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# Transition Radiation from Relativistic Charged Particles Interacting with Atomic Strings in a Crystal

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**Abstract**—The problem of transition radiation generated by relativistic particles incident on atomic strings in a crystal at a small angle is considered. Conditions are obtained under which the problem of transition radiation reduces to that of radiation generated by a collision with a filament-like target. It is shown that the angular distribution of transition radiation is symmetric with respect to the atomic-string axis.

## 1. INTRODUCTION

Transition radiation is known to be generated by a charged particle traversing the interface between two media (see [1–4] and references therein). For a relativistic particle, radiation of this type is concentrated in the region of small angles with respect to the direction of particle motion. At such angles, radiation is generated over a long segment along the particle-velocity direction, the length of this segment being referred to as the radiation coherence length [2, 5, 6]. If, within such a segment, the particle being considered traverses a few interfaces between different media, the interference between the waves generated by the particle upon traversing each interface is of importance. It was shown in [7] that, in the limit of long-wave transition radiation, both longitudinal and transverse dimensions of the domain where transition radiation is generated may be macroscopic. If, concurrently, the transverse dimensions of the target used satisfy the condition $L_\perp \le \gamma\lambda$ (where $\lambda$ is the wavelength of the radiation and $\gamma$ is the particle Lorentz factor), transition radiation depends strongly both on these transverse dimensions and on the shape of the target.

In this study, we consider the problem of transition radiation from relativistic particles incident on an atomic string in a crystal at a small angle $\psi$ with respect to the string axis (see Fig. 1). We study this radiation at high frequencies, in which case the dielectric permittivity can be represented in the form

$$\varepsilon_\omega = 1 - \omega_p^2/\omega^2, \quad \omega \gg \omega_p, \tag{1}$$

where $\omega_p^2 = 4\pi e^2 n_e(\mathbf{r})/m$ is the plasma frequency, $m$ is the electron mass, and $n_e(\mathbf{r})$ is the density of electrons in the target. Under such conditions, transition radiation is associated with the nonuniformity of the electron density in the atomic string. The problem under consideration is similar to the problem of the effect that the target boundaries exert on transition radiation, since electrons in the atomic string are concentrated near the string axis. We find conditions under which transition radiation is unaffected by the nonuniformity of the electron distribution along the string axis and determine the angular distribution of transition radiation for this case. We also study transition radiation generated by particle interaction with a set of atomic strings whose axes form a periodic or a chaotic structure in the transverse plane. (We use the system of units where the speed of light is equal to unity.)

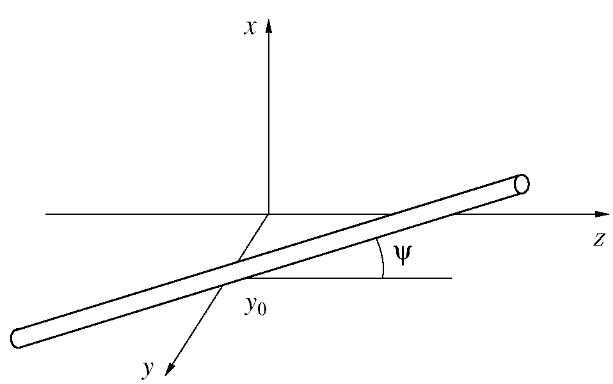


**Fig. 1.** Disposition of an atomic string (cylinder) and the trajectory of a particle incident on it ($z$ axis) in the transition-radiation problem being considered.

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## 2. TRANSITION RADIATION IN A COLLISION OF A PARTICLE WITH AN ISOLATED ATOMIC STRING

The spectral–angular distribution of transition radiation in a heterogeneous medium whose dielectric permittivity is given by (1) has the form

$$\frac{dE}{d\omega do} = \frac{1}{4\pi^2}|\mathbf{k}\times\mathbf{I}|^2, \qquad (2)$$

where

$$\mathbf{I} \approx \frac{1}{4\pi\omega}\int d^3r e^{i\mathbf{k}\cdot\mathbf{r}}\omega_p^2(\mathbf{r})\mathbf{E}_\omega(\mathbf{r}), \qquad (3)$$

$\mathbf{k}$ is the wave vector of the emitted wave, and $\mathbf{E}_\omega(\mathbf{r})$ is the Fourier transform of the particle field with respect to time. In perturbation theory in the deviation of the dielectric permittivity from unity, the first-order expression for the quantity $\mathbf{E}_\omega(\mathbf{r})$ appearing in (3) has the form

$$\mathbf{E}_\omega^{(0)}(\mathbf{r}) = \int \frac{d^3k}{\pi} ie\frac{\mathbf{k}-\omega\mathbf{v}}{\omega^2-\mathbf{k}^2}\delta(\omega-\mathbf{k}\cdot\mathbf{v})e^{i\mathbf{k}\cdot\mathbf{r}}. \qquad (4)$$

This quantity is the Fourier transform of the unperturbed Coulomb field of a particle moving at a constant velocity $\mathbf{v}$. Substituting (4) and (3) into (2), we obtain the angular distribution of transition radiation in the form

$$\frac{dE}{d\omega do} = \frac{e^6}{m^2}\left|\frac{\mathbf{k}}{\omega}\times\mathbf{J}_k\right|^2, \qquad (5)$$

where

$$\mathbf{J}_k = \int \frac{d^3q}{2\pi^2}n_q\frac{\mathbf{k}-\mathbf{q}-\omega\mathbf{v}}{\omega^2-(\mathbf{k}-\mathbf{q})^2}\delta(\omega-(\mathbf{k}-\mathbf{q})\cdot\mathbf{v}) \qquad (6)$$

and $n_q$ is the Fourier transform of the electron density in the medium,

$$n_q = \int d^3r n_e(\mathbf{r})e^{-i\mathbf{q}\cdot\mathbf{r}}. \qquad (7)$$

Let us consider the case where the coherence length is much greater than the length of the atomic string along the particle trajectory,

$$l_{\text{coh}} \sim \frac{2\gamma^2}{\omega}\frac{1}{1+\gamma^2\theta^2} \gg \frac{2R}{\psi}, \qquad (8)$$

where $R$ is the radius of atomic-potential screening and $\theta$ is the angle between the wave vector of the emitted wave and the particle velocity (it is assumed that the particles are incident on the string at a small angle $\psi$ with respect to the string axis). In this case, the atomic string can be considered as a homogeneous and infinitely thin dielectric filament; therefore, the spatial distribution of electrons can be represented in the form of a delta function:

$$n_e(\mathbf{r}) = n_e\delta(x-z\psi)\delta(y-y_0). \qquad (9)$$

Here, $n_e$ is the electron density per unit length of the string; the $z$ axis is parallel to the particle velocity; and $y_0$ is the distance between the particle trajectory and the string axis (see Fig. 1). The Fourier transform of this distribution has the form

$$n_q = 2\pi n_e e^{-iq_y y_0}\delta(q_x\psi + q_z). \qquad (10)$$

The spectral–angular distribution (5) must be averaged over all allowed values of the impact parameter $y_0$; that is,

$$\left\langle\frac{dE}{d\omega do}\right\rangle = \frac{1}{a_y}\int_{-\infty}^{\infty} dy_0\frac{dE(y_0)}{d\omega do}, \qquad (11)$$

where $a_y$ is the spacing between neighboring strings in the crystal along the $y$ axis.

For the averaged distribution, we obtain

$$\left\langle\frac{dE}{d\omega do}\right\rangle = \frac{e^6 n_e^2\gamma}{a_y m^2\omega\psi^2}F(\theta,\varphi), \qquad (12)$$

where $F(\theta,\varphi)$ is a function that describes the distribution of radiation with respect to the polar ($\theta$) and the azimuthal ($\varphi$) emission angle [the polar angle was defined in (8)],

$$F(\theta,\varphi) = \frac{1+2\left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1+\left(\gamma\theta\cos\varphi - \dfrac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^{3/2}}, \qquad (13)$$

where $\varphi$ is the angle between the $x$ axis and the projection of the wave vector onto the $xy$ plane; it is assumed that $\theta \ll 1$.

We will now focus on some special features of the angular distribution (12) of transition radiation intensity. First of all, we note that the distribution in (12) is symmetric with respect to the atomic-string axis ($\theta = \psi$, $\varphi = 0$). In order to demonstrate this explicitly, we go over from polar to Cartesian coordinates in (13) ($\theta_x = \theta\cos\varphi$, $\theta_y = \theta\sin\varphi$) and perform the transformation according to the formulas $\theta'_x = \psi - \theta_x$ and $\theta'_y = \theta_y$. For the function $F(\theta'_x, \theta'_y)$, this yields

$$F(\theta'_x,\theta'_y) = \frac{1+\dfrac{1}{2\gamma^2\psi^2}(1+\gamma^2(\theta'^2+\psi^2))^2}{\left[1+\dfrac{1}{4\gamma^2\psi^2}(1+\gamma^2(\theta'^2+\psi^2))^2\right]^{3/2}}. \qquad (14)$$

Since this function depends only on the polar angle $\theta' = \sqrt{\theta'^2_x + \theta'^2_y}$, the distribution of radiation in (12) is symmetric with respect to the atomic-string axis.

Formula (13) indicates that the minimum of the radiation intensity is somewhat shifted from the value

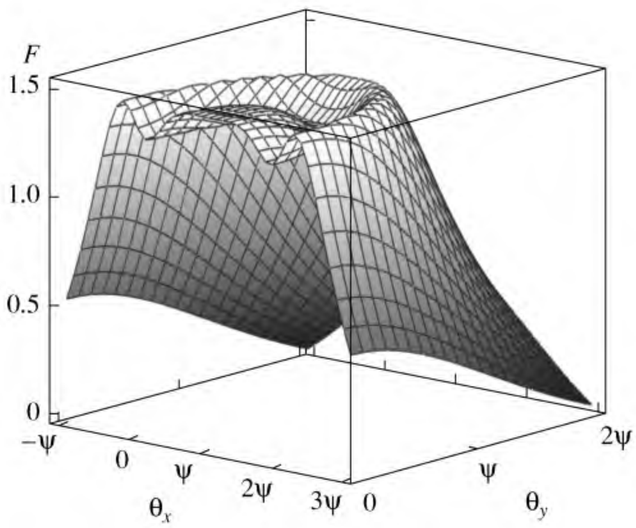


**Fig. 2.** Graph of the function $F(\theta, \varphi)$ that determines the angular distribution of transition radiation from a relativistic particle incident on a filament-like target for $\gamma = 2000$ and $\psi = 10^{-3}$; the values of $\theta_x = \theta \cos\varphi$ and $\theta_y = \theta \sin\varphi$ are plotted along the axes in the horizontal plane.

of $\theta = 0$, which corresponds to the direction of particle motion (see Fig. 2).

It should be noted that, in the vicinity of the symmetry axis of the distribution of radiation ($\theta = \psi$, $\varphi = 0$), the radiation intensity does not have a deep minimum, which often occurs there in transition-radiation problems; instead, it approaches a plateau that lies rather high.

## 3. TRANSITION RADIATION IN A COLLISION WITH A FEW ATOMIC STRINGS

We now proceed to consider special features of transition radiation generated in a collision of a particle with a few atomic strings.

If the atomic-string axes are disposed at random in the transverse plane (this corresponds to the chaotic motion of a particle in the periodic field of atomic strings in a crystal [8]), the interference between waves that are emitted in particle interaction with different atomic strings can be neglected. The total radiation will then be given by the sum of the contributions that arise from particle interactions with different atomic strings. Since the angular distribution of radiation is symmetric with respect to the axis of an atomic string, the resulting intensity of the radiation from a particle executing chaotic motion is proportional to the number of strings involved in the interaction with the particle, so that the form of the angular distribution of radiation will remain unchanged [that given by formula (12)].

But in the case where a particle executes a regular motion in a crystal, the effect of interference must be taken into account. By way of example, we indicate that, for a particle sequentially scattered by $N$ ($N \gg 1$) parallel strings at the same impact-parameter value $y_0$, expression (12) reduces to the corresponding result of the theory of transition radiation (or resonance radiation, according to the terminology used by Ter-Mikaelyan) for relativistic electrons in a crystal {see formula (28.160) in [2]}; that is,

$$\left\langle \frac{dE}{d\omega do} \right\rangle = \frac{e^6 n_e^2 \gamma}{a_y m^2 \omega \psi^2} F(\theta, \varphi) \tag{15}$$

$$\times N \frac{2\pi}{b} \sum_{n=-\infty}^{\infty} \delta\left(\frac{\omega}{2\gamma^2}(1 + \gamma^2\theta^2) - \frac{2\pi}{b} n\right),$$

where $b$ is the spacing between atomic strings along the $z$ axis. The presence of a delta function in formula (15) implies that the frequency of radiation at a given angle $\theta$ is given by

$$\omega_n = \frac{2\gamma^2}{1 + \gamma^2\theta_n^2} \frac{2\pi}{b} n. \tag{16}$$

Thus, the disposition of atomic strings in a crystal in the transverse plane and the character of particle motion have a pronounced effect on transition radiation.

## 4. COMPARISON WITH PARAMETRIC (RESONANCE) RADIATION

In the case of a three-dimensional periodic medium (for example, a crystal), the integral with respect to $\mathbf{q}$ in (6) can be replaced by a sum over a discrete set of reciprocal-lattice vectors $\mathbf{g}$. In this case, the substitution of expression (6) into (5) leads to the required result for the angular distribution of parametric radiation in a periodic three-dimensional medium {see formula (28.160) in [2]}, provided that the dielectric permittivity is taken in the form (1). The transition from the integral with respect to $\mathbf{q}$ in (6) to a sum over $\mathbf{g}$ suggests the emergence of a relation between the frequency and the emission angle. This relation arises in the case of particle motion along a straight line in a crystal and is due to the interference between waves generated by a particle interacting with atoms that occupy the sites in a periodic crystal lattice.

Formula (12) describes the angular distribution of radiation in the case of particle interaction with an isolated atomic string. In contrast to the corresponding result of the theory of parametric radiation, this formula features no relation between the frequency and the emission angle.

As was mentioned above, formula (12) is appropriate for describing the radiation from an electron moving in the field of many atomic strings if particle interactions with different strings do not interfere with one another. This type of situation is realized under the conditions of dynamical chaos accompanying particle motion in the periodic field of atomic strings in a crystal (see [8]).

Formula (12) can also be derived from the corresponding result of the theory of parametric x-ray radiation if, in this result, one considers a particle moving at a small angle with respect to a crystallographic axis (axis $z'$) and assumes that atomic strings parallel to this axis are widely spaced in the transverse direction. For this purpose, the lattice constants $a_{x'}$ and $a_{y'}$ along the $x'$ and $y'$ axes orthogonal to the crystallographic axis $z'$ must be made to tend to infinity in the Ter-Mikaelyan formula {formula (28.160) in [2]}. In this limit, summation over the components $q_{x'}$ and $q_{y'}$ in the Ter-Mikaelyan formula can then be replaced by relevant integration. If the condition in (8) is satisfied, the main contribution to the angular density of radiation then comes from the component $g_{z'} = 0$, this corresponding to the approximation of a continuous electron-density distribution along the string axis ($z'$ axis). It should be noted that the present analysis of the features of parametric x-ray radiation from a relativistic electron in a crystal is analogous to the analysis performed in Chapter 4 of [6] for coherent radiation generated by fast electrons in a crystal via various mechanisms.


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