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SHARING SET AND NORMAL FUNCTION OF HOLOMORPHIC FUNCTIONS

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Abstract. In this paper, we use the idea of sharing set to prove: Let $\mathfrak{F}$ be a family of holomorphic functions in the unit disc, a_1 and a_2 be two distinct finite numbers and $a_1 + a_2 \neq 0$. If for any $f \in \mathfrak{F}$, $E_f(S) = E_{f'}(S), S = \{a_1, a_2\}$, in the unit disc, then f is an α-normal function.

Keywords: entire functions, uniqueness, Nevanlinna theory, normal family.

1 Introduction and main results

Let D be a domain in $\mathbb{C}$ and let $\mathfrak{F}$ be a family of meromorphic functions defined in D. The family $\mathfrak{F}$ is said to be normal in D, in the sense of Montel, if each sequence $\{f_n\} \subset \mathfrak{F}$ contains a subsequence $\{f_{n_j}\}$ that converges, spherically locally uniformly in D, to a meromorphic function or to ∞.(see. [10])

In this paper, we assume that f, g are two meromorphic functions on D and S_1, S_2 are two sets. We denote $\overline{E}_f(S_1) \subset \overline{E}_g(S_2)$ by $f(z) \in S_1 \Rightarrow g(z) \in S_2$. If $\overline{E}_f(S_1) = \overline{E}_g(S_2)$, we denote this condition by $f(z) \in S_1 \Leftrightarrow g(z) \in S_2$. Similarly, if $E_f(S_1) = E_g(S_2)$, we denote this condition by $f(z) \in S_1 \rightleftharpoons g(z) \in S_2$. If the set S has only one element, say a, we denote $f(z) \in S$ by $f(z) = a$ (see [15]).

Schwick[14] was the first to draw a connection between values shared by functions in $\mathfrak{F}$ (and their derivatives) and the normality of the family $\mathfrak{F}$. Specially, he showed that if there exist three distinct complex numbers a_1, a_2, a_3 such that f and f' share $a_j (j = 1, 2, 3)$ in D for each $f \in \mathfrak{F}$, then $\mathfrak{F}$ is normal in D. Pang and Zalcman [9] extended this result as follows.

Theorem A. *Let $\mathfrak{F}$ be a family of meromorphic functions in a domain D, and let a, b, c, d be complex numbers such that $c \neq a$ and $d \neq b$. If for each $f \in \mathfrak{F}$ we have $f(z) = a \Leftrightarrow f'(z) = b$ and $f(z) = c \Leftrightarrow f'(z) = d$, then $\mathfrak{F}$ is normal in D.*

Definition 1.1 (see. [6, 7]) *A meromorphic function f is a normal function in the unit disc D if and only if there exists a constant $C(f)$ (which depends on f) such that*

$$(1 - |z|^2) f^{\sharp}(z) < C(f),$$

where $f^{\sharp}(z) = |g'(z)|/(1 + |g(z)|^2)$ is the spherical derivative of f.

In 2000, X.C. Pang [8] considered the normal function by using the condition of share values.

Theorem B. *Let $\mathfrak{F}$ be a family of meromrophic functions in the unit disc, a_1, a_2 and a_3 be three distinct finite numbers. If for any $f \in \mathfrak{F}$,*

$$\overline{E}_f(a_i) = \overline{E}_{f'}(a_i), \quad i = 1, 2, 3,$$

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in the unit disc, then there exists a positive M, such that for every $f \in \mathfrak{F}$, we have

$$(1-|z|^2)f^\sharp(z) < M,$$

where M depends on a_1, a_2 and a_3.

In fact, from the proof of Theorem B, one can get the following corollary.

Corollary 1.2 *Let $\mathfrak{F}$ be a family of holomorphic functions in the unit disc, a_1 and a_2 be two distinct finite numbers. If for any $f \in \mathfrak{F}$,*

$$\overline{E}_f(a_i) = \overline{E}_{f'}(a_i), \quad i = 1, 2,$$

in the unit disc, then the conclusion of Theorem B holds.

Recently, there exist a lot of studies in using the shared set to obtain the normal family(see. [2, 4, 5]). X.J. Liu obtained a normal function by using the share set $S = \{a_1, a_2, a_3\}$ corresponding Theorem B. Naturally, we ask whether there exists a normal function by using the shared set $S = \{a_1, a_2\}$ corresponding to Corolllary 1.2? In this paper, we study the question and get the following result.

Theorem 1.3 *Let $\mathfrak{F}$ be a family of holomorphic functions in the unit disc, a_1 and a_2 be two distinct finite numbers and $a_1 + a_2 \neq 0$. If for any $f \in \mathfrak{F}$,*

$$E_f(S) = E_{f'}(S), \quad S = \{a_1, a_2\},$$

in the unit disc, then there exists a positive M, such that for every $f \in \mathfrak{F}$, we have

$$(1-|z|^2)f^\sharp(z) < M,$$

where M depends on S.

In the following, we give a example to show the condition $a_1 + a_2 \neq 0$ is necessary.

Example 1.4 ([5]) *Let $S = \{-1, 1\}$. Set $\mathfrak{F} = \{f_n(z) : \ n = 2, 3, 4, \ldots\}$, where*

$$f_n(z) = \frac{n+1}{2n}e^{nz} + \frac{n-1}{2n}e^{-nz}, \quad D = \{z : |z| < 1\}.$$

Then, for any $f_n \in \mathfrak{F}$, we have

$$n^2[f_n^2(z) - 1] = f_n'^2(z) - 1.$$

Thus f_n and f_n' share S CM, but f_n is not a normal function in D.

From Case 1 in the proof of Theorem 1.3, we can easily get the following corollary.

Corollary 1.5 *Let $\mathfrak{F}$ be a family of functions holomorphic in a domain D, let a be a nonzero finite complex numbers. If for all $f \in \mathfrak{F}$, f and f' share $S = \{0, \ a\}$ IM, then the conclusion of the theorem 1.3 holds.*

The following example shows that it is necessary that the complex numbers a is finite.

Example 1.6 *Let $S = \{0, \infty\}$. Set $\mathcal{F} = \{e^{nz} : n = 1, 2, \ldots\}$ in the unite disc Δ, thus $f_n = e^{nz}$ and $f'_n = ne^{nz}$ share S, but f is not a normal function in Δ.*

Definition 1.7 ([11]) *Given $0 < \alpha < \infty$, if there exists a constant $C_\alpha(f)$ such that*

$$(1 - |z|^2)^\alpha f^\sharp(z) < C_\alpha(f),$$

for each $z \in D$, we say that f is an α-normal function in D.

α-normal functions may be viewed as the generalizations of normal functions. If we denote by N the class of the normal functions in D and denote by N^α the class of the α-normal functions in D, it is obvious that

$$N^{\alpha_1} \subset N \subset N^{\alpha_2}$$

for $0 < \alpha_1 < 1 < \alpha_2 < \infty$. The above inclusion relations are strict(see.[12]). Similarly, we can get the following generalized result.

Theorem 1.8 *Let $\alpha \geq 1$, and let $\mathcal{F}$ be a family of holomorphic functions in the unit disc, a_1 and a_2 be two distinct finite numbers and $a_1 + a_2 \neq 0$. If for any $f \in \mathcal{F}$,*

$$E_f(S) = E_{f'}(S), \quad S = \{a_1, a_2\},$$

in the unit disc, then there exists a positive M, such that for every $f \in \mathcal{F}$, we have

$$(1 - |z|^2)^\alpha f^\sharp(z) < M,$$

where M depends on S.

2 Lemmas

Lemma 2.1 ([9]) *Let $\mathcal{F}$ be a family of functions meromorphic on the unit disc, all of whose zeros have multiplicity at least k, and suppose that there exists $A \geq 1$ such that $|f^{(k)}(z)| \leq A$ whenever $f \in \mathcal{F}$ and $f(z) = 0$, $f \in \mathcal{F}$. Then if $\mathcal{F}$ is not normal, then there exist, for each $0 \leq \lambda \leq k$,*

(a) a number $0 < r < 1$;

(b) points z_n, $z_n < 1$;

(c) functions $f_n \in \mathcal{F}$, and

(d) positive number $\rho_n \to 0$ such that $\rho_n^{-\lambda} f_n(z_n + a_n\xi) = g_n(\xi) \to g(\xi)$ locally uniformly, where g is a nonconstant meromorphic function on C such that $g^\sharp(\xi) \leq g^\sharp(0) = A + 1$.

The normal lemma is for α-normal functions corresponding to Lemma 2.1.

Lemma 2.2 *Let $\mathcal{F}$ be a family of functions meromorphic on the unit disc, all of whose zeros have multiplicity at least k, and suppose that there exists $A \geq 1$ such that $|f^{(k)}(z)| \leq A$ whenever $f \in \mathcal{F}$ and $f(z) = 0$, $f \in \mathcal{F}$. Then if $\mathcal{F}$ is not an α-normal function, then there exist, for each $0 \leq \lambda \leq k$ and $1 \leq \alpha < \infty$, there exist a sequence of points $\{z_n\}$ in D and a sequence of positive numbers $\{\rho_n\}$ such that $|z_n| \to 1$, $\rho_n \to 0$, and the sequence of functions*

$$\{g_n(\zeta)\} = \rho_n^{-\lambda} f(z_n + (1 - |z_n|^2)^\alpha \rho_n \zeta)$$

converges spherically and locally uniformly to a non-constant Yosida function in the ζ-plane.

Remark. The case $0 \leq \lambda < k$ is first proved by Chen and Wulan, see [12, 13] for a detail. We can prove the above lemma by the similar method with [13].

3 Proof of Theorem 1.8

Suppose, to the contrary, that we can find $|z_n| < 1$ and $f_n \in \mathcal{F}$ such that

$$g_n(z) = f_n(z_n + (1 - |z_n|^2)^\alpha z) \tag{3.1}$$

satisfy

$$\lim_{n\to\infty} g^\sharp(0) = \lim_{n\to\infty}(1 - |z_n|^2)^\alpha f^\sharp(z_n) = \infty.$$

Hence $\{g_n(z)\}$ is not normal in the unit. By Lemma 2.1, we can find the positive number r, $0 < r < 1$; the complex numbers ζ_n, $|\zeta_n| < 1$; $\rho_n \to 0^+$ and $g_n \in \mathcal{F}$ such that

$$G_n(\zeta) = g_n(\zeta_n + \rho_n\zeta) = f_n(z_n + (1 - |z_n|^2)^\alpha\zeta_n + (1 - |z_n|^2)^\alpha\rho_n\zeta)$$

locally uniformly to a nonconstant entire function $G(\zeta)$ on C.

We know G is a nonconstant entire function. Without loss of generality, we can assume that $G - a_1$ has zeros in C. Let ζ_0 is a zero of $G - a_1$. Consider the family

$$\mathcal{H} = \{H_n(\zeta) : H_n(\zeta) = \frac{G_n(\zeta) - a}{(1 - |z_n|^2)^\alpha\rho_n}\}.$$

We claim $\mathcal{H}$ is not normal at ζ_0. In fact, $G(\zeta_0) = a_1$ and $G(\zeta) \not\equiv a_1$. From (3.1) and Hurwitz's Theorem, there exist ζ_n, $\zeta_n \to \zeta_0$ and $G_n(\zeta_n) = a_1$. Then $H_n(\zeta_n) = 0$. However, there exists a positive number δ such that $\Delta_\delta = \{z \in D : 0 < |\zeta - \zeta_0| < \delta\} \subset D$ and $G(\zeta) \neq a_1$ in Δ_δ. Thus for each $\zeta \in \Delta_\delta$, $G_n(\zeta) \neq a_1$ (for n sufficiently large). Therefore for each $\zeta \in \Delta_\delta$, we have $H(\zeta) = \infty$. Thus we have proved that $\mathcal{H}$ is not normal at ζ_0.

Noting that

$$H_n(\zeta) = 0 \Rightarrow H_n'(\zeta) = a_1 \text{ or } a_2,$$

and using the Lemma 2.1 again we can find $\tau_n \to \tau_0$, $\eta_n \to 0$ and $H_n \in \mathcal{H}$ such that

$$\begin{aligned} F_n(\xi) &= \frac{H_n(\tau_n + \eta_n\xi)}{\eta_n} = \frac{G_n(\tau_n + \rho_n\xi) - a_1}{\eta_n} \\ &= \frac{f_n(z_n + (1 - |z_n|^2)^\alpha\zeta_n + (1 - |z_n|^2)^\alpha\rho_n(\tau_n + \eta_n\xi)) - a_1}{(1 - |z_n|^2)^\alpha\rho_n\eta_n} \end{aligned}$$

locally uniformly convergence to $F(\xi)$ on C, where F is a nonconstant entire function such that $F^\sharp(\xi) \leq F^\sharp(0) = M$. In particular $\rho(F) \leq 1$.

We claim that

(1) F only has finitely many zeros.

(2) $F(\xi) = 0 \Leftrightarrow F'(\xi) = a_1$ or a_2.

We first prove Claim (1). Suppose ζ_0 is a zero of $G(\zeta) - a_1$ with multiplicity k. If $F(\xi)$ has infinitely many zeros, then there exist $k + 1$ distinct points ξ_j $(j = 1, \cdots, k + 1)$ satisfying $F(\xi_j) = 0$ $(j = 1, \cdots, k + 1)$. Noting that $F(\xi) \not\equiv 0$, by Hurwitz's Theorem, there exists N, if $n > N$, we have $F_n(\xi_{jn}) = 0$ $(j = 1, \cdots, k + 1)$ and $G_n(\tau_n + \eta_n\xi_{jn}) - a_1 = 0$. We have

$$\lim_{n\to\infty} \zeta_n + \eta_n\xi_{jn} = \zeta_0, \ (j = 1, \cdots, k + 1)$$

then ζ_0 is a zero of $G(\zeta) - a_1$ with multiplicity at least $k + 1$, which is a contradiction. Thus we have proved Claim (1).

Next we prove Claim (2). Suppose that $F(\xi_0)=0$, then by Hurwitz's Theorem, there exist ξ_n, $\xi_n\to\xi_0$, such that (for n sufficiently large)

$$F_n(\xi_n)=\frac{f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))-a_1}{(1-|z_n|^2)^\alpha\rho_n\eta_n}=0.$$

Thus $f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_1$. By the assumption, we have

$$f_n'(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi))=a_1 \text{ or } a_2,$$

hence

$$F'(\xi_0)=\lim_{n\to\infty}f_n'(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_1 \text{ or } a_2.$$

Thus we prove $F(\xi)=0\Rightarrow F'(\xi)=a_1$ or a_2.

In the following, we will prove $F'(\xi)=a_1$ or $a_2\Rightarrow F(\xi)=0$.

Suppose that $F'(\xi_0)=a_1$. Obviously $F'\not\equiv a_1$, for otherwise $F^\sharp(0)\leq|F'(0)|=|a_1|<M$, which is a contradiction. Then by Hurwitz's Theorem, there exist ξ_n, $\xi_n\to\xi_0$, such that (for n sufficiently large)

$$F_n'(\xi_n)=f_n'(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_1.$$

It follows that $F_n(\xi_n)=f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_1$ or a_2.

If there exists a positive integer N, for each $n>N$, we have

$$f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_2.$$

Then

$$F(\xi_0)=\lim_{n\to\infty}\frac{f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))-a_1}{(1-|z_n|^2)^\alpha\rho_n\eta_n}=\infty,$$

it contradicts with $F'(\xi_0)=a_1$. Hence there exists a subsequence of $\{f_n\}$(which, renumbering, we continue to denote by $\{f_n\}$) satisfying that

$$f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))=a_1.$$

Thus we derive

$$F(\xi_0)=\lim_{n\to\infty}\frac{f_n(z_n+(1-|z_n|^2)^\alpha\zeta_n+(1-|z_n|^2)^\alpha\rho_n(\tau_n+\eta_n\xi_n))-a_1}{(1-|z_n|^2)^\alpha\rho_n\eta_n}=0,$$

which implies $F'=a\Rightarrow F=0$. Similarly, we can get $F'=a_2\Rightarrow F=0$. Hence we have proved claim (2).

Since $\rho(F')=\rho(F)\leq 1$, then by the Nevanlinna's second fundamental theorem,

$$\begin{aligned} T(r,F') &\leq \overline{N}(r,\frac{1}{F'-a_1})+\overline{N}(r,\frac{1}{F'-a_2})+S(r,F')\\ &\leq \overline{N}(r,\frac{1}{F'-a_1})+\overline{N}(r,\frac{1}{F'-a_2})+O(\log r)\\ &\leq \overline{N}(r,\frac{1}{F})+O(\log r) \end{aligned} \tag{3.2}$$

From Claim (1), we get $\overline{N}(r, \frac{1}{F}) = O(\log r)$. Thus $T(r, F') = O(\log r)$, it is clear that F is a polynomial.

In the following, we consider two cases:

Case 1: $a_1a_2 = 0$. Without loss of generality we assume $a_1 = 0$. We know that F' has zeros, then F has multiple zeros. We assume $\deg(F) = n$, then $T(r, F') = (n-1)\log r$ and $S(r, F') = O(1)$. By (3.2) we get

$$T(r, F') = (n-1)\log r \le \overline{N}(r, \frac{1}{F}) + O(1) \le (n-1)\log r$$

Thus we derive that F only has one multiple zeros with multiplicity 2 and F' only has one zero with multiplicity 1, which yields that $n = 2$. Set $F' = B(\xi - \xi_0)$, then $F = (B/2)(\xi - \xi_0)^2$, which contradicts with $F' = a_2 \Rightarrow F = 0$. This completes the proof of Case 1.

Case 2: $a_1a_2 \neq 0$. We first prove $F = 0 \rightleftharpoons F' = a_1$ or a_2. From $a_1a_2 \neq 0$, we get $F = 0 \to F' = a_1$ or a_2. Thus we only need to prove $F' = a_1$ or $a_2 \to F = 0$.

Suppose ξ_0 is a zero of $F' - a_1$ with multiplicity m. By Rouché theorem, there exist m sequences $\{\xi_{in}\}(i = 1, 2 \cdots, m)$ on $D_{\delta/2} = \{\xi : |\xi - \xi_0| < \delta/2\}$ such that $F_n'(\xi_{in}) = a_1$. Then

$$f_n'(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})) = F_n'(\xi_{in}) = a_1 \ (i = 1, 2, \cdots, m).$$

By f and f' share $\{a_1, \ a_2\}$ CM, we get $f' - a_1$ only has simple zeros. That is $\xi_{in} \neq \xi_{jn}(1 \le i \neq j \le m)$. We obtain

$$f_n(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})) = a_1 \text{ or } a_2 \ \ (i = 1, 2, \cdots, m).$$

We claim that there exist infinitely many n satisfying

$$f_n\big(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})\big) = a_1 \ \ (i = 1, 2, \cdots, m). \qquad (3.3)$$

Otherwise we may assume that for all n, there exist $j \in (1, \dots, m)$ satisfying

$$f_n(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})) = a_2.$$

We take a fixed number $l \in (1, ..., m)$ satisfying (for infinitely many n)

$$f_n(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})) = a_2.$$

Hence

$$\begin{aligned} F(\xi_0) &= \lim_{n\to\infty} \frac{f_n'(z_n + (1-|z_n|^2)^\alpha \zeta_n + (1-|z_n|^2)^\alpha \rho_n(\tau_n + \eta_n \xi_{in})) - a_1}{(1-|z_n|^2)^\alpha \rho_n \eta_n} \\ &= \lim_{n\to\infty} \frac{a_2 - a_1}{(1-|z_n|^2)^\alpha \rho_n \eta_n} = \infty, \end{aligned}$$

which contradicts with $F'(\xi_0) = a_1$. This proves (3.3). Therefore,

$$F_n(\xi_{in}) = 0, \ \ (i = 1, 2, \cdots, m)$$

and $\xi_{in} \neq \xi_{jn}$ $(1 \leq i \neq j \leq m)$. As $n \to \infty$, we get ξ_0 is a zero of F with multiplicity at least m. This proves $F' = a_1 \to F = 0$. Similarly we can get $F' = a_2 \to F = 0$. Thus we have proved

$$F = 0 \rightleftharpoons F' = a_1 \text{ or } a_2.$$

From this we know $F' - a_1$ and $F' - a_2$ only have simple zeros. Suppose that $\deg(F) = n$, then $n = 2(n-1)$ and $n = 2$. Set $F = A(\xi - \xi_1)(\xi - \xi_2)$, then $F' = A(2\xi - \xi_1 - \xi_2)$.

Without loss of generality, we assume that $F'(\xi_1) = a_1$ and $F'(\xi_2) = a_2$, we get $a_1 + a_2 = 0$. It is a contradiction.

Thus we complete the proof of Theorem 1.3.

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РАЗДЕЛЕННОЕ МНОЖЕСТВО И НОРМАЛЬНАЯ ФУНКЦИЯ ГОЛОМОРФНЫХ ОТОБРАЖЕНИЙ

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Аннотация. В работе идея разделенного множества применяется к описанию нормальных функций для семейства мероморфных функций в единичном круге.

Ключевые слова: целая функция, единственность, теория Неванлинна.