# Sound waves in foams

K.B. Kann

*Belgorod State University, ul. Pobedy, 85 Belgorod, Russia*

**Abstract**

A new (film) model is proposed for the propagation of sound waves in gas–liquid foams. The model explains the effect of sound retardation in foams by foam films inertness. It qualitatively agrees with the experimental data on the velocity of sound propagation in foams of various structures. The high absorption of sound in foams is explained by hydrodynamic losses in foam films. The experimental dependence of the sound absorption coefficient on the foam expansion is explained. The calculated expansion corresponding to the absorption maximum is close to the experimental value.

*Keywords:* Foams; Film model; Sound velocity; Sound absorption; Hydrodynamic losses

Gas–liquid foams differ from other media in terms of many physical properties. Acoustic properties of foams are not the exception. The velocity of sound in foams is less than that in gas and in liquid. It is not surprising, since according to known dependence the sound velocity in the homogeneous medium depends on density $\rho$ of media under the correlation

$$c = \sqrt{\frac{\kappa}{\rho}} \tag{1}$$

where $\kappa$ is the module of volumetric elasticity of material.

Substituting the average density of gas–liquid mixes $\rho$ into (1) authors [1] have received the following expression for the sound velocity

$$c^{-2} = \rho\left(\frac{\varphi}{\rho_0 c_0^2} + \frac{1-\varphi}{\rho_1 c_1^2}\right) \tag{2}$$

where $\rho_0$ and $\rho_1$ are the densities of gas and liquid, $c_0$ and $c_1$ are the sound velocities in these media, and $\varphi$ the volume gas content in the mixture. It is essentially the Wood's formula, provided in 1944 [2].

Dependence (2) is illustrated graphically in Fig. 1. It is well carried out for gas–liquid emulsions, suspensions of gas bubbles in liquid, volume gas content which does not exceed $\sim$0.5. Some authors (for example, [3]) also use this dependence for foams. However, as shown in experiments (see [4]), at gas contents $\varphi > 0.9$ the dependence (2) displays errors. For example in Fig. 2 experimental dependence of the velocity of sound in foam from foam expansion $K$, received by authors [5], is shown by curve 1. Dependence (2) in Fig. 2 is displayed by a curve 2. This demonstrates that divergences between the curves do not get into the range of measurements errors. It is not possible to explain these divergences within the framework of homogeneous (molecular) model of sound propagation in foams.

The abnormally large absorption of sound in foams is even more surprising. The absorption of sound by foams exceeds the absorption of sound in liquid and gas in $10^7$–$10^{10}$ times (!).

In 2003 we offered (see [4]) the new model (the so called "film model") of sound propagation in foams.[1] The given report is devoted to the description of this model.

Foam differs from homogeneous media that it is the structured medium. The two-sided liquid films, which separate gas bubbles, are the basic structural elements of foams. The liquid-films chaotic structure essentially hampers the molecular conduction of sound energy in liquid phase. Sound oscillations in foams are transferred from film to film through gas

*E-mail address:* kkann@belnet.ru.

[1] The model has been developed for "liquid" foams. It is unsuitable when the foam-forming solution forms structure (for example, gel) in volume.

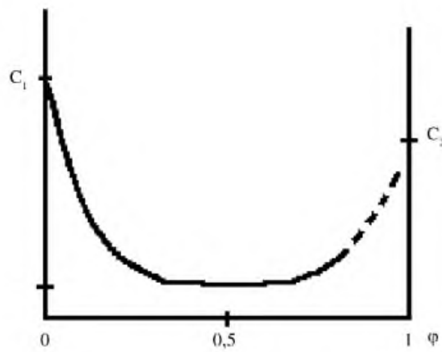


Fig. 1. Dependence of sound velocity in a gas–liquid mixture $C$ from gas volume content $\varphi$ [1].

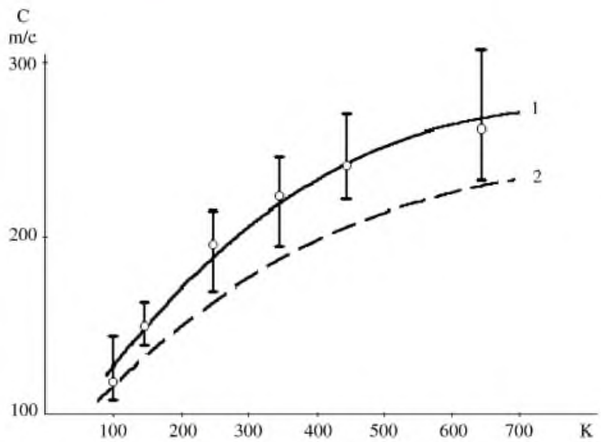


Fig. 2. Comparison of the experimental dependence of sound velocity in foams from expansion [4] with theoretical dependence (2) built according to homogeneous model.

bubbles. Let's examine the simplified model of gas–liquid foams, consisting of cubic "bubbles" of linear size $d$ (Fig. 3). Let us assume that the flat sound wave of pressure $p = p_{\mathrm{m}} \exp[-\mathrm{i}(\omega t - k_0 z)]$ with cyclic frequency $\omega$ and wave number $k_0 = 2\pi/\lambda_0$ ($\lambda_0$ is the length of a sound wave in air) falls on such foam in direction Z. Since the thickness of foam films $\lambda \ll \lambda_0$ the film can be considered as sound resistance

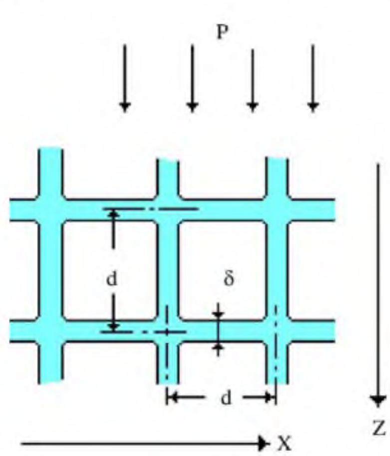


Fig. 3. Model of "cubic foam".

of the concentrated weight $\sigma_1 = -\mathrm{i}\omega\mu$ with surface density $\mu = \rho_1\delta$ where $\rho_1$ is density of the liquid. Together with the sound resistance of bubble gas $\sigma_0 = \rho_0 c_0$ they form sound resistance of one cell of foam

$$\sigma' = \sigma_0 + \sigma_1 = \rho_0 c_0 - \mathrm{i}\omega\mu = r\,\mathrm{e}^{-\mathrm{i}\varepsilon'}$$

where $\rho_0$ is gas density, $r = \sqrt{\rho_0^2 c_0^2 + \omega^2\mu^2} \approx \rho_0 c_0$, and $\varepsilon' = \operatorname{arctg}(\omega\mu/\rho_0 c_0 \approx \omega\mu/\rho_0 c_0)$, since for sound frequencies $\omega\mu \ll \rho_0 c_0$.

The speed of gas particles in the sound wave that have passed through the film, is

$$v_{\mathrm{f}} = \frac{p}{\sigma'} = \frac{p_{\mathrm{m}} \exp[-\mathrm{i}(\omega t - k_0 z)]}{r\,\mathrm{e}^{-\mathrm{i}\varepsilon'}} \approx \frac{p_{\mathrm{m}}}{\rho_0 c_0}\,\mathrm{e}^{-\mathrm{i}(\omega t - k_0 z - \varepsilon')} \tag{3}$$

It can be observed, that the sound wave that has passed through a liquid film, stays behind on a phase of a falling wave on size $\varepsilon'$. If the sound wave meets $n = z/d$ films on a way $z$ it stays behind a primary wave on $\varepsilon' n = \varepsilon' z/d$. The pressure in such wave will be expressed by dependence

$$p = p_{\mathrm{m}} \exp\left[-\mathrm{i}\left(\omega t - k_0 z - \frac{\varepsilon' z}{d}\right)\right] \tag{4}$$

On the other hand the wave of sound pressure in the foam can be presented as

$$p = p_{\mathrm{m}} \exp[-\mathrm{i}(\omega t - k_{\mathrm{f}} z)] \tag{5}$$

where $k_{\mathrm{f}}$ is the wave number for sound wave in the foam.

Comparing (5) with (4), we conclude, that the wave number for sound in the foam is

$$k_{\mathrm{f}} = k_0 + \frac{\varepsilon'}{d} \tag{6}$$

Then the sound velocity in the foam

$$c_{\mathrm{f}} = \frac{\omega}{k_{\mathrm{f}}} = \frac{\omega}{k_0 + \varepsilon'/d} = \frac{\omega/k_0}{1 + \varepsilon'/(k_0 d)} = \frac{c_0}{1 + \varepsilon'/(k_0 d)} \tag{7}$$

where $c_0$ is the sound velocity in gas.

Substituting values $\varepsilon' = \omega\mu/\rho_0 c_0$ and $\mu = \rho_1\delta$ in (7), we finally receive

$$c_{\mathrm{f}} = \frac{c_0}{1 + \rho_1\delta/(\rho_0 d)} \tag{8}$$

So it is the expression for sound velocity according to film model of sound propagation in foams.

The comparison of experimental values of sound velocity in foams of various structures [5,6,9] with calculations on formulas of a cellular model of foam structure (see [4]) confirms the sound velocity's dependence on structural parameters of foam, thickness $\delta$ and the size $d$ foam films, which reflects the Eq. (8). It is not possible to receive quantitative results yet, since now there are no certain notions about the form and the sizes of foam films. (It is important to note that the Eq. (8) includes effective values of parameters $d$ and $\delta$ for real foams).

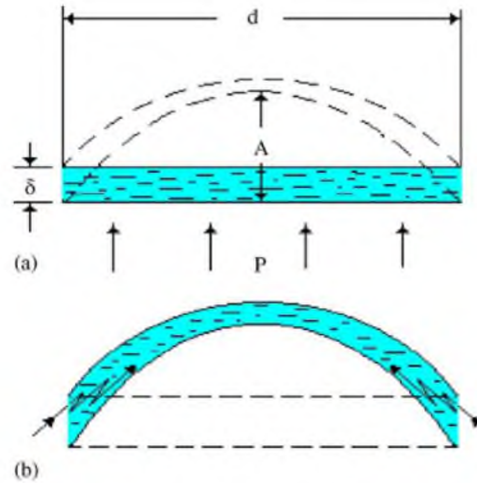


Fig. 4. The deformation of liquid film under the influence of sound pressure (a) and process of "sound pumping" of foam films (b).

According to dependence (8) sound velocity in foams does not depend on frequency. It will correspond to the results of measurements (see [5–7]) at small sound frequencies (up to 1 kHz). We shall notice, that the analysis of a homogeneous model [8] gives a large dispersion of a sound in all a range of frequencies.

Let us examine the issue of absorption of energy of sound waves in foams. Sound oscillations on foam films (see Fig. 3) force them to oscillate at the frequency of the sound wave. During these oscillations the film is bent and the area of its surface periodically (with frequency $\omega_1 = 2\omega$) changes. Because of inertness the volume of liquid in a film changes insignificantly for the period. So, when at a curvature the area of a film grows, its thickness in the film centre decreases (see Fig. 4a). Capillary forces create rarefaction $p_1$ in the centre of a film under which action the liquid from periphery directs in a film. This movement of liquid has a reciprocatory character that is shown in Fig. 4b, i.e. during some period of time foam films, influenced by sound, should be filled with liquid—become thicker. This phenomenon was discovered experimentally by our colleagues—authors [10]. They found that "sound pumping" of foam films proceeds within several seconds after the beginning of sound influence. During this time the films are making many thousands of oscillations.

On the other hand, repeated back and forth motions of liquid in films demand a lot of hydrodynamic expenses of energy. In our opinion, it is the explanation for the abnormally large absorption of sound energy by gas–liquid foams.

Let's examine the process of dissipation of energy of a sound wave in a film model (see Fig. 3). It is obvious, that the maximal rarefaction $p_{m1}$ in the center of a film grows with the growth of the amplitude of its oscillations and, therefore is directly dependent on the amplitude of sound pressure $p_m$. When sound is propagated in the foam (as well as in any other media) the amplitude of sound pressure decreases under the dependence $p_m(z) = p_{m0}\,e^{-\beta z}$, and the intensity of a wave falls

$$u(z) = u_0\,e^{-2\beta z} \tag{9}$$

where $\beta$ is the factor of absorption of sound in the foam. Substituting the obvious parity $z = c_f t$ in (9) and integrating it, it is possible to produce the equation for factor of absorption of sound through the intensity of a sound wave

$$\beta = -\frac{1}{2uc_f}\frac{\partial u}{\partial t} \tag{10}$$

Solution of the problem of liquid flow in flat clearance between two parallel plates is known (see, for example, [11]). If the width of clearance is $\delta$ then the power that operates on the unit of a plate surface in the direction of flow $X$ is

$$f = \frac{\delta}{2}\frac{dp_1}{dx} \tag{11}$$

and the average on cross-section velocity of flow is

$$v = \frac{\delta^2}{12\eta}\frac{dp_1}{dx} = \frac{v_{m1}}{\sqrt{2}} \tag{12}$$

where $\eta$ is the dynamic viscosity of liquid, and $v_{m1}$ the maximal velocity of current in the flow.

Excluding $dp_1/dx$ from (11) and (12), we find, that

$$f = \frac{6\eta v}{\delta}$$

Multiplying this value by the area of a film surface ($2d^2$), we find the power of viscous friction in a film $F = 12\eta d^2 v/\delta$, and then we also find the capacity of losses within the limits of one cell of foam

$$W = \frac{Fv}{2} = \frac{6\eta d^2 v^2}{\delta}$$

The capacity losses of a sound wave in the unit of foam volume can be received, by dividing this expression by the volume of a foam cell ($d^3$)

$$-\frac{\partial u}{\partial t} = \frac{W}{d^3} = \frac{6\eta v^2}{d\delta} \tag{13}$$

Substituting (13) for (10), we find the equation for the absorption factor of sound in foam

$$\beta = \frac{3\eta v^2}{uc_f d\delta} \tag{14}$$

The average velocity of liquid flow in foam films is defined by parity (12). Since $p_1$ is the rarefaction in the film centre (in relation to pressure upon peripheries) it is possible to assume

$$\frac{dp_1}{dx} \approx -\frac{p_{m1}}{d/2}, \qquad \text{then } v = \frac{\delta^2 p_{m1}}{6\eta d} = \frac{v_{m1}}{\sqrt{2}} \tag{15}$$

With the film thickness's increase (with reduction of foam expansion $K$) the velocity of liquid flow in films grows. But this velocity cannot surpass the velocity of sound propagation in the medium acoustic velocity

$$v_{ak} = \frac{p_{m1}}{\rho_1 c_1} \tag{16}$$

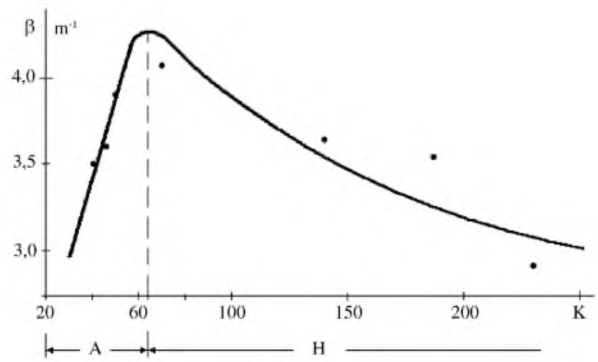


Fig. 5. Experimental dependence of sound absorption factor $\beta$ on foam expansion [9].

At some (critical) of a film thickness $\delta_k$ the maximum velocity of flow $v_{m1}$ achieves the acoustic velocity and avoids its depending on the thickness. The value of this critical thickness of foam films can be found from parities (15) and (16). From the equality $v_{m1} = v_{ak}$ we find, that

$$\delta_k = \sqrt{\frac{6\eta d}{\sqrt{2}\rho_1 c_1}} \approx 2\sqrt{\frac{\eta d}{\rho_1 c_1}} \tag{17}$$

For aqueous foams with the bubble sizes of 0.1–1.0 mm critical film thickness is equal to 0.5–1.7 μm.

Thus, in relation to the mechanism of liquid flow in films, all foams are divided into two types:

1. If the thickness of foam films $\delta \ll \delta_k$ the velocity of liquid flow in a film grows with the thickness according to the hydrodynamic parity (15). We name such foams H-foams (foams of a hydrodynamic range). High-expansion ("dry") foams are in this range.
2. For foams with film thickness $\delta \geq \delta_k$ the maximum velocity of liquid flow in films is constant, it is equal to the acoustic velocity (16) and does not depend on $\delta$. We name such foams A-foams—foams from an acoustic range. They are "wet", low-expansion foams (see Fig. 5).

Since the absorption of sound energy in foams depends on the velocity of liquid flow in films (see (14)) the dependence of the absorption factor on foam expansion would vary in different ranges. Let us find these dependences.

Substituting the value of hydrodynamic velocity (15) for (14), we receive

$$\beta = \frac{3\eta}{uc_f d\delta}\left(\frac{\delta^2 p_{m1}}{6\eta d}\right)^2 = \frac{p_{m1}^4 \delta^3}{12uc_f d^3 \eta} \tag{18}$$

Foam expansion is proportional to the attitude $d/\delta$

$$K = \frac{ad}{\delta} \tag{19}$$

where $a$ is the empirical coefficient.[2] The substitution (19) for (18) shows, that in the H-range the factor of sound absorption quickly decreases with the growth of foam $K$

$$\beta = \frac{p_{m1}^4 a^3}{12uc_f \eta}\frac{1}{K^3} \sim \frac{1}{K^3} \tag{20}$$

Similar dependence has been experimentally received by authors [5] within the range of foam expansion 80–1700.

The dependence for factor of absorption of sound by foams in the A-range has been received by substituting (16) for (14). Taking (19) into account we find, that

$$\beta = \frac{3\eta}{uc_f d\delta}\left(\frac{p_{m1}}{\rho_1 c_1}\right)^2 = \frac{3\eta p_{m1}^2}{uc_f \rho_1^2 c_1^2 d^2 a} K \sim K \tag{21}$$

So, for low-expansion foams the absorption grows with the foam expansion increase. The experimental dependence $\beta(K)$ received by the results of our work [12] is shown in Fig. 5. This dependence varies in different ranges. At expansion $K_m \approx 60$ the absorption is maximum. This maximum stays between the ranges when the hydrodynamic velocity of liquid current in films reaches the acoustic velocity that is at $\delta = \delta_k$. The value of the foam expansion corresponding to a maximum of absorption, is possible to estimate, substituting the expression (17) for (19)

$$K_m = a\sqrt{\frac{\rho_1 c_1 d}{6\eta}} \tag{22}$$

For the conditions of our experiment the dependence (22) gives $K_m \approx 63$ which is close enough to experimental.

In summary, we notice that gas–liquid foam is, apparently, a unique object in which the absorption of sound is caused by the powerful hydrodynamic mechanism of dissipation of energy. If we take into account that the degree of sound absorption in foams is easy to operate, it is possible to predict, that in some time gas–liquid foams will become the unique and perspective object of acoustic research and of interesting practical applications.

Theoretical and experimental investigations of film model of sound propagation in gas–liquid foams are to be continued.

## Acknowledgements

The author acknowledges the financial support from the Organizing Committee of the EUFOAM 2004 Conference and Russian FFI.

## References

[1] V.E. Nakoryakov, B.G. Pokusaev, I.R. Shreiber, Volnovaya dinamika gazo- i parozhidkostnyh sred, Energoatomizdat, Moskva, 1990.
[2] A.B. Wood, A Textbook of Sound-Bell and Sons, London, 1944, pp. 360–363.
[3] B.S. Gardiner, B.Z. Dlugogorski, J. Jameson, J. Rheol. 42 (1998) 1437.
[4] K.B. Kann, A.A. Kislitsyn, Kolloidnyi zhurnal 65 (2003) 26.

[2] The estimation of $a$-parameter under the results of work [5] gives $a = 0.23$. In our experiments [12], this parameter equals to 0.22.

[5] V.V. Zamaschikov, N.A. Kakutkina, Akusticheskii zhurnal 37 (1991) 484.
[6] G.A. Shushkov, V.N. Feklistov, K.B. Kann, Z.M. Orenbah, Kolloidnyi zhurnal 55 (1993) 163.
[7] F.I. Vafina, I.I. Gol'dfarb, I.R. Shreiber, Akusticheskii zhurnal 38 (1992) 5–11.
[8] N. Mujica, S. Fauve, Phys. Rev. E 66 (2002) 021404.
[9] N.T. Moxon, A.C. Torrance, S.B. Richardson, Appl. Acoustics 24 (1988) 193.
[10] V.V. Zamaschikov, N.A. Kakutkina, Kolloidnyi zhurnal 54 (1992) 64.
[11] L.D. Landau, E.M. Lifshits, Gidrodinamika, Nauka, Moskva, 1988.
[12] K.B. Kann, V.N. Feklistov, G.A. Shushkov, Akusticheskii zhurnal 40 (1994) 147.