Radiation on a Semi-Infinite Plate and the Smith–Purcell Effect in the X-Ray Range

V. V. Syshchenko[a] and N. F. Shulga[b]

[a] *Belgorod State University, Belgorod, Russia*
e-mail: syshch@yandex.ru, syshch@bsu.edu.ru
[b] *Akhiezer Institute for Theoretical Physics, National Science Center, Kharkiv Institute of Physics and Technology, Kharkiv, Ukraine*

Abstract—Smith–Purcell radiation is usually detected at the motion of an electron above a grating of metal strips. However, for high frequencies (exceeding the plasma frequency of the grating material) no substance can be considered a conductor, but instead should be considered a dielectric with plasma-type permeability. Therefore, new approaches are required to describe the Smith–Purcell radiation in the high frequency range. In this article a previously developed variant of eikonal approximation is applied for description of the radiation on a set of parallel semi-infinite plates. The derived equations describe the radiation generated both by the particles passing across the plates (usually referred to as "transition radiation") and by the particles traveling above the grating formed by the edges of the plates (usually referred to as "diffraction radiation," or, taking into account the periodicity of the grating, as the Smith–Purcell radiation).

INTRODUCTION

The Smith–Purcell effect consists of emission of electromagnetic waves at the motion of an electron above periodic grating [1]. In recent years there has been increasing interest in this effect as a promising method of generating radiation in the terahertz [2] to soft X-ray [3] range, as well as a new method of monitoring a bunch [4]. The article [3] discusses the parameters of the electron bunch required for efficient generation of X-ray beams according to the Smith–Purcell mechanism. The estimation of the radiation intensity was aided by the concepts of the theory of diffraction radiation on opaque screen [5]. However, in the high frequency range (exceeding the plasma frequency of the target material), no substance can be considered as an ideally opaque conductor, but instead should be considered as a dielectric with permeability $\varepsilon_\omega = 1 - \omega_p^2/\omega^2$. Therefore, new approaches are required to describe the Smith–Purcell radiation in the high frequency range.

A straightforward method of eikonal approximation for description of transition radiation on targets of complex configuration has been proposed elsewhere [6,7]. In this article we apply this method for radiation on a set of parallel semi-infinite plates. The derived equations describe in a proper manner the radiation appearing both upon the motion of a particle across the plates (conventionally referred to as "transition radiation") and upon the motion of a particle in vacuum above the grating formed by the edges of the plates (conventionally referred to as "diffraction radiation," or, taking into account the periodicity of the spatial arrangement of the plates, as the Smith–Purcell radiation). It will be shown that, in the discussed frequency range, for sufficiently low values of impact parameter (as in all articles devoted to the application of the Smith–Purcell effect), the intensity of the Smith–Purcell radiation is comparable in terms of order of magnitude with the intensity of the transition radiation. This confirms the possibility of applying the Smith–Purcell effect in development of new sources of radiation in the soft X-ray range. Here a system of units is used in which the speed of light in a vacuum is $c = 1$.

EXPERIMENTAL

Let us consider a particle with charge e moving at constant velocity $\mathbf{v}$ in heterogeneous medium with dielectric permeability $\varepsilon_\omega(\mathbf{r})$. It can be shown [6, 7] that the spectral-angular density of the resulting radiation will be described as follows:

$$\frac{dE}{d\omega do} = \frac{\omega^2}{(8\pi^2)^2}|\mathbf{k}\times\mathbf{I}|^2, \tag{1}$$

where ω and $\mathbf{k}$ are the frequency and wave vector of the radiated wave,

$$\mathbf{I} = \int d^3r \exp(-i\mathbf{kr})(1-\varepsilon_\omega(\mathbf{r}))\mathbf{E}_\omega(\mathbf{r}), \tag{2}$$

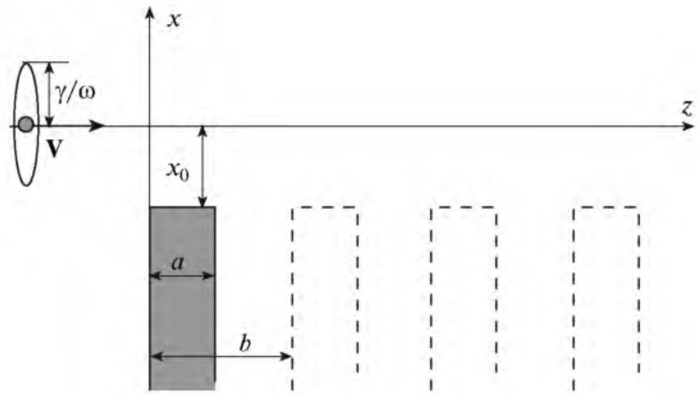

Fig. 1. Particle motion in the vicinity of plate edge or a set of parallel plates. The parameter γ/ω is the characteristic transversal value of the Fourier component of particle field corresponding to the frequency ω. Positive values of the impact parameter x_0 correspond to the motion of a particle across the plates.

$\mathbf{E}_\omega(\mathbf{r})$ is the Fourier component of the electric field generated in the target substance as particle passes,

$$\mathbf{E}_\omega(\mathbf{r}) = \int_{-\infty}^{\infty} \mathbf{E}(\mathbf{r}, t)\exp(i\omega t)dt.$$

Let us consider the transition radiation in the high frequency range, when the dielectric permeability of the target is defined as follows:

$$\varepsilon_\omega(\mathbf{r}) \approx 1 - \omega_p^2/\omega^2, \omega \gg \omega_p,$$

where $\omega_p = \sqrt{4\pi e^2 n(\mathbf{r})/m}$ is the plasma frequency; m and e are the weight and charge of an electron, respectively; and $n(\mathbf{r})$ is the density of electrons in target. In the first order, in terms of small quantity $(1 - \varepsilon_\omega)$ the precise value of the field in the target substance in Eq. (2) can be substituted by the nondisturbed Coulomb field of a particle moving consistently in vacuum, which will be similar to the Born approximation in the quantum theory of scattering [8]. Although this approximation makes it possible to calculate the properties of the transition radiation on targets with a complex configuration, its applicability is limited to the range of very high frequencies,

$$\omega \gg \gamma\omega_p, \quad (3)$$

where $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor of the particle.

The transition radiation in the range of softer photons can be investigated by simple variant of eikonal approximation proposed elsewhere [6, 7]. In this approximation the component **I**, perpendicular to the particle velocity **v**, can be rewritten as follows:

$$\mathbf{I}_\perp^{(eik)} = i\frac{4e}{v^2\gamma}\int d^2\rho e^{-i\mathbf{k}_\perp\boldsymbol{\rho}} \times \frac{\boldsymbol{\rho}}{\rho}K_1\left(\frac{\omega\rho}{v\gamma}\right)\left\{\exp\left[-i\frac{\omega}{2}\int_{-\infty}^{\infty}(1 - \varepsilon_\omega(\mathbf{r}))dz\right] - 1\right\}, \quad (4)$$

where $K_1(x)$ is the modified Bessel function of the third kind (the Macdonald function), the z axis is directed along **v**, and $\boldsymbol{\rho}$ is the component **r** perpendicular to **v**. The eikonal approximation is valid at

$$\omega \gg \omega_p \quad (5)$$

but only for low angles of radiation,

$$\theta \ll (a\omega)^{-1/2}, \quad (6)$$

where a is the target thickness. The latter restriction provides the capability of taking into account only transversal component of the vector **I** upon calculation of spectral-angular density of the radiation.

RESULTS AND DISCUSSION

Let us consider the radiation appearing at interaction of a moving particle with semi-finite dielectric plate, a in thickness (Figure 1). In this case Eqs. (1) and (4) gives the following result for spectral-angular density of radiation:

$$\frac{dE}{d\omega do} = \frac{e^2\gamma^2}{2\pi^2}\left\{1 - \cos\left[\frac{\omega_p^2}{\omega^2}\frac{\omega a}{2}\right]\right\}F(\theta_x, \theta_y), \quad (7)$$

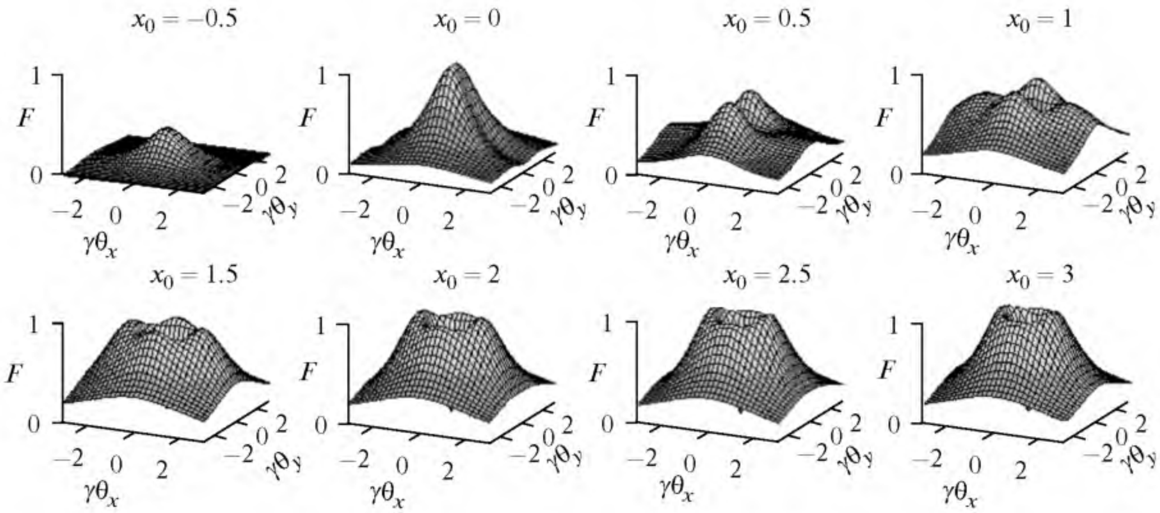

Fig. 2. Angular distribution of radiation on a semi-infinite plate according to Eq. (8). The impact parameter is expressed in terms of γ/ω.

where θ_x and θ_y are the components on two-dimensional angle of radiation $\boldsymbol{\theta}$,

$$F(\theta_x, \theta_y) = \frac{1 + 2\gamma^2\theta_y^2}{\left(1 + \gamma^2\theta_y^2\right)\left(1 + \gamma^2\theta^2\right)} \exp\left(-2|x_0|\frac{\omega}{\gamma}\sqrt{1 + \gamma^2\theta_y^2}\right) \quad (8a)$$

at $x_0 \leq 0$ and

$$F(\theta_x, \theta_y) = \frac{4\gamma^2\theta^2}{\left(1 + \gamma^2\theta^2\right)^2} + \frac{1 + 2\gamma^2\theta_y^2}{\left(1 + \gamma^2\theta_y^2\right)\left(1 + \gamma^2\theta^2\right)} \exp\left(-2|x_0|\frac{\omega}{\gamma}\sqrt{1 + \gamma^2\theta_y^2}\right) - \frac{4\exp\left(-|x_0|\frac{\omega}{\gamma}\sqrt{1 + \gamma^2\theta_y^2}\right)}{\left(1 + \gamma^2\theta^2\right)^2} \times \left(\frac{\gamma\theta_x \sin(x_0\omega\theta_x)}{\sqrt{1 + \gamma^2\theta_y^2}} + \gamma^2\theta^2\cos(x_0\omega\theta_x)\right) \quad (8b)$$

at $x_0 > 0$. In Eq. (8b) the first term describes the transition radiation on semi-finite plate and the second term describes the effect of the plate edge (diffraction radiation, as in Eq. (8a)), the third term describes the interference of these two radiations. Equations (7) and (8) are derived in accordance with the results [9] attained by the method given in [10]. The plots of the function $F(\theta_x, \theta_y)$ for certain values of the impact parameter x_0 are illustrated in Fig. 2. Within the scope of the approach we proposed, the angular distribution of the radiation is symmetrical relative to the plane (y, z) both for positive and negative values of x_0.

Now let us consider the radiation on periodic grating consisting of such plates. It is impossible to take into account directly the periodic structure of such target as a result of integration along the z axis in the index of the exponent in Eq. (4). Nevertheless, if the length of radiation generation (the length of coherence) [5] does not exceed the distance between two plates, then

$$l_{\text{coh}} = \frac{2\gamma^2/\omega}{1 + \gamma^2\theta^2 + \gamma^2\omega_p^2/\omega^2} < b - a, \quad (9)$$

the influence of the target periodicity will consist of interference of electromagnetic waves emitted upon interaction of a particle with separate plates. In this case the radiation will be described by Eq. (7) multiplied by the factor

$$2\pi N \sum_{m=-\infty}^{\infty} \delta\left\{\omega b\left(\frac{1}{v} - \cos\theta\right) - 2\pi m\right\}, \quad (10)$$

where N is the total number of the plates, $N \gg 1$ (as in most works devoted to transition radiation we neglect diffraction of the emitted radiation in the target substance). The delta-functions in Eq. (10) mean that the radiation at angle θ can exist only for the frequencies meeting the condition

$$\omega = \frac{2\pi m}{b\left(\frac{1}{v} - \cos\theta\right)} \quad (11a)$$

(where m is a positive integer number) or, for small angles of radiation,

$$\omega_m = \frac{2\gamma^2}{1 + \gamma^2\theta_m^2} \frac{2\pi}{b} m. \qquad (11b)$$

Equation (11) is the well-known Smith–Purcell condition [1].

Therefore, Eqs. (7), (8a), and (10) describe the Smith–Purcell radiation for a high frequency range: $\omega \gg \omega_p$. The exponent in Eq. (8a) describes the radiation intensity as a function of the impact parameter. The characteristic values of the latter when the radiation is significant are in the following range:

$$|x_0|_{\text{eff}} \leq \gamma/2\omega. \qquad (12)$$

Let us note that such an exponential coefficient is included in the equations describing the Smith–Purcell effect, irrespective of the considered frequency range and method of calculations [11]: the results of various models vary from each other only in the preexponential factor.

It can be seen from Figure 2 that, in the range of impact parameters satisfying Eq. (12), the intensity of the Smith–Purcell radiation is comparable with the intensity of the transition radiation. Thus, it is possible to conclude that the Smith–Purcell effect can be applied for generation of soft X-ray radiation.

CONCLUSIONS

On the basis of a simple variant of eikonal approximation in the theory of transition radiation, a unified description of the Smith–Purcell radiation and transition radiation on a set of semi-infinite plates in the range of high radiation frequencies is presented. It is demonstrated that the intensity of the Smith–Purcell radiation in the soft X-ray range is comparable with the intensity of transition radiation. This feature can be applied for development of new radiation sources.

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