# Transition radiation of high energy particles on fiber-like targets

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**Abstract**

The problem of transition radiation under the impact of relativistic particles at small angle with a fiber-like target such as an atomic string in a crystal or a nanotube is considered. The conditions under which the non-uniformity of the electron density along the string does not matter are obtained. The formulae for the spectral-angular distribution of transition radiation under both regular and random collisions of a particle with the set of fiber-like targets are obtained. The peculiarities of radiation on an infinitely thin target and on a target with finite transverse dimensions are discussed. The study of the radiation process is carried out in the framework of perturbation theory on the interaction of the particle with the target.

## 1. Introduction

Transition radiation arises when a charged particle crosses the boundary between two media with different dielectric properties (see [1–4] and references therein). For relativistic particle this radiation is concentrated in the region of small angles along the direction of the particle motion. The process of radiation develops in a large spatial region along the particle velocity that is called the coherence length [2,5,6]. If the particle in the limits of this region crosses some other boundaries, the interference between the radiation emitted at each boundary is substantial. It was demonstrated in [7] that for long waves not only longitudinal, but also transverse dimensions of the region of radiation formation could have macroscopic sizes. If the transverse size of the target satisfies the condition $L_\perp \leqslant \gamma\lambda$, where $\lambda$ is the length of the radiated wave and $\gamma$ is the particle's Lorentz-factor, then the transverse sizes of the target and its geometrical shape have substantial influence on the transition radiation.

Of special interest is the case of transition radiation on fiber-like targets when relativistic particles are incident under small angle $\psi$ with their axes. Attention to some peculiarities of the radiation process in this case, in the approximation where the electron density in transverse plane can be expressed in delta-function form, was paid in [8]. Particularly, it was demonstrated that in this case the angular distribution of radiation is symmetrical about the axis of the fiber.

In the present Letter the influence of transverse dimensions of the fiber on the transition radiation is

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investigated. We consider some simple distributions of the electron density in transverse plane, corresponding to the cases of atomic string in a crystal and of a long nanotube. The conditions under which the transverse distribution of the electron density of the fiber is important for the transition radiation are discussed. It is shown that account of transverse dimensions of the target leads to asymmetry of radiation in high frequency region.

## 2. Transition radiation on a dielectric fiber

Let us consider the radiation of relativistic electron moving rectilinearly with the velocity $\vec{v}$ through the medium with non-uniform dielectric function $\varepsilon_\omega(\vec{r})$ in the region of frequencies satisfying the condition $\omega \gg \omega_{\rm p}$, where $\omega_{\rm p} = \sqrt{4\pi e^2 n(\vec{r})/m}$ is the plasma frequency, $m$ and $e$ are the electron's mass and its charge, $n(\vec{r})$ is the electron density in the target. It is well known [1–4] that in this region of frequencies the dielectric function is determined by relation

$$\varepsilon_\omega(\vec{r}) \approx 1 - \frac{\omega_{\rm p}^2}{\omega^2}, \quad \omega \gg \omega_{\rm p}. \tag{2.1}$$

To first order in $\gamma^2\omega_{\rm p}^2/\omega^2$ the spectral-angular density of transition radiation can be written in the form (see, for example, [7,8])

$$\frac{dE}{d\omega\, do} = \frac{e^4}{(2\pi)^2 m^2}|\vec{e}\vec{J}_k|^2, \tag{2.2}$$

where $\vec{e}$ is the polarization vector and

$$\vec{J}_k = \int d^3r\, e^{-i\vec{k}\vec{r}} n(\vec{r})\vec{E}_\omega(\vec{r}). \tag{2.3}$$

Here $\vec{k}$ is the wave vector in the direction of radiation ($k = \omega$), $n(\vec{r})$ is the electron density in the target, and $\vec{E}_\omega$ is the Fourier component by time of the Coulomb field of the incident electron:

$$\vec{E}_\omega(\vec{r}) = \frac{2e\omega}{v^2\gamma} e^{i\frac{\omega}{v}z}\left\{\frac{\vec{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right) - i\frac{\vec{v}}{v}\frac{1}{\gamma}K_0\left(\frac{\omega\rho}{v\gamma}\right)\right\}$$
$$= \frac{e}{\pi}\int \frac{\vec{k} - \omega\vec{v}}{\omega^2 - \vec{k}^2}\delta(\omega - \vec{k}\vec{v})e^{i\vec{k}\vec{r}}\, d^3k. \tag{2.4}$$

For some problems it could be convenient to use in (2.3) the Fourier transforms of the electron density $n(\vec{r})$,

$$n_q = \int d^3r\, n(\vec{r})e^{-i\vec{q}\vec{r}},$$

and the Coulomb field. In this case we can write the value (2.3) in the form [8]

$$\vec{J}_k = \int \frac{d^3q}{\pi} n_q \frac{\vec{k} - \vec{q} - \omega\vec{v}}{\omega^2 - (\vec{k} - \vec{q})^2}\delta\big(\omega - (\vec{k} - \vec{q})\vec{v}\big). \tag{2.5}$$

(We use the system of units in which the velocity of light is taken equal to unity, $c = 1$.) The spectral-angular distribution of radiation summed over polarizations is equal to

$$\frac{dE}{d\omega\, do} = \frac{e^4}{(2\pi)^2 m^2}\left|\frac{\vec{k}}{\omega} \times \vec{J}_k\right|^2. \tag{2.6}$$

Consider transition radiation of a relativistic particle incident on a thin dielectric fiber at small angle $\psi \ll 1$ with its axis. An atomic string in a crystal [6] or a nanotube [9,10] can be treated as such fiber.

The coherence length of radiation for relativistic electrons has the following form [6]:

$$l_{\rm coh} = \frac{2\gamma^2}{\omega}\frac{1}{1 + \gamma^2\theta^2 + (\gamma\omega_{\rm p}/\omega)^2}, \quad \theta \ll 1, \tag{2.7}$$

where $\theta$ is the angle between the wave vector of the radiated wave and the particle velocity. If the particle interacts with large number of atomic electrons within the coherence length one can use the electron density distribution in the fiber averaged along its axis:

$$n(\vec{\rho}\,') = \frac{1}{L}\int dz'\, n(\vec{r}\,'), \tag{2.8}$$

where $L$ is the length of the fiber, the $z'$ axis is parallel to the fiber axis, $\vec{\rho}\,' = (x', y')$ are the coordinates in the transverse plane.

If, in addition, the conditions

$$l_{\rm coh} \gg \frac{2R}{\psi}, \qquad \frac{\gamma}{\omega} \gg R, \tag{2.9}$$

where $R$ is the transverse size of the fiber, are satisfied then the target can be treated as an uniform infinitely thin fiber. The electron density distribution in this case can be written using delta-function

$$n(\vec{r}\,') = n_e\delta(x')\delta(y'),$$

where $n_e$ is the electron density per unit length of the fiber.

When the electron is incident under small angle $\psi$ to the fiber axis, it is convenient for calculating the spectral-angular density of radiation to transform the system of coordinates $(x', y', z')$ connected with the fiber to the system of coordinates $(x, y, z)$ in which the $z$ axis is parallel to the particle velocity $\vec{v}$. In the new system of coordinates the electron density distribution can be written in the form

$$n(\vec{r}) = n_e \delta(x - z\psi)\delta(y - y_0) \tag{2.10}$$

taking into account that the electron moves at distance $y_0$ to the fiber axis. The coordinate $y$ here is perpendicular to the fiber axis $z'$ and the particle velocity vector $\vec{v}$.

Substituting (2.10) to the formula for spectral-angular distribution of radiation, we obtain after integration (2.6) over impact parameters $y_0$ and $x_0$ the following expression for radiation efficiency [8]:

$$\frac{dK}{d\omega do} = L\psi \int_{-\infty}^{\infty} dy_0 \frac{dE}{d\omega do} = \frac{Le^6 n_e^2 \gamma}{m^2 \omega \psi} F(\theta, \varphi), \tag{2.11}$$

where $L$ is the length of the fiber and $F(\theta, \varphi)$ is the function that determines the angular distribution of radiation,

$$F(\theta, \varphi) = \frac{1 + 2\left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1 + \left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^{3/2}}. \tag{2.12}$$

Here $\varphi$ is the azimuth angle (the angle between $x$ axis and the projection of wave vector $\vec{k}$ to the plane $(x, y)$). The surface plot of the function (2.12) for $\psi = 10^{-3}$, $\gamma = 2000$ is presented on Fig. 1 (the upper plot). It is easy to see that the angular distribution of radiation intensity possesses the axial symmetry relatively to the fiber axis ($\theta = \psi$, $\varphi = 0$), that can be demonstrated analytically from (2.12). Near the axis of symmetry the intensity of radiation is rather high. With increasing of the incidence angle $\psi$ a local minimum of intensity arises in the center of the distribution (see Fig. 2, the upper plot). The angular distribution of intensity takes the shape of a narrow double ring of the radius $\psi$ for $\psi \geqslant 10\gamma^{-1}$. Note in connection with this, that the radiation on the fiber can be interpreted as the radiation produced by the perturbation created in the fiber by the relativistically compressed Coulomb field of the incident particle. Such perturbation moves along the fiber with the velocity exceeding the velocity of light. Analogous situation arises in the case of incidence of electromagnetic wave under some angle to thin linear conductor [11]. In this case the electromagnetic wave creates in the conductor a perturbation moving faster than the speed of light. This situation leads to a radiation analogous to Cherenkov one.

Consider now the polarization of radiation on dielectric fiber. Choosing the polarization vectors in

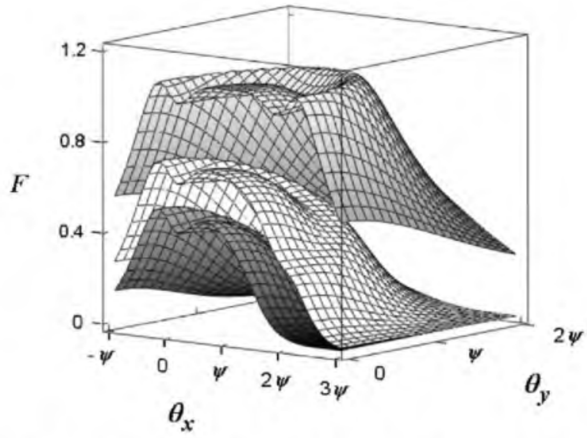


Fig. 1. Surface plots of the function $F(\theta, \varphi)$ (Eq. (2.12); upper surface) and $F_g(\theta, \varphi)$ (Eq. (3.5); $R\omega/\gamma = 0.1\psi\gamma$, middle surface, $R\omega/\gamma = 0.2\psi\gamma$, lower surface) that determine the angular distribution of transition radiation of relativistic particle on thin fiber-like target for the case $\psi = 10^{-3}$, $\gamma = 2000$ ($\theta_x = \theta\cos\varphi$, $\theta_y = \theta\sin\varphi$).

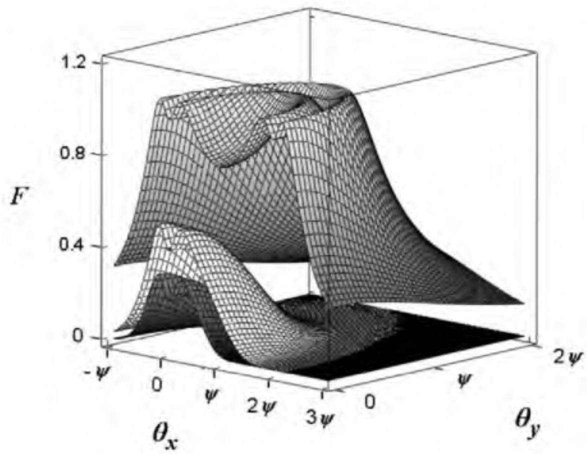


Fig. 2. The same as on Fig. 1 for the case $\psi = 2 \times 10^{-3}$.

the form

$$\vec{e}^{(1)} = \frac{\vec{k} \times \vec{e}_x}{|\vec{k} \times \vec{e}_x|}, \qquad \vec{e}^{(2)} = \frac{\vec{k} \times \vec{e}^{(1)}}{\omega}, \tag{2.13}$$

we find that the radiation is partially polarized in $\vec{e}^{(1)}$ direction with polarization

$$P \equiv \frac{I^{(1)} - I^{(2)}}{I^{(1)} + I^{(2)}} = \frac{1}{1 + 2\left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}, \tag{2.14}$$

where $I^{(1)}$ and $I^{(2)}$ are the spectral-angular densities of radiation of electromagnetic waves with polarization vectors $\vec{e}^{(1)}$ and $\vec{e}^{(2)}$.

## 3. Radiation on a fiber-like target of finite thickness

Now let us investigate the influence of transverse dimensions of the fiber on the transition radiation. At first, consider the simplest case of uniform distribution of electron density in a cylinder of the radius $R$. For such distribution we obtain the result (2.11) with the factor $F_c(\theta, \varphi)$ instead of $F(\theta, \varphi)$:

$$F_c(\theta, \varphi) = \frac{2}{\pi} \int_{-\infty}^{\infty} dq \left( \frac{2J_1\left(\frac{R\omega}{\gamma}\sqrt{q^2 + \left(\frac{1+\gamma^2\theta^2}{2\psi\gamma}\right)^2}\right)}{\frac{R\omega}{\gamma}\sqrt{q^2 + \left(\frac{1+\gamma^2\theta^2}{2\psi\gamma}\right)^2}} \right)^2 \times \Phi(q, \theta, \varphi), \tag{3.1}$$

where $J_1(x)$ is the Bessel function of the first kind and

$$\Phi(q, \theta, \varphi) = \frac{(\gamma\theta\sin\varphi - q)^2 + \left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}{\left[1 + (\gamma\theta\sin\varphi - q)^2 + \left(\gamma\theta\cos\varphi - \frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2\right]^2}. \tag{3.2}$$

The first factor in the integrand in (3.1) leads to violation of the axial symmetry of the angular distribution of radiation about the axis of the fiber, considered in Section 2. However, the symmetry of intensity distribution respectively to the plane containing the $z$ axis and the axis of the fiber is conserved. The surface plots of $F_c(\theta, \varphi)$ for two different values of $R$ are pictured on Fig. 3. According to (3.1), radiation in the region of large angles $\theta$ is suppressed.

Analogous situation arises in the case of a fiber with Gaussian distribution of electron density in transverse plane:

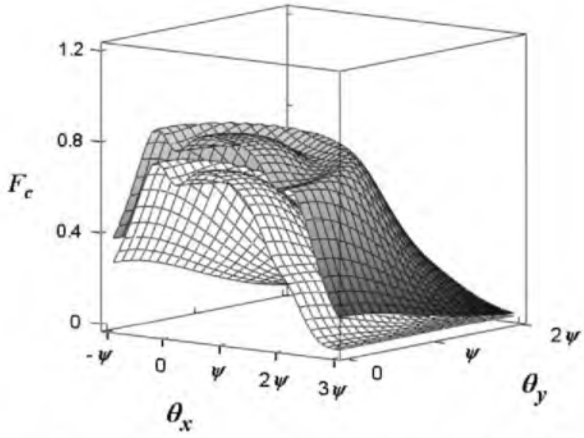


Fig. 3. Surface plots of the function $F_c(\theta, \varphi)$ (Eq. (3.1)) for $\psi = 10^{-3}$, $\gamma = 2000$, $R\omega/\gamma = 0.1\psi\gamma$ (upper surface) and $R\omega/\gamma = 0.2\psi\gamma$ (lower surface).

$$n(\vec{r}) = \frac{n_e}{2\pi R^2} \exp\left[-\frac{(x')^2 + (y - y_0)^2}{2R^2}\right]. \tag{3.3}$$

Such a distribution of electron density takes place in the case of single atomic string, for example. The parameter $R$ is equal by the order of magnitude to the Thomas–Fermi screening radius of the potential of a single atom.

Fourier transformation of distribution (3.2) in the system of coordinates $(x, y, z)$, in which the $z$ axis is directed along the particle velocity vector (in this case for small $\psi$ we have $x' \approx x - \psi z$), has the form

$$n_q = 2\pi n_e e^{i q_y y_0} \delta(q_x \psi + q_z) \times \exp\left[-\frac{(q_x^2 + q_y^2) R^2}{2}\right]. \tag{3.4}$$

Substituting this equation in (2.6), we obtain after integrating over impact parameter $y_0$ the formula (2.11), which contains the function

$$F_g(\theta, \varphi) = \frac{2}{\pi} \exp\left[-\left(\frac{R\omega}{2\psi\gamma^2}\left(1 + \theta^2\gamma^2\right)\right)^2\right] \times \int_{-\infty}^{\infty} dq \exp\left[-\left(\frac{R\omega}{\gamma} q\right)^2\right] \Phi(q, \theta, \varphi), \tag{3.5}$$

instead of function (2.12).

The presence of exponential factors in this formula also leads to violation of the axial symmetry of the angular distribution of radiation, as described above. However, the conditions (2.9) are satisfied in the region, where frequencies $\omega$ and radiation angles $\theta$ are small enough. In this case the exponential factors in (3.5) (or additional factor in (3.1)) can be replaced by unity; after that the functions (3.1) and (3.5) coincide with (2.12). In other words, the details of the transverse distribution of electrons become unimportant for transition radiation under these conditions.

At least, consider the case when the distribution of electrons in the transverse plane has the form

$$n(\vec{r}) = \frac{n_e}{2\pi\sqrt{\pi}RR_n}\exp\left[-\frac{(\rho' - R_n)^2}{R^2}\right]. \qquad (3.6)$$

Nanotube has such distribution of electron density. In this case $R_n$ is the nanotube radius ($R_n \sim 5 \times 10^{-8}$ cm) and $R$ is the Thomas–Fermi radius ($R \sim 10^{-9}$ cm) [9,10], so $R_n \gg R$. In the limitation case $R \to 0$ we can approximate (3.6) by the formula

$$n(\vec{r}) = \frac{n_e}{2\pi R_n}\delta(\rho' - R_n). \qquad (3.7)$$

For such distribution of electron density after transformation to $(x, y, z)$ coordinates we obtain the result (2.11) with the function $F_n(\theta, \varphi)$ instead of (2.12):

$$F_n(\theta, \varphi) = \frac{2}{\pi}\int_{-\infty}^{\infty} dq\left[J_0\left(\frac{R_n\omega}{\gamma}\sqrt{q^2 + \left(\frac{1+\gamma^2\theta^2}{2\gamma\psi}\right)^2}\right)\right]^2 \times \Phi(q, \theta, \varphi), \qquad (3.8)$$

where $J_0(x)$ is the Bessel function of the first kind and $\Phi(q, \theta, \varphi)$ is determined by (3.2). The function $F_n(\theta, \varphi)$ is plotted on Fig. 4.

Note that the lower surface on Fig. 1 (that corresponds to Gaussian distribution of electron density in transverse plane of the fiber), the lower surface on Fig. 3 (corresponding to uniform distribution of electron density in cylindrical fiber), and upper surface on Fig. 4 (for a tube with thin walls) are plotted for the same characteristic transverse size of a target (and other parameters). Comparing these three figures we can see that the angular distribution of radiation intensity depends on details of the target structure. This fact creates new possibilities in diagnostics of nanostructures.

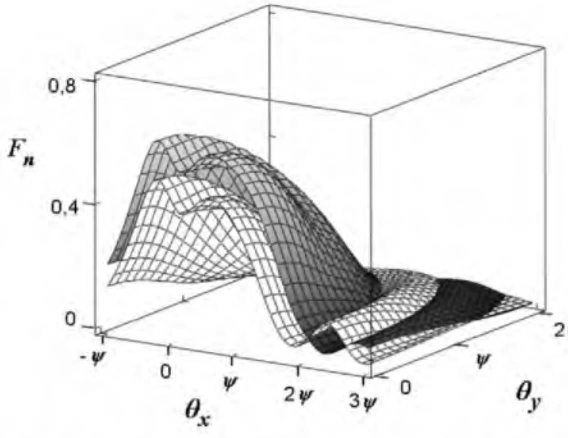


Fig. 4. Surface plots of the function $F_n(\theta, \varphi)$ (Eq. (3.8)) for $\psi = 10^{-3}$, $\gamma = 2000$, $R_n\omega/\gamma = 0.2\psi\gamma$ (upper surface) and $R_n\omega/\gamma = 0.3\psi\gamma$ (lower surface).


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