КРАТКИЕ СООБЩЕНИЯ

ABOUT THE PLACING OF TRANSPORT COEFFICIENTS UNDER THE CONTROL OF THE STRUCTURE OF DIFFERENTIAL-DIFFERENCE EQUATIONS

L.A. Uvarova

Moscow state technological university "STANKIN",
Moscow, Russia, e-mail: uvar11@yandex.ru

1. Introduction. The problem under consideration is connected with the substantial selection of dependencies of kinetic coefficients in non-linear transport differential or difference equations on desired parameters. It is well-known that dependencies of coefficients in non-linear transport equations on functions which are required leads to the existence of special solutions. For example, the equation of diffusion transport with quadratic dependence of diffusion coefficient on substance concentration

$$D_i = D_{i0} + D_{i1}c_i + D_{i2}c_i^2$$

leads to the existence of cnoidal solution

$$c_i = c_{i1} - (c_{i1} - c_{i2})\mathrm{sn}^2\left(\frac{x}{d_i}, k_i\right).$$

Here, $k_i^2 = \dfrac{c_{i1} - c_{i2}}{c_{i1} - c_{i3}}$, $d_i = \left(\dfrac{6D_0^2}{\pm D_2(c_{i1} - c_{i3})}\right)^{1/2}$, I is the power of mass source, c_{i1}, c_{i2}, c_{i3} are roots of cubic equation:

$$c_i^3 + 1.5\frac{D_{i1}}{D_{i2}}c_i^2 \pm 3\frac{D_{i0}}{D_{i2}}c_i \mp 1.5\frac{A_i^2 D_{i0}^2}{D_{i2}} = 0,$$

A is integration constant that depends on boundary condition. Such solutions may exist really. However, for other dependencies $D(C)$, it is possible to obtain a great number of different solutions in phase space. Therefore, the question about real coefficients is very important. It is possible to assume that dependence $f(U)$ must obey to differential or difference equations. Such equations as a rule are found on the basis of fundamental laws. The given problem was discussed in [1] partially.

2. Modeling of the viscosity coefficient by difference equation. We consider the viscosity coefficient at the flow of multilayer liquid in cylinder. It is assumed that the velocity in each layer depends on coordinates r and z. The layers are preserved due to difference of densities. It is realized by special methods of filling or in the case of the presence of thin punched partitions. On the external border of cylinder when $r = R$, the velocity is equal to zero (k is number of external layer), $u_{kn} = 0$.

It is assumed that the crossing of molecules from one layer to another may be occur with transition probabilities $P_{i,j}, P_{j,i}$ (i, j are numbers of layers, $j = i+1, i-1$). The length of cylinder exceeds its diameter significantly. It permits to disregard of non-homogenous for the pressure p on radial coordinate compared with non-homogenous on z in each layer. The flow is isothermal. Therefore, the density and the viscosity are functions on pressure only:

$$\rho_i = \rho_i(p_i),\ \eta_i = \eta_i(p_i).$$

The work was supported by the Russian Foundation for Fundamental Research (Grant №12-01-00874-a).

The appropriate system of equations has the following view:

$$\rho_i u_i = M_i - (G_{i,i+1}P_{i,i+1} + G_{i,i-1}P_{i,i-1}) + G_{i+1,i}P_{i+1,i} + G_{i-1,i}P_{i-1,i}\,, \tag{1}$$

$$\frac{dp_i}{dz} = \frac{1}{r}\frac{\partial}{\partial r}\left(\eta_i r\frac{\partial}{\partial r}\left(\frac{M_i}{\rho_i} + \frac{\Delta(G_iP_i)}{\rho_i}\right)\right) + \frac{4}{3r}\frac{\partial}{\partial z}\left(\eta_i r\frac{\partial}{\partial z}\left(\frac{M_i}{\rho_i} + \frac{\Delta(G_iP_i)}{\rho_i}\right)\right). \tag{2}$$

Here, u_i is the velocity, M_i, $G_{i,j}$ are mass flow in ith layer and mass flow of $i-j$ transition. The mass flow of $i-j$ transition depends on mechanism of transition. For example, in the case of diffusion mechanism it equals to $G_{i,j} = -m_i D_{ij}\dfrac{dc_i}{dr}$, m is molecule mass. The change of the substance in the i-layer at the expense of neighboring layers has following view:

$$\Delta(G_iP_i) \equiv -(G_{i,i+1}P_{i,i+1} + G_{i,i-1}P_{i,i-1}) + G_{i+1,i}P_{i+1,i} + G_{i-1,i}P_{i-1,i}\,.$$

The equation (2) is obtained on the basis Navier-Stokes' equations. After transformation of equations (1)-(2) we find:

$$\begin{gathered}\frac{\rho_i dp_i}{\eta_i dz} = \frac{d^2M_i}{dr^2} + \frac{dM_i}{r\,dr} + \frac{1}{r}\frac{d}{dr}(\Delta(G_iP_i)) + \frac{d^2}{dr^2}(\Delta(G_iP_i)) + \\ \frac{4}{3}\left(\rho_i\frac{d^2}{dr^2}\left(\frac{1}{\rho_i}\right) + \frac{\rho_i}{\eta_i}\frac{d\eta_i}{dz}\frac{d}{dz}\left(\frac{1}{\rho_i}\right)\right)(M_i + \Delta(G_iP_i))\,.\end{gathered} \tag{3}$$

The equation (3) is equation of the following view:

$$F_i(r) + \varphi_i(r)\psi_i(z) = Z_i(z)\,. \tag{4}$$

Then it must be fulfilled in each layer:

1. $F(r) = \text{const}\,, \qquad \varphi(r) = \text{const}\,.$
2. $Z(z) = \text{const}\,, \qquad \psi(z) = \text{const}\,.$

The first of formulas of the couple pointed out may lead to some difference expressions for M in different layers. In homogenous cylinder such solutions are trivial ones because of boundary conditions for the velocity. The second formulas are most interesting for our investigation. Then, we receive from (3)-(4):

$$\begin{gathered}\frac{\rho_i dp_i}{\eta_i dz} = b_{1i}\,, \\ \rho_i\frac{d^2}{dr^2}\left(\frac{1}{\rho_i}\right) + \frac{\rho_i d\eta_i}{\eta_i dz}\frac{d}{dz}\left(\frac{1}{\rho_i}\right) = b_{2i}\,, \\ b_{1i} = \text{const}\,, \qquad b_{2i} = \text{const}\,.\end{gathered} \tag{5}$$

It is necessary to consider the dependence $\rho_i(p_i)$. We consider the Tait's equation. It is well known that the evolution of many liquids obeys to this equation up to the pressure near thousand atm. The Tait's equation has the differential form:

$$\frac{1}{\rho_i^2}\frac{d\rho_i}{dp_i} = \frac{\xi_i}{p_i + w_i}\,. \tag{6}$$

Here, ξ_i is constant value, w_i is characteristic value of the liquid. It depends on the temperature T (T is constant in our case). The formula for the viscosity dependence on the density is received by the using of expressions (5)-(6) for each layer i:

$$\begin{aligned}
&\eta = \left(\eta_0^2 A(p,\rho) + (\eta_1^2 - \eta_0^2 A(p_1,\rho_1))\frac{A(p,\rho)(p_0-p)}{A(p_1,\rho_1)(p_0-p_1)}\right)^{0,5}, \\
&A(p,\rho) = \frac{(p+w)\rho}{(p_1+w_1)\rho_1}\,.
\end{aligned} \tag{7}$$

Here, $\eta_k = \eta(p_k)$, $\rho_k = \rho(p_k)$, $k = 0, 1$, p_k are fixed values of pressure.

Calculations on the basis of the formula (7) give good tallying with experimental results connected with different liquids (for example, relative error for oil constitutes near 1%).

From expressions (3), (5) one can find

$$\begin{aligned}
&\frac{d^2 M_i}{dr^2} + \frac{dM_i}{r\,dr} + \frac{1}{r}\frac{d}{dr}(\Delta(G_i P_i)) + \frac{d^2}{dr^2}(\Delta(G_i P_i)) + \\
&\frac{4}{3} b_{i2}(M_i + \Delta(G_i P_i)) = b_{1i}\,.
\end{aligned} \tag{8}$$

Solution of the equation (8) depends on b_{i2} substantially. Next expression follows from the famous Bachinskii formula for the density and the viscosity:

$$\frac{\Delta\eta_i}{\eta_{i0}} > \frac{\Delta\rho_i}{\rho_{i0}}\,. \tag{9}$$

Let (9) be fulfilled. Then value b_{i2} is negative

$$b_{i2} = 2(p_0')^2 \kappa_{i0}^2 \left(1 - \frac{\rho_{i0}\Delta\eta_i}{\eta_i \Delta\rho_i}\right), \quad p_0' = \frac{dp}{dz}(0)$$

where κ_i is the coefficient of liquid compressing. In this case the solution of equation (8) with the account of mass differences in different flows is found in terms of Bessel's functions of imaginary argument. In particular, the solution for the velocity u in homogenous cylinder has the form:

$$u = -\frac{r_0^2}{\eta}\left(1 - \frac{I_0(r/r_0)}{I_0(R/r_0)}\right)\frac{dp}{dz}\,, \qquad r_0 = -\sqrt{3} b_2/2\,.$$

The given formula transforms to the Poiseuille formula when the inequality $R \ll r_0$ is fulfilled.

3. Conclusion. It is demonstrated the method by which the acceptable dependence on unknown function for the kinetic coefficient by non-linear differential-difference transport equation. It permits to obtain the equation under study. Calculations of viscosity on the basis of obtained formula (7) give good tallying with experimental data for different liquids (for example, relative error for oil constitutes near 1%).

References

1. Anikonov D.S. Unique joint determination of two coefficients of transport equation // RAS USSR. – 1984. – 277. – P.777-779.