\gamma = \frac{E}{m}$, $\kappa^2 = \frac{1}{2} \cdot \chi^2_{\mathrm{av}} \cdot L$, $y = \frac{\omega}{E}$, $f(y)$ is effective function of detector, $v = \frac{y}{1-y} \cdot \frac{m \cdot R}{2 \cdot \beta} \cdot (1 + \alpha^2 + \beta^2 - 2 \cdot \alpha \cdot \beta \cdot \cos(\chi - \phi))$, χ^2_{av} is mean square azimuthal scattering angle per unit length, $\Phi\Phi(x)$ is error function integral.

Fig. 6 presents the calculated by formula (1) angular distributions of coherent radiation generated by 300 MeV electrons in Si crystal of 70 μm thickness (solid line). The experimental dependences (points) is presented without the background of incoherent radiation (radiation in disoriented crystal). As can be seen from Fig. 6, the presented model describes correctly the maximum position as function of orientation angle Φ (angle between directions of crystallographic axis and incident electron beam).

For qualitative explanation observed phenomena let us estimate the mean-square value of electron azimuthal scattering angle $\kappa = \sqrt{\chi^2_{\mathrm{av}} \cdot L}$ on crystal exit. χ^2_{av} is average square angle of azimuthal scattering per unit length. In according

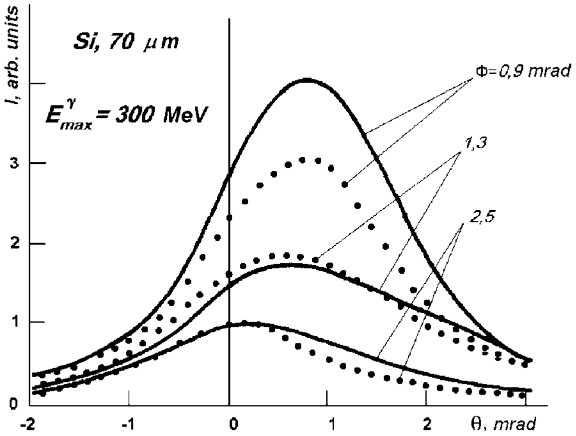

Fig. 6. The angular distribution of gamma-radiation of 300 MeV electrons in Si crystal of 70 μm thickness for different values of orientation angle Φ_i: (—) calculation by (1); (●●●) – experimental evidence.

with [7] under $E = 300$ MeV $L = 70$ μm for $\langle 1\,1\,1 \rangle$ axis κ is about $\frac{2 \cdot \pi}{\Phi}$ (κ and θ is measured in milliradian). Under $\Phi = 0.9$ mrad parameter $\kappa > 2 \cdot \pi$, therefore azimuthal scattering of electron is practically homogeneous, i.e. the center of radiating electron flow is in direction of crystal axis. The case of $\Phi = 1.3$ mrad is intermediate – under a such orientation of the crystal axis the centroid of scattered electron flow is situated in direction between axes of crystal and initial electron beam.

In case of $\Phi = 2.5$ mrad parameter $\kappa \approx \frac{\pi}{2}$. At this rate the coherent azimuthal scattering leads to insignificant displacement of the centroid of scattered electron flow in relation to incident beam direction. As the results the maximum in angular distribution is directed parallel to incident electron beam.

Let us note, that dependence of radiation angle distribution density $I(\Phi) \sim \frac{1}{\Phi}$ is observed in the experiment in a range of $\phi > \Phi_c$, which completely corresponds to range of application of the used model (see curves in Fig. 6 under $\Phi = 1.3$ and 2.5 mrad). Some divergency of calculated and measured data is because of the model not takes into account the growing restriction of function $I(\Phi)$ in the range $\Phi \leqslant \Phi_c$ (in discussed case the critical angle of axis channeling $\Phi_c = 0.7$ mrad). The other reason of the divergency have to do with the incoherent multiplicity scattering of relativistic electrons on separate atoms of medium, unsuspected in the model and leading to the changing of motion direction of radiating electrons. It is light to be convinced, that the condition of thin crystal

$$L \ll L_{\mathrm{R}} \cdot \frac{E^2 \cdot \Phi^2}{E_k}, \tag{2}$$

where L_{R} is radiation length, $E_k = 21$ MeV, is quite well satisfied in the experiment with crystal of 70 μm thickness only for orientation angles $\Phi > 1$ mrad.

Incoherent scattering and angular dependence of coherent radiation probability cause orientation effects in angular distribution of relativistic electron radiation in thick crystal basically.

In Fig. 7 measured angle distributions of gamma-radiation generated by 300 MeV electrons in Si crystal of 3 mm thickness (dashed lines) and the corresponding calculations (solid lines) is presented without taking into account coherent scattering. The calculations were carried out by [9]

$$\frac{\mathrm{d}N_n}{\mathrm{d}\Omega} = \frac{2 \cdot \pi \cdot \mathrm{e}^6 \cdot n_{\mathrm{norm}} \cdot R \cdot \gamma^3 \cdot \exp\left(\frac{u^2}{R^2}\right) \cdot L}{a^2 \cdot m^2 \lambda^2} \cdot N_2, \tag{3}$$

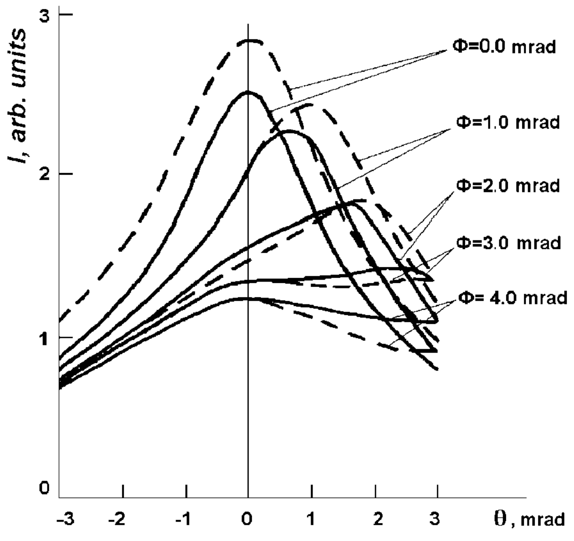

Fig. 7. Angular distribution $I(\theta)$ of gamma-radiation generated by 300 MeV electrons in Si crystal of 3 mm thickness: (—) calculated by (3); (– – –) obtained in experiment.

where

$$N_2 = \int_0^\infty g(\beta) \cdot \int_{-\pi}^{\pi} F1(\chi) \cdot \int_0^1 \frac{f(y)}{\sqrt{1-y^2}} \times (A_1 \cdot B_1 + A_2 \cdot B_2)\,\mathrm{d}y\,\mathrm{d}\chi\,\mathrm{d}\beta,$$

$$F1(\chi) = \frac{E\left(\frac{\beta^2 - \beta_1^2 - 2 \cdot \beta \cdot \beta_1 \cdot \cos(\chi)}{\lambda^2}\right)}{(1 + \alpha^2 + \beta^2 - 2 \cdot \alpha \cdot \beta \cdot \cos(\chi - \phi))^2},$$

where $\lambda = \Phi_{\mathrm{av}}^2 \cdot \gamma \cdot L$, $g(\beta) = 1$ under $\beta = \gamma \cdot \Phi_{\mathrm{c}}$, $g(\beta) = \frac{\beta}{\gamma \cdot \Phi_{\mathrm{c}}}$ under $\beta < \gamma \cdot \Phi_{\mathrm{c}}$, $E(x) = \int_x^\infty \frac{\mathrm{e}^{-t}}{t}\,\mathrm{d}t$.

The rest denominations are coincided with ones in formula (1).

It is important to underline that the crystal thickness restriction (2) characterizes influence of the electron incoherent scattering on the structure of angular distribution of the electrons passing through the crystal. The corresponding distribution of the radiation also significantly depends on parameter γ^{-1} defining the angle width of the relativistic particle radiation cone. That is why the details of angle distribution of electrons scattered by the crystal, which angular size is less then γ^{-1}, cannot manifest in angular distribution of gamma-radiation generated by these electrons. In particular under $\Phi \leqslant \gamma^{-1}$ the angular distribution of radiation of electron beam in crystal will have only one maximum under any thickness. In case of $\Phi > \gamma^{-1}$ the multiple azimuthal electron scattering can lead to the appearance of second maximum under $\theta = 2\Phi$ due to forming of a ring structure of scattered electrons. But for electron energy of 300 MeV the mechanism of azimuthal scattering cannot lead to the angular distribution with two maxima. To form a closed ring structure in condition of consideration the inequality L (μm) $\gg 70\Phi^3$ (mrad) must be really met (it can be obtained from condition $\kappa \gg 2\pi$ ($\kappa = (\langle\theta^2\rangle/\Phi^2)$). On the other hand, condition of neglecting of incoherent scattering (2), which causes the smoothing of ring structure, is reduced to inequality L (μm) $\ll 45\Phi^2$ (mrad). Both of these conditions are simultaneous only under Φ (mrad) $\ll 0.66$, i.e. in range of orientation angles in which the necessary condition $\Phi > \gamma^{-1}$ not meets under electron energy 300 MeV.

The maximum under $\theta = 2\Phi$ really was not observed in the experiment for any used thickness of crystals. But in the cases of enough thick crystals ($L = 1500$ and 3000 μm) the second maximum is observed under $\theta = \Phi$, i.e. in direction of crystal axis. The appearance of this maximum evidently cannot be explained by multiple azimuthal scattering on chains of atoms in crystal, as was supposed for example in work [4].

The appearance of second maximum in angular distribution of radiation in crystal of big thickness is conditioned by significant increasing of intensity of coherent radiation of electrons hitting by means of incoherent scattering to region of small angles in respect to crystal axis (in according of (1) the intensity of coherent radiation of electrons is proportional to Φ^{-1}).

Some misalignment of the direction of the pointed radiation intensity maximum and the crystal axis can be explained by the significant difference of scattered electron distribution and the radiation intensity as functions of the angle Φ. Whereas the function of angular electron distribution is bell-shaped and have a maximum in direction of incident electron beam, the coherent radiation intensity of electrons as function of Φ have a maximum in direction of crystal axis. Integration of the product of these functions by formula (3) leads to the removal of the radiation intensity maximum in respect to crystal axis aside the electron beam direction. The attempt to connect the formation of a maximum in electron angular distribution in the thick crystal along of crystal axis with the radiation of channeled electrons, undertaken in [5,10], is not able to explain the misalignment of the direction of pointed radiation intensity maximum and crystal axis observed in the experiment so far as the radiation of channeled particles is knowingly symmetrical in respect to axis of the channel.

The data calculated and measured (Fig. 7) was joined in magnitude on point $\theta = 0$, $\Phi = 4$ mrad. As is easy to ascertain, the angle of multiple scattering θ_s in this case significantly exceed the initial orientation angle Φ that is a necessary condition of the calculating model workability. As expected, the bifurcation of the maximum in the angular distribution of radiation is observed only in the orientation range $\Phi > 1/\gamma$. The agreement of the experimental and theoretical results confirms the validity elected model of the investigated process.

So, analysis executed shows that for a crystal of middle thickness the displacements of radiation maximum in direction of crystal axis observed in the experiment is well described in frame of the abovebarrier electron radiation theory with the taking to account multiple scattering on the atomic chains. For the thick crystal remembered displacement of the maximum and the effect of it is bifurcation are well described under taking to account of incoherent scattering of electrons and the dependence of electron radiation intensity on the angle θ at which the electron momentum is directed in reference to crystal axis.

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References

[1] M.A. Kumakhov, X.G. Trikalinos, JETP 78 (1980) 1623 (in Russian).

[2] N.F. Shulga, V.I. Truten', S.P. Fomin, Preprint AN USSR, KPTI 80-42, Kharkov, 1980 (in Russian);
N.F. Shul'ga, JETP Lett. 32 (1980) 166;
N.F. Shul'ga, V.I. Truten', S.P. Fomin, Sov. Phys. JETP 60 (1984) 145.

[3] S.A. Vorobijov, A.A. Kurkov, A.P. Potylitcyn, Izvestija VUZov i SSD SSSR Fizika (9) (1986) 32 (in Russian).

[4] A.N. Alejnik, S.A. Vorobijov, B.N. Kalinin, A.A. Kurkov, A.A. Potylitcyn, JETP 56 (Vyp7) (1986) 1416 (in Russian).

[5] C.V. Beslaneeva, V.I. Telegin, JETP 92 (Vyp4) (1987) 101 (in Russian).

[6] G.L. Bochek, V.I. Vit'ko, I.A. Grishaev, et al., Proceeding VI All Union Conference on Interaction of Charged Particles with Monocrystals M, MSU, 1975, p. 219 (in Russian).

[7] V.I. Truten', S.P. Fomin, N.F. Shul'ga, Preprint AS USSR, KFTI, 32-11, Kharkov, 1982, p. 28 (in Russian);
A.I. Akhiezer, N.F. Shul'ga, Sov. Phys. Usp. (1982) 541;
A.I. Akhiezer, V.I. Truten', N.F. Shul'ga, Phys. Rep. 203 (1991) 289.

[8] V.S. Malyshevsky, V.I. Truten', N.F. Shul'ga, JETP 93 (Vyp2(8)) (1987) 570 (in Russian).
[9] A.P. Antipenko, S.V. Blazhevich, G.L. Bochek, et al., Preprint KPTI 90-33, Kharkov, 1990, p. 29 (in Russian).
[10] G.A. Andrejashkin, S.A. Vorobijov, V.I. Zabaev, et al., Book of Abstracts of XIX All Union Conference on Physics of Interaction of Charged Particles with Crystals M, Prod. SU, 1989, p. 87 (in Russian).