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Computational method for increasing the accuracy of determining coordinates

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Abstract. The article discusses a computing system for determining the location of an object by using satellite monitoring in real time with high accuracy. The computing system proposed by the authors is based on the determination of triple phase differences. High coordinate accuracy is ensured by entering additional data, such as tropospheric and ionospheric models. The resulting system is currently being tested.

1. Introduction

Recently, special attention is paid to improving the accuracy of measurements carried out by using global satellite systems, including GPS. Popular decision-making methods [1-4] are poorly suited for solving the problems associated with accurate satellite positioning, in which all calculable corrections are required to be entered. In real situations, when it is necessary to observe the object location and track its state, and it is in an indefinite place, it is difficult to choose the best method for determining its location. Examples of such objects are configurations of complex technical schemes, vehicles, engineering structures, electrical networks, and the like. Additional difficulties arise with the choice of the location determination algorithm, the environment of the object location, sources of interference and the absence of a single coordinate base [5]. The signals transmitted by satellites travel through the Earth's atmosphere and interact with charged particles, neutral atoms and molecules in the atmosphere, as the result of which their speed and propagation changes. The most characteristic areas of distribution are the ionosphere and troposphere. To reduce these factors in order to improve the satellite positioning accuracy and efficiency, in the published literature it is concluded that taking into account ionospheric and tropospheric delays with particularly accurate global navigation satellite measurement systems is mandatory [6].

The required positioning accuracy can be achieved with minimal errors in the following ways: by using triple phase differences or by introducing ionospheric and tropospheric corrections into the calculations.

2. Proposed method

The carrier phase equations for measurements from a certain point A to the satellite i are written in the form:

$$\begin{aligned} \Phi_A^i(t) = & p_A^i(t, t - \tau_A^i) - I_A^i + T_A^i + \delta m_A^i + c[dt_A(t) - dt^i(t - \tau_A^i)] + \\ & + c[\delta_A(t) + \delta^i(t - \tau_Z^i)] + \lambda[\phi_A(t_0) - \phi^i(t_0)] + \lambda N_A^i + \varepsilon_A^i \end{aligned} \quad (1)$$



where $p_A^i(t, t - \tau_A^i)$ is geometric range, that is, the true distance between the receiver at the time of signal reception and the satellite at the time of signal output; I_A^i is an ionospheric delay; T_A^i is a tropospheric delay; δm_A^i is the effect of multipath on phase measurements; dt_A, dt^i are the time corrections for the point and for the satellite, respectively; δ_A, δ^i are the signal delays in the phase measurement circuits in the receiver and on the satellite; ϕ_A, ϕ^i are the initial phases of the satellite and receiver generators; τ_A^i is a signal transit time; N_A^i is the phase integer-valued initial ambiguity; c is the speed of light; λ is the length of the carrier wave; ε_A^i is the phase measurement noise.

The phase observation is interconnected with the rest of the satellite measurements and must be performed continuously. While maintaining a constant capture of the satellite signal, it becomes possible to make high-precision kinematic measurements. All phase observations for one satellite Φ_A^i contain the same integer-valued initial ambiguity N_A^i .

To eliminate the difference in errors, we obtain the differences between the results of measurements from two points A and B on two satellites i and j related to epochs:

$$\Phi_{AB}^{ij} = p_{AB}^{ij} - I_{AB}^{ij} + T_{AB}^{ij} + \lambda N_{AB}^{ij} + \delta m_{AB}^{ij} + \varepsilon_{AB}^{ij} \quad (2)$$

To determine the baseline vector, the phase ambiguities need to be only integer values, therefore, the processing of a separate baseline on phase data will be based on a triple difference equation:

$$\Phi_{AB}^{ij}(t_1, t_2) = p_{AB}^{ij}(t_1, t_2) - I_{AB}^{ij}(t_1, t_2) + T_{AB}^{ij}(t_1, t_2) + \lambda N_{AB}^{ij} + \varepsilon_{AB}^{ij}(t_1, t_2) \quad (3)$$

In this case, the equation of corrections has the form:

$$-dR_B(\lambda_B^{ij}(t_1, t_2)) + l_{AB}^{ij}(t_1, t_2) = v_{AB}^{ij}(t_1, t_2) \quad (4)$$

Where dR_B is the vector of corrections; l_{AB}^{ij} is an intercept term; v_{AB}^{ij} is disparity.

$$\lambda_B^{ij}(t_1, t_2) = (u_B^i(t_2) - u_B^j(t_2)) - (u_B^i(t_1) - u_B^j(t_1)) \quad (5)$$

The intercept term is calculated by using the formula:

$$l_{AB}^{ij}(t_1, t_2) = (p_{AB}^{ij}(t_2))^0 - (p_{AB}^{ij}(t_1))^0 - I_{AB}^{ij}(t_2) + I_{AB}^{ij}(t_1) + T_{AB}^{ij}(t_2) - T_{AB}^{ij}(t_1) - \Phi_{AB}^{ij}(t_1, t_2) \quad (6)$$

It is necessary to determine the satellites coordinates at the time of the signal output by using the true anomaly, as well as the Kepler equation for the eccentric anomaly. The got satellites positions vectors are corrected by introducing a correction that takes into account the rotation of the Earth. Then we proceed to calculating the direction cosines of the directions to the satellites from each station and the a priori geometric ranges. For this, it is necessary to form double differences in each epoch of the phase between the satellites i and j and the receivers A and B . To do this, we form the equation of the single difference between the satellites Φ_B^{ij} , observed from the point B :

$$\Phi_B^{ij} = p_B^{ij} - cdt^i + cdt^j - I_B^{ij} + T_{AB}^{ij} + d^j - d^i + \lambda N_B^{ij} + \lambda[\phi^j(t_0) - \phi^i(t_0)] + \varepsilon_B^{ij} \quad (7)$$

Subtracting the difference Φ_A^{ij} from it, the satellite time errors go away, and we get the equation of the double difference:

$$\Phi_{AB}^{ij} = p_{AB}^{ij} - I_{AB}^{ij} + T_{AB}^{ij} + \lambda N_{AB}^{ij} + \varepsilon_{AB}^{ij} \quad (8)$$

In measurement cycles, it is necessary that the ambiguities N remain intact. For this, we introduce the ambiguity parameter into the equation

$$N_{AB}^{ij} = N_B^j - N_B^i - N_A^j - N_A^i \quad (9)$$

To determine the delay of the navigation satellite system signals in the ionosphere, we calculate the oblique delay for each satellite by using the formula:

$$I_{\text{накл.}}^i(i) = \int_{i=i_H}^i \Delta I_{\text{накл.}}^j(i) di + I_{\text{верт.}}(i_H^j) \cdot OF^j(\gamma^j(i_H)) \quad (10)$$

The initial data for calculating the oblique delay of the signal in the ionosphere are the satellite elevation angle, pseudorange measured by the phase of the signal carrier, and the slope factor, where i is the current moment in time; j is the number of the currently visible navigation spacecraft, $j = 1, 2, \dots, n$; i_H is the initial moment of tracking the signal of the j -th navigation satellite vehicle (hereinafter - NSV); $\Delta I_{\text{накл.}}^j(i)$ is the increment in the navigation satellite signal slant delay of the j -th satellite in the ionosphere; $I_{\text{верт.}}(i_H^j) \cdot OF^j(\gamma^j(i_H))$ is the estimate of the ionospheric signal delay value, formed at the initial stage of tracking the signal of the j -th NSV, taking into account the value of the satellite elevation angle and assessing the vertical delay of the navigation satellite system signals in the ionosphere, formed by the time of the signal of the j -th NSV tracking beginning; $OF^j(\gamma^j(i_H))$ is the slope factor of the j -th NSV at the initial moment of time; $\gamma^j(i_H)$ is the elevation angle of the NSV at the initial moment of time, degrees.

The phase pseudorange at frequencies L_1 and L_2 and the slope factor of each j -th NSV are described by the following equations:

$$L_{21}^j(i) = p^j(i) - N_2^j \lambda_2^j - I_{\text{накл.2}}^j(i) + T^j(i) + c\tau(i) + s_2^j(i) \quad (11)$$

$$L_{21}^j(i) = p^j(i) - N_2^j \lambda_2^j - I_{\text{накл.2}}^j(i) + T^j(i) + c\tau(i) + s_2^j(i) \quad (12)$$

$$OF^j(\gamma(i)) = \frac{1}{\sqrt{1 - \left(\frac{R_e}{R_e + h} \cdot \cos(\gamma^j(i)) \right)^2}} \quad (13)$$

Where $p^j(i)$ is distance to the j -th NSV; N_1^j , N_2^j is the phase ambiguity; λ_1^j , λ_2^j , is the wavelength; $I_1^j(i)$, $I_2^j(i)$ are the signal delay of the j -th NSV in the ionosphere; $T^j(i)$ is the signal delay of the j -th NSV in the troposphere; τ is the signal transit time; $s_1^j(i)$, $s_2^j(i)$ are random error; c is the speed of radio waves propagation in vacuum; $\gamma^j(i)$ is the elevation angle of the j -th NSV; R_e is the radius of the Earth; h is the average height of the ionospheric layer.

The phase pseudo-ranges increments measured at the carrier frequency L_2 and L_1 and the increment of the slope factor are calculated by:

$$\Delta L_1^j(i) = L_1^j(i) - L_1^j(i - M) \quad (14)$$

$$\Delta L_2^j(i) = L_2^j(i) - L_2^j(i - M) \quad (15)$$

$$\Delta OF^j(\gamma^j(i)) = OF^j(\gamma^j(i)) - OF^j(\gamma^j(i - M)) \quad (16)$$

Next, we calculate the slope delay increment and repeat the algorithm until the loop condition becomes false, that is $i > M$.

$$\Delta I_{\text{накл.}}^j(i) = \frac{f_2^2}{f_1^2 - f_2^2} \cdot (\Delta L_1^j(i) - \Delta L_2^j(i)) \quad (17)$$

As soon as the condition of the cycle becomes false, we proceed to the execution of the next cycle, where the variable k is reset to zero and proceed to checking the condition for the end of the cycle. This cycle is responsible for rejecting satellites for which the slant delay increment was not calculated.

At the next step, the variable k is reset to zero and we check the condition for the end of the cycle. If the current moment of time is the initial moments of tracking the signal, then the estimate of the initial value of the ionospheric delay of the signal C is calculated, otherwise the oblique delay of the signal in the ionosphere $I_{\text{накл.}}$ is calculated.

$$C = I_{\text{верт.}}(i_H^j) \cdot OF^j(\gamma^j(i_H)) \quad (18)$$

$$I_{\text{накл.}}^j(i) = \int_{i=i_H}^i \Delta I_{\text{накл.}}^j(i) di + C \quad (19)$$

Tropospheric delay is due to the refractive index, which is related to air density P (kg/m³), temperature T (K) and water vapor pressure e (mb) by the following equation:

$$N = (n-1) \cdot 10^6 = 77,689 \cdot \frac{P}{T} + 71,2952 \cdot \frac{e}{T} + 375463 \cdot \frac{e}{T^2} \quad (20)$$

where Z_w is the coefficient of compressibility of water vapor, close to unity.

Since the delay in the atmosphere strongly depends on the zenith angle of the satellite transmitting radio signals, the delay in the zenith direction serves as a universal characteristic of the neutral atmosphere effect:

$$ZTD = 10^{-6} \cdot \int_{h_1}^{h_2} N \cdot dh \quad (21)$$

where h_1 and h_2 are the heights of the receiver and the satellite, respectively.

Since the radio paths of signals from satellites are observed in an oblique direction, the actual phase measurements contain oblique tropospheric delays STD . To connect the latter with the zenith tropospheric delay, a mapping function m is used:

$$STD = ZTD \cdot m(z) \quad (22)$$

where z is the zenith angle of the satellite transmitting radio signal.

The zenith tropospheric delay is determined by the profiles of meteorological quantities and is expressed by using two components: hydrostatic (ZHD , m) and wet (ZWD , m). At the same time, separate mapping functions are selected for them, in this regard, the oblique tropospheric delay will be calculated as follows:

$$STD = ZHD \cdot m_H(z) + ZWD \cdot m_W(z) \quad (23)$$

where $m_H(z)$, $m_W(z)$ are "hydrostatic" and "wet" mapping functions, respectively.

To simulate tropospheric delays, we represent the lodging functions as a continuous fractional decomposition like the Martini decomposition:

$$m(z) = \frac{1 + \frac{a_1}{\cos(z)} + \frac{a_2}{\cos^2(z)} + \frac{a_3}{\cos^3(z)} + \dots}{\cos(z) + \frac{a_1}{\cos(z)} + \frac{a_2}{\cos^2(z)} + \frac{a_3}{\cos^3(z)} + \dots} \quad (24)$$

where a_i are the coefficients that are determined from radio sounding data or from numerical weather fields.

The most accurate is the Mendes mapping function (FCULb) which is defined as follows:

$$a_i = a_{i0} + (a_{i1} + a_{i2} \cdot B^2) \cdot \cos\left(\frac{2 \cdot \pi}{365,25} \cdot (DOY - 28)\right) + a_{i3} \cdot H + a_{i4} \cdot \cos(B) \quad (25)$$

where $a_{i1}, a_{i2}, a_{i3}, a_{i4}$ are constant empirical coefficients; i is the coefficient number, $i = 1, 2, 3$

DOY is the number of the day of the year; B, H are latitude (degrees) and height (m) of the station.

After we determine all the unknown variables for the double difference equation, we calculate them in every 5th epoch, while observing discreteness. We form triple differences by using double phase differences for two different epochs t_1 and t_2 :

$$\Phi_{AB}^{ij}(t_1, t_2) = \Phi_{AB}^{ij}(t_2) - \Phi_{AB}^{ij}(t_1) \quad (26)$$

All the resulting triple differences are considered independent and, without taking into account the correlation, we form and determine the weight matrix P :

$$P = \begin{pmatrix} p_1 & 0 & \dots & 0 \\ 0 & p_2 & \dots & 0 \\ \dots & & & \\ 0 & 0 & \dots & p_n \end{pmatrix} \quad (27)$$

Then we create a matrix of coefficients A , forming an observation equation

$$A = \begin{bmatrix} a_X^{Sk}(t_1) & a_Y^{Sk}(t_1) & a_Z^{Sk}(t_1) & 1 & 0 & 0 \\ a_X^{Sl}(t_1) & a_Y^{Sl}(t_1) & a_Z^{Sl}(t_1) & 0 & 1 & 0 \\ a_X^{Sm}(t_1) & a_Y^{Sm}(t_1) & a_Z^{Sm}(t_1) & 0 & 0 & 1 \\ a_X^{Sk}(t_2) & a_Y^{Sk}(t_2) & a_Z^{Sk}(t_2) & 1 & 0 & 0 \\ a_X^{Sl}(t_2) & a_Y^{Sl}(t_2) & a_Z^{Sl}(t_2) & 0 & 1 & 0 \\ a_X^{Sm}(t_2) & a_Y^{Sm}(t_2) & a_Z^{Sm}(t_2) & 0 & 0 & 1 \end{bmatrix} \quad (28)$$

The matrix of the system of normal equations is formed in accordance with the formula:

$$B = A^T P A \quad (29)$$

where A^T is the matrix transposed with respect to the matrix A ;

The vector of intercept terms of the system of normal equations is formed by the formula:

$$C = A^T P L \quad (30)$$

where L is the vector of intercept term of the errors equations, which is calculated as the difference between the measured value, calculated from the approximate values of the parameters and the measurement result.

We solve the system of normal equations and calculate the equalized values of the parameters. The roots of the normal equations system can be found by inverting the coefficient matrix B :

$$t = B^{-1} C \quad (31)$$

The system of normal equations is solved by the method of successive elimination of unknown variable (by the Gauss method). The adjusted values of the parameters are expressed by the sum of the approximate values of the parameters t^0 and t . We find the adjusted parameters by the formula:

$$r = t^0 + t \quad (32)$$

3. Conclusion

The location technology working out and development is a priority. Various methods and technologies for determining coordinates open up new possibilities for continuous accurate positioning

of objects. The use of the satellite signal in the calculations of the ionospheric and tropospheric delay is the most expedient method for determining the coordinates of the object, which allows to determine the object's location more accurately. The proposed method improves the accuracy of determining the location of objects with the accuracy of up to mm. Now the prototype that implements the proposed method is tested.

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