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DYNAMIC THEORY COHERENT X-RAY RADIATION BY BEAM OF RELATIVISTIC ELECTRONS IN SINGLE-CRYSTAL

S.V. Blazhevich, A.V. Noskov

Belgorod State University, Belgorod, Russia.
E-mail: noskovbupk@mail.ru

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Resume. The dynamic theory of coherent X-ray radiation by divergent beam of relativistic electrons crossing a single-crystal plate has been developed in Bragg scattering geometry. Numerical calculations of parametric X-ray and diffracted transition radiation angular densities has been made using their averaging by two-dimensional Gauss distribution as the angular distribution of electrons in the beam. It has been shown that the influence of divergence of the high-energy electron beam on DTR is much stronger than on PXR.

1.Introduction

In the physics of interaction of relativistic electrons with matter, to know spatial and angular distributions of particles in the incident beam is important for the experimental data interpretation. That is why working out the express methods to get information about the characteristics of the beam used in the experiment is actual problem. One of the approaches is to use different types of radiation excited by relativistic charged particles in matter. The possibility of the use of parametric X-radiation (PXR) for the diagnostics of relativistic electron beams recently was experimentally studied in [1, 2]. In [3] it was suggested to use PXR generated in a thin crystal to get operative information on spatial position of relativistic electron beam. The applicability of transition radiation (TR) of vacuum ultraviolet range to measure the electron beam cross dimensions was demonstrated in [4]. The authors of [5] offer the use of X-ray Cherenkov radiation by ultrarelativistic charged particles in the photon energy range, which includes K-absorption edges for some of the materials, to reveal the beam cross-dimensions.

In all of the works above listed, the beam parameters estimation was carried out in the framework of kinematic PXR theory, therefore studying the influence of dynamic effects on the characteristics of coherent radiation by relativistic electron beams remains an important task.

As known PXR appears due to the scattering of a relativistic electron Coulomb field on a system of parallel crystal atomic planes [6-8]. When a charged particle crosses the crystal plate surface, the transition radiation (TR) takes place [9, 10]. TR appearing on the border diffracts then on a system of parallel atomic planes of the crystal that forms DTR in a narrow spectral range [11-14]. DTR photons move near the Bragg scattering direction. The process of coherent X-ray radiation by a single relativistic electron in a crystal is described in the framework of the dynamical theory of x-rays diffraction in [15-18]. In these papers, the dynamic theory of coherent X-ray radiation generated by a relativistic electron in a crystal has been built for general case of asymmetric relative to the crystal (target) surface reflection of the electron Coulomb field. In this case, the system of the parallel reflecting atomic planes in the target can be located at arbitrarily given angle to the target surface. Under these conditions, coherent X-radiation in the direction of the Bragg scattering appears because of two coherent radiation mechanisms, namely PXR and DTR. In work [19] dynamical theory of coherent X-ray radiation generated in a single-crystal target by the finite divergence beam of relativistic electrons has been developed in the scattering Laue geometry. The significant difference of the effects of electron beam divergence in PXR and DTR is shown. The possibilities of practical use of DTR from a single-crystal target for indication of beam divergence of ultrarelativistic electrons are investigated.

In the present work, a dynamical theory of coherent X-ray radiation generated in a single-crystal target by a divergent beam of relativistic electrons is developed in the scattering Bragg geometry. We have obtained the expressions describing spectral-angular distributions of PXR and DTR generated by a relativistic electron moving rectilinearly through the crystal plate at a predetermined angle relative to the axis of the electron beam. Further on the expression for the spectral-angular density of the radiation generated by an electron beam has been derived using the averaging of the radiation cross section over the angular distribution of electrons moving in straight lines in the beam. The influence of the electron beam divergence on spectral and angular characteristics of the coherent radiation has been investigated.One of the goals of this paper is to demonstrate a significant effect of reduction of DTR contribution under increase of the electron beam divergence. This effect allows investigating PXR of a relativistic electron beam in a single crystal target without DTR background. Another goal of this work is to define the possibilities of indication of electron beam divergence in accelerators with use of DTR. It is important to note that the effect of the beam divergence in the considered coherent radiation does not depend on cross sizes of the electron beam at the target-radiator.

2. Radiation amplitude

Let us consider a beam of relativistic electrons crossing a monocrystalline plate (Fig.1). Let us involve the angular variables $\boldsymbol{\psi}$, θ and $\boldsymbol{\theta}_0$ in accordance with the definition of relativistic electron velocity $\mathbf{V}$ and unit vectors in direction of momentum of the photon radiated in the direction near electron velocity vector $\mathbf{n}$ and in the of Bragg scattering direction $\mathbf{n}_\mathbf{g}$:

$$\mathbf{V} = \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \qquad \mathbf{e}_1\boldsymbol{\psi} = 0$$

$$\mathbf{n} = \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \qquad \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \qquad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B,$$

$$\mathbf{n}_\mathbf{g} = \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \ \mathbf{e}_2\boldsymbol{\theta} = 0, \tag{1}$$

θ – is the radiation angle, counted from direction of axis of radiation detector $\mathbf{e}_2$, $\boldsymbol{\psi}$ – is the incidence angle of an electron in the beam counted from the electron beam axis $\mathbf{e}_1$, $\boldsymbol{\theta}_0$ – is the angle between the incident photon movement direction and axis $\mathbf{e}_1$, ψ_0 – is the divergence of the beam of radiating electrons, $\gamma = 1/\sqrt{1-V^2}$ - Lorentz-factor of the particle, θ_B – is the angle between the electron velocity and the system of crystallographic planes (Bragg angle), δ – is the angle between the input surface of the target and the crystallographic plane, $\mathbf{k} = k\mathbf{n}$, $\mathbf{k}_\mathbf{g} = k_\mathbf{g}\mathbf{n}_\mathbf{g}$ - are the wave vectors of the radiated photons.

The angular variables are decomposed into the components parallel and perpendicular to the figure plane: $\boldsymbol{\theta} = \boldsymbol{\theta}_{||} + \boldsymbol{\theta}_{\perp}$, $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0||} + \boldsymbol{\theta}_{0\perp}$, $\boldsymbol{\psi} = \boldsymbol{\psi}_{||} + \boldsymbol{\psi}_{\perp}$.

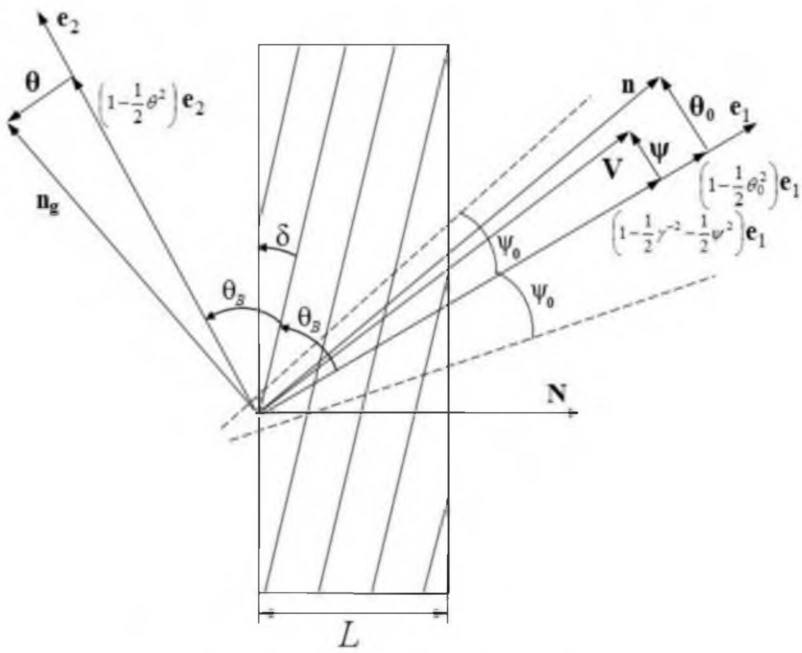

Рис.1 Geometry of the emission process

Let us consider electromagnetic processes in the crystalline medium characterized by complex permittivity

$$\varepsilon(\omega,\mathbf{r}) = 1+\chi(\omega,\mathbf{r}), \tag{2}$$

where $\chi(\omega,\mathbf{r}) = \chi_0(\omega)+\sum_{\mathbf{g}}{}' \chi_{\mathbf{g}}(\omega)\exp(i\mathbf{gr})$, $\chi(\omega,\mathbf{r})$ is the dielectric susceptibility, $\chi_{\mathbf{g}}(\omega)=\chi'_{\mathbf{g}}(\omega)+i\chi''_{\mathbf{g}}(\omega)$ is the Fourier coefficient of the expansion of the dielectric susceptibility of the crystal in reciprocal lattice vectors $\mathbf{g}$, and $\chi_0(\omega)$ is the average dielectric susceptibility. The permeability of the substance in the range of relatively high X-ray frequencies we are interested in is unity, and the Maxwell equations in this case have the form

$$\begin{aligned} rot\,\mathbf{H}(\mathbf{r},t) &= 4\pi\,\mathbf{J}(\mathbf{r},t)+\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \\ rot\,\mathbf{E}(\mathbf{r},t) &= -\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t}. \end{aligned} \tag{3}$$

The expansions of electric field strength $\mathbf{E}$ and magnetic field strength $\mathbf{H}$, as well as on the electric induction $\mathbf{D}$ of the field in the crystal and current density $\mathbf{J}$ of the emitting electron into the Fourier integral in frequency have the form

$$\mathbf{E}(\mathbf{r},t) = \frac{1}{2\pi}\int \mathbf{E}(\omega,\mathbf{r})\exp(-i\omega t)d\omega,$$

$$\mathbf{H}(\mathbf{r},t) = \frac{1}{2\pi}\int \mathbf{H}(\omega,\mathbf{r})\exp(-i\omega t)d\omega,$$

$$\mathbf{D}(\mathbf{r},t) = \frac{1}{2\pi}\int \mathbf{D}(\omega,\mathbf{r})\exp(-i\omega t)d\omega,$$

$$\mathbf{J}(\mathbf{r},t)=\frac{1}{2\pi}\int \mathbf{J}(\omega,\mathbf{r})\exp(-i\omega t)d\omega \,. \tag{4}$$

Substituting these expansions into Eqs. (3) and using the well-known relation $\mathbf{D}(\omega,\mathbf{r})=\varepsilon(\omega,\mathbf{r})\mathbf{E}(\omega,\mathbf{r})$, we obtain the following expression

$$\begin{aligned} &rot\,\mathbf{H}(\omega,\mathbf{r})=4\pi\mathbf{J}(\omega,\mathbf{r})-i\omega\,\varepsilon(\omega,\mathbf{r})\mathbf{E}(\omega,\mathbf{r}),\\ &rot\,\mathbf{E}(\omega,\mathbf{r})=i\omega\mathbf{H}(\omega,\mathbf{r}), \end{aligned} \tag{5}$$

Substituting into Eq. (5) the Fourier expansions of fields and electron current in wave vector $\mathbf{k}$,

$$\mathbf{E}(\omega,\mathbf{r})=\frac{1}{(2\pi)^3}\int \mathbf{E}(\mathbf{k},\omega)\exp(i\mathbf{kr})d^3\mathbf{k}\,,$$

$$\mathbf{H}(\omega,\mathbf{r})=\frac{1}{(2\pi)^3}\int \mathbf{H}(\mathbf{k},\omega)\exp(i\mathbf{kr})d^3\mathbf{k}\,,$$

$$\mathbf{J}(\omega,\mathbf{r})=\frac{1}{(2\pi)^3}\int \mathbf{J}(\mathbf{k},\omega)\exp(i\mathbf{kr})d^3\mathbf{k}\,, \tag{6}$$

and performing simple transformations, we obtain the well-known expression for Fourier transform of electric field $\mathbf{E}(\mathbf{k},\omega)$ in the crystal, generated by a relativistic electron:

$$(k^2-\omega^2(1+\chi_0))\mathbf{E}(\mathbf{k},\omega)-\mathbf{k}(\mathbf{kE}(\mathbf{k},\omega))-\omega^2{\sum_{\mathbf{g}}}'\chi_{-\mathbf{g}}\mathbf{E}(\mathbf{k}+\mathbf{g},\omega)=4\pi i\omega\mathbf{J}(\mathbf{k},\omega)\,, \tag{7}$$

where $\mathbf{J}(\mathbf{k},\omega)=2\pi e\mathbf{V}\delta(\omega-\mathbf{kV})$ is the Fourier transform of the current density $\mathbf{J}(\mathbf{r},t)=e\mathbf{V}\delta(\mathbf{r}-\mathbf{V}t)$ of a relativistic electron crossing the target. It should be noted that for $\chi_{-\mathbf{g}}=0$ expression (7) describes the electric field in the amorphous medium.

In this work the two-wave approach of dynamic diffraction theory are used, in which both the incident and diffracted waves are considered as equitable in process of self repumping one into another in crystalline target.

The strengths of electromagnetic fields, excited by electron in the crystal are

$$\begin{aligned} \mathbf{E}(\mathbf{k},\omega)&=E_0^{(1)}(\mathbf{k},\omega)\mathbf{e}_0^{(1)}+E_0^{(2)}(\mathbf{k},\omega)\mathbf{e}_0^{(2)},\\ \mathbf{E}(\mathbf{k}+\mathbf{g},\omega)&=E_{\mathbf{g}}^{(1)}(\mathbf{k},\omega)\mathbf{e}_1^{(1)}+E_{\mathbf{g}}^{(2)}(\mathbf{k},\omega)\mathbf{e}_1^{(2)}, \end{aligned} \tag{8}$$

$\mathbf{e}_0^{(1)}\perp\mathbf{k}$, $\mathbf{e}_0^{(2)}\perp\mathbf{k}$, $\mathbf{e}_1^{(1)}\perp\mathbf{k}_{\mathbf{g}}$ и $\mathbf{e}_1^{(2)}\perp\mathbf{k}_{\mathbf{g}}$, $\mathbf{k}_{\mathbf{g}}=\mathbf{k}+\mathbf{g}$. Vectors $\mathbf{e}_0^{(2)}$, $\mathbf{e}_1^{(2)}$ are situated on the plane of vectors $\boldsymbol{k}$ и $\boldsymbol{k}_{\mathrm{g}}$ (π-polarization) and $\mathbf{e}_0^{(1)}$, $\mathbf{e}_1^{(1)}$ are perpendicular to this plane (σ-polarization); g is vector of the reciprocal lattice, defining a set of reflecting atomic planes.

In the two-wave approximation of the dynamic theory, Eq. (7) combined with expressions (8) can be reduced to the well-known system of equations [20]

$$\begin{cases} (\omega^2(1+\chi_0)-k^2)E_0^{(s)}+\omega^2\chi_{-\mathbf{g}}C^{(s,\tau)}E_{\mathbf{g}}^{(s)}=8\pi^2ie\omega\mathbf{e}_0^{(s)}\mathbf{V}\delta(\omega-\mathbf{kV}),\\ \omega^2\chi_{\mathbf{g}}C^{(s,\tau)}E_0^{(s)}+(\omega^2(1+\chi_0)-k_{\mathbf{g}}^2)E_{\mathbf{g}}^{(s)}=0. \end{cases} \tag{9}$$

$$\chi'_{\mathbf{g}}=\chi'_0(F(g)/Z)(S(\mathbf{g})/N_0)\exp(-g^2u_\tau^2/2)\,, \tag{10a}$$

$$\chi''_{\mathbf{g}}=\chi''_0\exp\left(-\frac{1}{2}g^2u_\tau^2\right), \tag{10b}$$

where $\chi_0 = \chi_0' + i\chi_0''$ – is the average dielectric susceptibility, *F(g)* – is the form factor of atom containing Z electrons, $S(\mathbf{g})$ is the structural factor of a unit cell containing N_u atoms, u_τ is the r.m.s. amplitude of thermal vibrations of crystal atoms. The work addresses the X-ray frequency range, where $\chi_g' < 0$, $\chi_0' < 0$. We will consider a crystal with the following symmetry $(\chi_g = \chi_{-g})$.

The quantity $C^{(s)}$ and $P^{(s)}$ in system (9) are defined in the following way:

$$C^{(s,\tau)} = \mathbf{e}_0^{(s)}\mathbf{e}_1^{(s)} = (-1)^\tau C^{(s)},\ C^{(1)} = 1,\ C^{(2)} = |\cos 2\theta_B|,\ P^{(1)} = \sin\varphi,\ P^{(2)} = \cos\varphi,$$

$$\mathbf{e}_0^{(1)}\mathbf{V} = (\theta - \psi)P^{(1)} = \theta_\perp - \psi_\perp,\quad \mathbf{e}_0^{(2)}\mathbf{V} = (\theta + \psi)P^{(2)} = \theta_{//} + \psi_{//}, \tag{11}$$

where φ is the azimuthal radiation angle measured from the plane formed by the vectors V и g, the value of the reciprocal lattice vector is determined by the $g = 2\omega_B \sin\theta_B / V$, where ω_B is the Bragg frequency. The system of equations (9) for $s = 1$ and $\tau = 2$ describes the σ - polarized fields. For $s = 2$, the system of equations (9) describes π - polarized fields; note that if $2\theta_B < \frac{\pi}{2}$, then $\tau = 2$, otherwise $\tau = 1$.

Let us solve the dispersion equation for x-waves in crystal following from the system (9):

$$(\omega^2(1+\chi_0) - k^2)(\omega^2(1+\chi_0) - k_g^2) - \omega^4\chi_{-g}\chi_g C^{(s)^2} = 0, \tag{12}$$

using standard methods of dynamic theory [20].

Let us search for the wave vectors projection $\mathbf{k}$ and $\mathbf{k}_g$ to the axle X, coinciding with the vector *n* (see fig. 1.) as:

$$k_x = \omega\cos\psi_0 + \frac{\omega\chi_0}{2\cos\phi_0} + \frac{\lambda_0}{\cos\phi_0},\ k_{gx} = \omega\cos\phi_g + \frac{\omega\chi_0}{2\cos\phi_g} + \frac{\lambda_g}{\cos\phi_g}. \tag{13}$$

For this purpose we will use the well-known relation, connecting dynamic addition agents λ_0 and λ_g for x-waves [20].

$$\lambda_g = \frac{\omega\beta}{2} + \lambda_0\frac{\gamma_g}{\gamma_0}, \tag{14}$$

where $\beta = \alpha - \chi_0\left(1 - \frac{\gamma_g}{\gamma_0}\right)$, $\alpha = \frac{1}{\omega^2}(k_g^2 - k^2)$, $\gamma_0 = \cos\phi_0$, $\gamma_g = \cos\phi_g$, ϕ_0 is the angle between accident wave vector $\mathbf{k}$ and vector normal to the plate surface $\mathbf{N}$, ϕ_g is the angle between wave vector $\mathbf{k}_g$ and the vector $\mathbf{N}$ (see fig.1). The modules of vectors $\mathbf{k}$ and $\mathbf{k}_g$ are:

$$k = \omega\sqrt{1+\chi_0} + \lambda_0,\ k_g = \omega\sqrt{1+\chi_0} + \lambda_g. \tag{15}$$

Let us place (13) to (12), taking into account (14), $k_{||} \approx \omega\sin\phi_0$ and $k_{g||} \approx \omega\sin\phi_g$. As a result, we will obtain the expression for dynamic addition agents:

$$\lambda_{\mathbf{g}}^{(1,2)}=\frac{\omega}{4}\left(\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right),\ \lambda_0^{(1,2)}=\omega\frac{\gamma_0}{4\gamma_{\mathbf{g}}}\left(-\beta\pm\sqrt{\beta^2+4\chi_{\mathbf{g}}\chi_{-\mathbf{g}}C^{(s)^2}\frac{\gamma_{\mathbf{g}}}{\gamma_0}}\right). \tag{16}$$

As $|\lambda_0|<<\omega$, $|\lambda_g|<<\omega$, we can show that $\theta\approx\theta'$ (see fig.1), and therefore later on θ will be used for all occasions.

The solution of the system (9) for the field located in vacuum forward of the crystal will be:

$$E_0^{(s)\text{vac}}=\frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{1}{-\chi_0-\frac{2}{\omega}\lambda_0}\delta(\lambda_0^*-\lambda_0)=\frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{|\gamma_{\mathbf{g}}|}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}}+\beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}), \tag{17}$$

where $\lambda_{\mathbf{g}}^*=\frac{\omega\beta}{2}+\frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda_0^*$, $\lambda_0^*=\omega\left(\frac{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_{//}+\psi_{//})^2-\chi_0}{2}\right)$, $\delta(\lambda_0^*-\lambda_0)=\frac{|\gamma_{\mathbf{g}}|}{\gamma_0}\delta(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}})$,

$\gamma=1/\sqrt{1-V^2}$ is Lorenz factor of the particle.

It was suitable for us to express the solution of the system (9) for diffracted field in crystal as:

$$E_{\mathbf{g}}^{(s)\text{cr}}=\frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s,\tau)}}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)}\right)\left(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}\right)}\delta(\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}})+E^{(s)(1)}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(1)})+E^{(s)(2)}\delta(\lambda_{\mathbf{g}}-\lambda_{\mathbf{g}}^{(2)}), \tag{18}$$

where, $E^{(s)(1)}$ and $E^{(s)(2)}$ are free fields, corresponding to two solutions (16) of dispersion equation (12).

The diffracted field in vacuum can be written in the form

$$E_{\mathbf{g}}^{(s)\text{vac}}=E_{\mathbf{g}}^{(s)Rad}\delta\left(\lambda_{\mathbf{g}}+\frac{\omega\chi_0}{2}\right), \tag{19}$$

where $E_{\mathbf{g}}^{(s)Rad}$ is the required radiation field.

The expression connecting the diffracted and incident fields in the crystal follows from the second equation of system (9):

$$E_0^{(s)\text{cr}}=\frac{2\omega\lambda_{\mathbf{g}}}{\omega^2\chi_g C^{(s,\tau)}}E_{\mathbf{g}}^{(s)\text{cr}}. \tag{20}$$

The boundary conditions on the inlet and outlet surfaces of the crystal plate in the given geometry have the form

$$\int E_0^{(s)\text{vac}}d\lambda_{\mathbf{g}}=\int E_0^{(s)\text{cr}}d\lambda_{\mathbf{g}},$$

$$\int E_{\mathbf{g}}^{(s)\text{cr}}d\lambda_{\mathbf{g}}=\int E_{\mathbf{g}}^{(s)\text{vac}}d\lambda_{\mathbf{g}},$$

$$\int E_{\mathbf{g}}^{(s)\text{cr}}\exp\left(i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}L\right)d\lambda_{\mathbf{g}}=0, \tag{21}$$

we will obtain the following expression for radiation field:

$$E_{\text{Rad}}^{(s)}=\frac{8\pi^2 ieV\Theta P^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}C^{(s,\tau)}}{2\omega\left(\lambda_{\mathbf{g}}^{(2)}\exp\left(i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(2)}}{\gamma_{\mathbf{g}}}L\right)-\lambda_{\mathbf{g}}^{(1)}\exp\left(i\frac{\lambda_{\mathbf{g}}^*-\lambda_{\mathbf{g}}^{(1)}}{\gamma_{\mathbf{g}}}L\right)\right)}\times$$

$$\times\left[\left(\frac{1}{\frac{\gamma_0}{|\gamma_g|}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g^*+\beta\frac{\gamma_0}{\gamma_g}\right)}-\frac{2\omega\exp\left(i\frac{\lambda_g^*-\lambda_g^{(2)}}{\gamma_g}L\right)}{4\frac{\gamma_0^2}{\gamma_g^2}\left(\lambda_g^*-\lambda_g^{(1)}\right)}\right)\left(1-\exp\left(i\frac{\lambda_g^*-\lambda_g^{(1)}}{\gamma_g}L\right)\right)-\right.$$
$$\left.-\left(\frac{1}{\frac{\gamma_0}{|\gamma_g|}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g^*+\beta\frac{\gamma_0}{\gamma_g}\right)}-\frac{2\omega\exp\left(i\frac{\lambda_g^*-\lambda_g^{(1)}}{\gamma_g}L\right)}{4\frac{\gamma_0^2}{\gamma_g^2}\left(\lambda_g^*-\lambda_g^{(2)}\right)}\right)\left(1-\exp\left(i\frac{\lambda_g^*-\lambda_g^{(2)}}{\gamma_g}L\right)\right)\right]. \tag{22}$$

As the radiation field of relativistic electron which rectilinearly crosses a crystal contains the contributions of PXR and DTR we will represent the amplitude $E_{\mathrm{Rad}}^{(s)}$ as a sum of PXR and DTR amplitudes:

$$E_{\mathrm{Rad}}^{(s)}=E_{PXR}^{(s)}+E_{DTR}^{(s)}, \tag{23a}$$

$$E_{PXR}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_g C^{(s,\tau)}}{2\omega\left(\lambda_g^{(2)}\exp\left(i\frac{\lambda_g^*-\lambda_g^{(2)}}{\gamma_g}L\right)-\lambda_g^{(1)}\exp\left(i\frac{\lambda_g^*-\lambda_g^{(1)}}{\gamma_g}L\right)\right)}\times$$
$$\times\left[\left(\frac{2\omega\exp\left(i\frac{\lambda_g^*-\lambda_g^{(1)}}{\gamma_g}L\right)}{4\frac{\gamma_0^2}{\gamma_g^2}\left(\lambda_g^*-\lambda_g^{(2)}\right)}+\frac{\omega}{2\frac{\gamma_0}{|\gamma_g|}\lambda_0^*}\right)\left(1-\exp\left(i\frac{\lambda_g^*-\lambda_g^{(2)}}{\gamma_g}L\right)\right)-\right.$$
$$\left.-\left(\frac{2\omega\exp\left(i\frac{\lambda_g^*-\lambda_g^{(2)}}{\gamma_g}L\right)}{4\frac{\gamma_0^2}{\gamma_g^2}\left(\lambda_g^*-\lambda_g^{(1)}\right)}+\frac{\omega}{2\frac{\gamma_0}{|\gamma_g|}\lambda_0^*}\right)\left(1-\exp\left(i\frac{\lambda_g^*-\lambda_g^{(1)}}{\gamma_g}L\right)\right)\right], \tag{23b}$$

$$E_{DTR}^{(s)}=\frac{8\pi^2 ieV\theta P^{(s)}}{\omega}\frac{\omega^2\chi_g C^{(s,\tau)}}{2\omega\left(\lambda_g^{(2)}\exp\left(-i\frac{\lambda_g^{(2)}}{\gamma_g}L\right)-\lambda_g^{(1)}\exp\left(-i\frac{\lambda_g^{(1)}}{\gamma_g}L\right)\right)}\times$$
$$\times\left[\frac{1}{\frac{\gamma_0}{|\gamma_g|}\left(-\chi_0-\frac{2}{\omega}\frac{\gamma_0}{\gamma_g}\lambda_g^*+\beta\frac{\gamma_0}{\gamma_g}\right)}+\frac{\omega}{2\frac{\gamma_0}{|\gamma_g|}\lambda_0^*}\right]\left(\exp\left(-i\frac{\lambda_g^{(2)}}{\gamma_g}L\right)-\exp\left(-i\frac{\lambda_g^{(1)}}{\gamma_g}L\right)\right). \tag{23c}$$

In accordance with the expression (23b) there are possible two branches (16) of dispersion relation (12) which give the contributions into PXR yield. These branches correspond to two radiated X-ray waves formed together with the equilibrium electromagnetic field of the fast particle. A larger contribution to the radiation gives the branch of RXR, for which the real part of the denominator in the formula (23b) can vanish ($\mathrm{Re}\left(\lambda_{\mathbf{g}}^{*}-\lambda_{\mathbf{g}}^{(1,2)}\right)=0$): For further analyses it is convenient to represent $\lambda_{\mathbf{g}}^{*}$ and $\lambda_{\mathbf{g}}^{(1,2)}$ in such a form:

$$\lambda_{\mathbf{g}}^{(1,2)}=\frac{\omega\left|\chi_{\mathbf{g}}'C^{(s)}\right|}{2}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1+\varepsilon)}{2}\pm\sqrt{\xi^{(s)2}-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^{2}}{4}-\kappa^{(s)2}\varepsilon\right)}\right),$$

$$\lambda_{\mathbf{g}}^{*}=\frac{\omega\left|\chi_{\mathbf{g}}'C^{(s)}\right|}{2}\left(2\xi^{(s)}-i\rho^{(s)}-\varepsilon\sigma^{(s)}\right), \qquad (24)$$

where

$$\xi^{(s)}=\xi^{(s)}(\omega)=\eta^{(s)}(\omega)+\frac{(1+\varepsilon)}{2\nu^{(s)}},\quad \nu^{(s)}=\frac{|\chi_{\mathbf{g}}'C^{(s)}|}{|\chi_{0}'|},\quad \rho^{(s)}=\frac{\chi_{0}''}{|\chi_{\mathbf{g}}'C^{(s)}|},\quad \varepsilon=\frac{|\gamma_{\mathbf{g}}|}{\gamma_{0}},\quad \kappa^{(s)}=\frac{\chi_{\mathbf{g}}''C^{(s)}}{\chi_{0}''}$$

$$\eta^{(s)}(\omega)=\frac{\alpha}{2|\chi_{\mathbf{g}}'C^{(s)}|}=\frac{2\sin^{2}\Theta_{B}}{V^{2}|\chi_{\mathbf{g}}'C^{(s)}|}\left(\frac{\omega_{B}(1+\theta\cos\varphi\cot\Theta_{B})}{\omega}-1\right),$$

$$\sigma^{(s)}=\frac{1}{|\chi_{\mathbf{g}}'C^{(s)}|}\left((\theta_{\perp}-\psi_{\perp})^{2}+(\theta_{//}+\psi_{//})^{2}+\gamma^{-2}-\chi_{0}'\right). \qquad (25)$$

Since the inequality $2\sin^{2}\Theta_{B}/V^{2}|\chi_{\mathbf{g}}'C^{(s)}|>>1$ is fulfilled in the range of X-ray frequencies, $\eta^{(s)}(\omega)$ is a fast function of frequency ω , and it is convenient for the further analysis of the properties of the PXR and DTR spectrum to consider $\eta^{(s)}(\omega)$ as a spectral variable. Let us note that the resultant formulas contain $\xi^{(s)}(\omega)$ and not $\eta^{(s)}(\omega)$. The second term of the last equation appears due to the refraction effect.

When deducing the formula (24) we took into account that in the considered geometry of radiation the angle between the momentum of diffracted photon and normal vector to the crystal surface is blunt which means that $\gamma_{\mathbf{g}}=\cos\phi_{\mathbf{g}}<0$.

The parameter ε from Eq. (25) can be presented in the form $\varepsilon=\sin(\Theta_{B}-\delta)/\sin(\Theta_{B}+\delta)$, where δ is the angle between the inlet surface of the target and the crystallographic plane. For a fixed value of Θ_{B} , quantity ε defines the orientation of the inlet surface of the crystal plate relative to the system of diffracting atomic planes (Fig. 2). As the angle of incidence $(\Theta_{B}+\delta)$ of an electron on the target decreases, parameter δ becomes negative and then increases in magnitude (in the limiting case $\delta\rightarrow-\Theta_{B}$), which leads to an increase in ε . On the contrary, upon an increase in the angle of incidence, ε decreases (in the limiting case $\delta\rightarrow\Theta_{B}$).

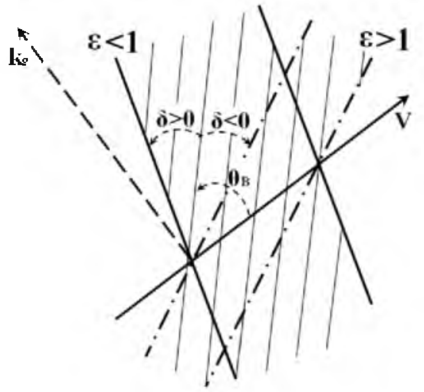

Fig. 2 Asymmetric ($\varepsilon > 1, \varepsilon < 1$) reflections of radiation from a crystal plate.

The case $\varepsilon = 1$ ($\delta = 0$) corresponds to symmetric reflection.

3. Spectral-angular density of PXR and DTR

By substitution of (23) to the expression for x-radiation spectra-angular density [21]:

$$\frac{d^2W}{d\omega d\Omega} = \omega^2 (2\pi)^{-6} \sum_{s=1}^{2} \left| E_{Rad}^{(s)} \right|^2 , \tag{26}$$

we will obtain the formula for spectral-angular density of PXR, DTR and the item which is the result of these mechanisms interference

$$\omega \frac{d^2 N_{\text{PXR}}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\Omega^{(s)2}}{(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0')^2} R_{\text{PXR}}^{(s)} , \tag{27a}$$

$$R_{PXR}^{(s)} = \left| \frac{\Omega_+^{(s)}}{\Delta^{(s)}} \frac{1 - \exp\left(-ib^{(s)}\Delta_+^{(s)}\right)}{\Delta_+^{(s)}} - \frac{\Omega_-^{(s)}}{\Delta^{(s)}} \frac{1 - \exp\left(-ib^{(s)}\Delta_-^{(s)}\right)}{\Delta_-^{(s)}} \right|^2 , \tag{27b}$$

$$\omega \frac{d^2 N_{DTR}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \Omega^{(s)2} \left(\frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2} - \frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'} \right)^2 R_{DTR}^{(s)} , \tag{28a}$$

$$R_{DTR}^{(s)} = \varepsilon^2 \left| \frac{\exp\left(-ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right) - \exp\left(ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right)}{\left(\xi^{(s)} - K^{(s)} - i\rho^{(s)} \frac{1+\varepsilon}{2}\right) \exp\left(-ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right) - \left(\xi^{(s)} + K^{(s)} - i\rho^{(s)} \frac{1+\varepsilon}{2}\right) \exp\left(ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right)} \right|^2 , \tag{28b}$$

$$\omega \frac{d^2 N_{INT}^{(s)}}{d\omega d\Omega} = \frac{e^2}{\pi^2} \frac{\Omega^{(s)2}}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'} \times$$
$$\times \left(\frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2 - \chi_0'} - \frac{1}{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_{//} + \psi_{//})^2} \right) R_{INT}^{(s)} , \tag{29a}$$

$$R_{INT}^{(s)} = 2\varepsilon \operatorname{Re}\left(\left(\frac{\Omega_+^{(s)}}{\Delta^{(s)}} \cdot \frac{1 - \exp\left(-ib^{(s)}\Delta_+^{(s)}\right)}{\Delta_+^{(s)}} - \frac{\Omega_-^{(s)}}{\Delta^{(s)}} \cdot \frac{1 - \exp\left(-ib^{(s)}\Delta_-^{(s)}\right)}{\Delta_-^{(s)}} \right) \times \right.$$

$$\left. \times \left(\frac{\exp\left(-ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right) - \exp\left(ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right)}{\left(\xi^{(s)} - K^{(s)} - i\rho^{(s)} \frac{1+\varepsilon}{2}\right) \exp\left(-ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right) - \left(\xi^{(s)} + K^{(s)} - i\rho^{(s)} \frac{1+\varepsilon}{2}\right) \exp\left(ib^{(s)} \frac{K^{(s)}}{\varepsilon}\right)} \right)^* \right) , \tag{29b}$$

where the sign "*" indicates complex conjugation. The table of symbols in the formulas is:

$$\Omega^{(1)}=\theta_{\perp}-\psi_{\perp},\ \Omega^{(2)}=\theta_{//}+\psi_{//},\ \sigma^{(s)}=\frac{1}{|\chi'_g|C^{(s)}}\left(\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi'_0\right),$$

$$\Delta^{(s)}=\left(\xi^{(s)}-K^{(s)}-i\rho^{(s)}\frac{1+\varepsilon}{2}\right)\exp\left(-ib^{(s)}\Delta_{+}^{(s)}\right)-\left(\xi^{(s)}+K^{(s)}-i\rho^{(s)}\frac{1+\varepsilon}{2}\right)\exp\left(-ib^{(s)}\Delta_{-}^{(s)}\right),$$

$$\Omega_{\pm}^{(s)}=\varepsilon\left((\sigma^{(s)}-i\rho^{(s)})\cdot\exp\left(-ib^{(s)}\Delta_{\mp}^{(s)}\right)+\Delta_{\pm}^{(s)}\right),$$

$$\Delta_{\pm}^{(s)}=\frac{\xi^{(s)}\pm K^{(s)}}{\varepsilon}-\sigma^{(s)}+i\frac{\rho^{(s)}(\varepsilon-1)}{2\varepsilon},\ b^{(s)}=\frac{1}{2\sin(\theta_B+\delta)}\frac{L}{L_{ext}^{(s)}},$$

$$K^{(s)}=\sqrt{\xi^{(s)2}-\varepsilon-i\rho^{(s)}((1+\varepsilon)\xi^{(s)}-2\kappa^{(s)}\varepsilon)-\rho^{(s)2}\left(\frac{(1+\varepsilon)^2}{4}-\kappa^{(s)2}\varepsilon\right)}. \quad (30)$$

Parameter $b^{(s)}$ characterizing the thickness of the crystal plate is the ratio of half of the path of the electron in the target $L_e=L/\sin(\theta_B+\delta)$ to the extinction length $L_{ext}^{(s)}=1/\omega|\chi'_g|C^{(s)}$. Parameter $\rho^{(s)}$ characterizing the degree of absorption of X-waves in the crystal, is the ratio of the extinction length $L_{ext}^{(s)}$ to the length of the absorption $L_{abs}=1/\omega\chi''_0$ of the X- rays: $\rho^{(s)}=L_{ext}^{(s)}/L_{abs}$. The functions $R_{PXR}^{(s)}$ and $R_{DTR}^{(s)}$ describe the spectra of PXR and DTR.

The expressions (27-29) describe the spectral-angular density of PXR and DTR of the relativistic electron crossing a crystal plate at an angle ψ relative to the axis of the electron beam $\mathbf{e}_1$ and their interference. The expressions are obtained in the framework of the two-wave approximation of dynamical diffraction theory taking into account the angle between the reflecting system of parallel atomic planes of the crystal and the target surface (angle δ). The expressions (27-29) can be used for investigation of spectral and angular properties of PXR and DTR generated in a crystal by beams of relativistic electron. In the present time the indication of angular divergence of the super high energy electron beams, for example, at the planning international linear collider ILC is actual problem. Because of the fact that the direction of radiation can be considerable diverged from the direction of the electron beam these mechanisms of the radiation could be contenders for use in this task. The beam indication at accelerator must be carried out in real time with minimal effect on the beam characteristics. In connection with it we will consider further the spectral-angular densities of PXR and DTR for the case of a thin monocrystalline target, when the multiple scattering of the electrons on atoms can be neglected.

4. Spectral-angular density of PXR and DTR in a thin crystal

Let us take up (consider) a thin crystal. Assuming in expressions (27-28) $\rho^{(s)}=\frac{\chi''_0}{|\chi'_g|C^{(s)}}=0$, we will obtain the following expression for spectral-angular density of PXR and DTR

$$\omega\frac{d^2N_{\rm PXR}^{(s)}}{d\omega d\Omega}=\frac{e^2}{\pi^2}\frac{\Omega^{(s)2}}{(\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^2+(\theta_{//}+\psi_{//})^2-\chi'_0)^2}R_{\rm PXR}^{(s)}, \quad (31a)$$

$$R_{\mathrm{PXR}}^{(s)}=\frac{\left(\xi^{(s)}+\sqrt{\xi^{(s)2}-\varepsilon}\right)^{2}}{\xi^{(s)2}-\varepsilon+\varepsilon\sin^{2}\left(\dfrac{b^{(s)}\sqrt{\xi^{(s)2}-\varepsilon}}{\varepsilon}\right)}\frac{\sin^{2}\left(\dfrac{b^{(s)}}{2}\left(\dfrac{\xi^{(s)}+\sqrt{\xi^{(s)2}-\varepsilon}}{\varepsilon}-\sigma^{(s)}\right)\right)}{\left(\dfrac{\xi^{(s)}+\sqrt{\xi^{(s)2}-\varepsilon}}{\varepsilon}-\sigma^{(s)}\right)^{2}},\qquad(31b)$$

$$\omega\frac{d^{2}N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega}=\frac{e^{2}}{\pi^{2}}\Omega^{(s)2}\times$$
$$\times\left(\frac{1}{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^{2}+(\theta_{//}+\psi_{//})^{2}}-\frac{1}{\gamma^{-2}+(\theta_{\perp}-\psi_{\perp})^{2}+(\theta_{//}+\psi_{//})^{2}-\chi_{0}'}\right)^{2}R_{DTR}^{(s)},\qquad(32a)$$

$$R_{DTR}^{(s)}=\frac{\varepsilon^{2}}{\xi^{(s)2}-\left(\xi^{(s)2}-\varepsilon\right)\coth^{2}\left(\dfrac{b^{(s)}\sqrt{\varepsilon-\xi^{(s)2}}}{\varepsilon}\right)}.\qquad(32b)$$

Let us define the angular densities of PXR and DTR by integration of expressions (31) and (32) by the frequency function $\xi^{(s)}(\omega)$ with use of the relation $\frac{d\omega}{\omega}=-\frac{|\chi_{\mathrm{g}}'|C^{(s)}}{2\sin^{2}\theta_{B}}d\xi^{(s)}$ which is followed from expression for $\xi^{(s)}(\omega)$ in (25). Then the angular density of PXR will take such a view

$$\frac{dN_{PXR}^{(s)}}{d\Omega}=\frac{e^{2}}{2\pi^{2}\sin^{2}\theta_{B}|\chi_{\mathrm{g}}'|C^{(s)}}\frac{\Omega^{(s)2}}{\sigma^{(s)2}}\int_{\sqrt{\varepsilon}}^{\infty}R_{PXR}^{(s)}d\xi^{(s)}(\omega).\qquad(33)$$

Since the spectral peak of PXR is very narrow then under condition $b^{(s)}>>1$ the well-known approximation $\frac{\sin^{2}(ax)}{x^{2}}\to\pi a\delta(x)$ can be used for integration. In this case the expression (33) will take a view

$$\frac{dN_{PXR}^{(s)}}{d\Omega}=\frac{e^{2}}{8\pi\sin^{2}\theta_{B}|\chi_{\mathrm{g}}'|C^{(s)}}\frac{\Omega^{(s)2}}{\sigma^{(s)2}}\frac{\varepsilon^{2}(\sigma^{(s)2}\varepsilon-1)}{\left(\dfrac{\sigma^{(s)2}\varepsilon-1}{2\sigma^{(s)}}\right)^{2}+\varepsilon\sin^{2}\left(\dfrac{\sigma^{(s)2}\varepsilon-1}{2\sigma^{(s)}\varepsilon}b^{(s)}\right)}b^{(s)}.\qquad(34)$$

The angular density of DTR will take the view

$$\frac{dN_{DTR}^{(s)}}{d\Omega}=\frac{e^{2}\chi_{0}'^{2}}{2\pi^{2}\sin^{2}\theta_{B}|\chi_{\mathrm{g}}'|C^{(s)}}\frac{\Omega^{(s)2}}{\sigma^{(s)2}\left(|\chi_{\mathrm{g}}'|C^{(s)}\sigma^{(s)}+\chi_{0}'\right)^{2}}\varepsilon\sqrt{\varepsilon}\pi\cdot\tanh\left(\frac{b^{(s)}}{\sqrt{\varepsilon}}\right).\qquad(35)$$

These expressions are derived with a glance of possibility of deviation $\psi(\psi_{\perp},\psi_{//})$ of the direction of the electron velocity $\mathbf{V}$ in relation to electron beam axis $\mathbf{e}_1$. The obtained expressions take into account the reflection asymmetry of the electron field relative to the target surface (parameter ε) are characterized by the angle δ between target surface and a system of diffracting atomic planes in the crystal.

5. Influence of electron beam divergence on the angular densities of PXR and DTR

Let us consider the effect of the electron beam divergence on the spectral and angular characteristics of the radiation. For that, let us average radiation of an electron over all of its possible straight trajectories in the beam. As an example, we will carry out the averaging of the spectral-angular density of PXR and DTR for the electron beam with the Gauss angular distribution:

$$f(\psi) = \frac{1}{\pi\psi_0^2} \cdot \exp\left\{-\frac{\psi^2}{\psi_0^2}\right\}, \tag{36}$$

where ψ_0 is the divergence of the beam of radiating electrons (see Figure 1). In this case the formulas for averaged angular density (11) and (12) will take the view:

$$\left\langle \frac{dN^{(s)}_{\text{PXR,DTR}}}{d\Omega} \right\rangle = \frac{1}{\pi\psi_0^2} \iint d\psi_\perp d\psi_{//} \exp\left\{-\frac{\psi_\perp^2 + \psi_{//}^2}{\psi_0^2}\right\} \frac{dN^{(s)}_{PXR,DTR}}{d\Omega}. \tag{37}$$

Let us carry out the numerical calculations of the angular densities of PXR and DTR using the derived expressions (34) and (35) and expression (37). For example let us consider the normalized to one electron angular density of the radiation of the beam of relativistic electron crossing a single crystal plate of tungsten $W(110)$ of thickness $L = 2\mu m$. The angle between electron beam axis $\mathbf{e}_1$ and the system of parallel diffracting atomic planes (110) of the crystal is chosen as $\theta_B \approx 20{,}5^o$, the Bragg frequency $\omega_B \approx 8keV$. The angle between the crystal surface and system of diffracting atomic planes $\delta \approx -10^o$, i.e. the case of symmetric reflection of asymmetric parameter $\varepsilon = \sin(\theta_B - \delta)/\sin(\theta_B + \delta) \approx 2{,}7$ is considered.

In Fig.3 the curves describing the angular density of PXR of relativistic electron under different values of initial divergence ψ_0 of electron beam are cited. The curves are built for the case when the relativistic electrons have energy of 150 GeV. Under this condition the dependence of angular density of PXR on electron energy is saturated and angular density do not change with increase of electron energy.

The curves demonstrate a weak dependence of PXR angular density on initial divergence, that is connected with the broad angular distribution of PXR. That is why the use of PXR for the estimation of divergence of the electron beams of high energy will not be possible.

One can see, that angular density of PXR weakly depends on divergence of the beam at such value of energy of relativistic electrons. It is connected with the fact that the angular distribution of PXR in this case is broader than that of electron beam.

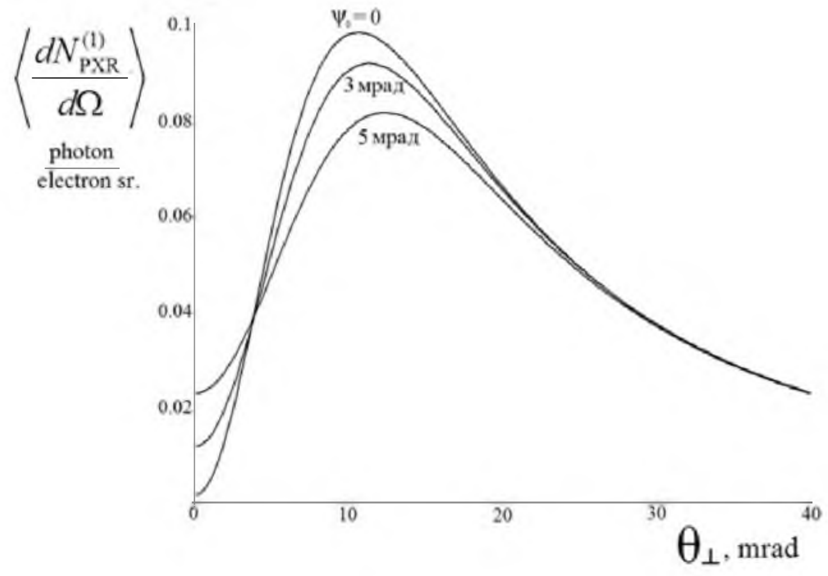

Fig. 3 The influence of electron beam divergence on the angular density of PXR.

Energy of electron is $E > 150 МэВ$, ψ_0 is electron beam.

In Fig.4 the curves describing the angular density od DTR of relativistic electron with energy of $E = 500 МэВ$ are presented for different values of divergence ψ_0. One can see that the angular density of DTR strongly depends on initial divergence of the electron beam. This fact can be explained by narrow angular distribution of DTR, which leads to considerable dependence on the divergence of the electron beam.

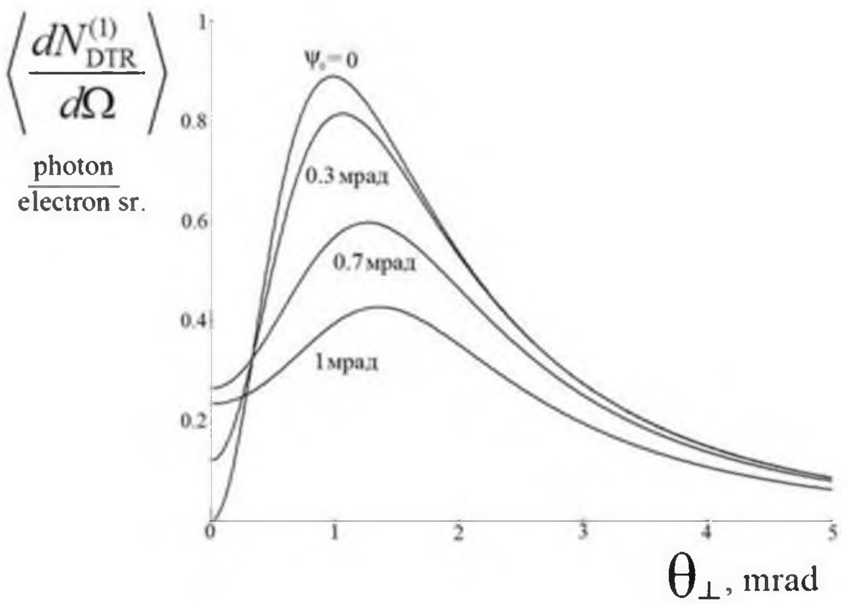

Fig. 4 The influence of electron beam divergence on angular density of DTR.

Energy of electron beam is $E = 500 МэВ$.

On can see that electron beam divergence more appreciably influence on angular density of DTR. It can be explained by the fact that DTR is narrower than PXR under high energy of radiating electron and becomes even narrower when energy of the electron increases. That is why DTR is more sensitive to a change of divergence of electron beam than PXR, which angular distribution almost do not depend on the electron energy. One can see in the fig. 4 that the angular distribution of DTR becomes narrower and the dependence of DTR characteristics on electron beam divergence becomes stronger when the energy of radiating electron increases. It are demonstrated also in Fig. 5 and Fig.6 for energy of electrons $E = 10 ГэВ$ and $E = 200 ГэВ$ correspondingly.

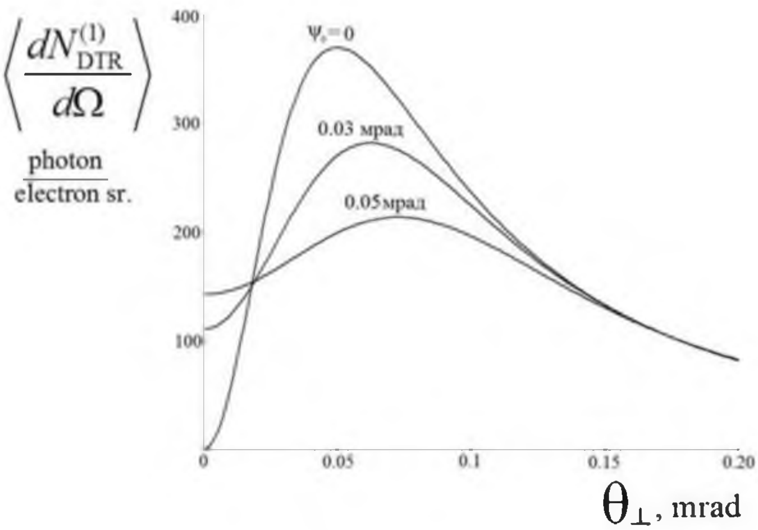

Fig. 5 The same, that in Fig. 4 under value of relativistic electron energy $E = 10\, ГэВ$.

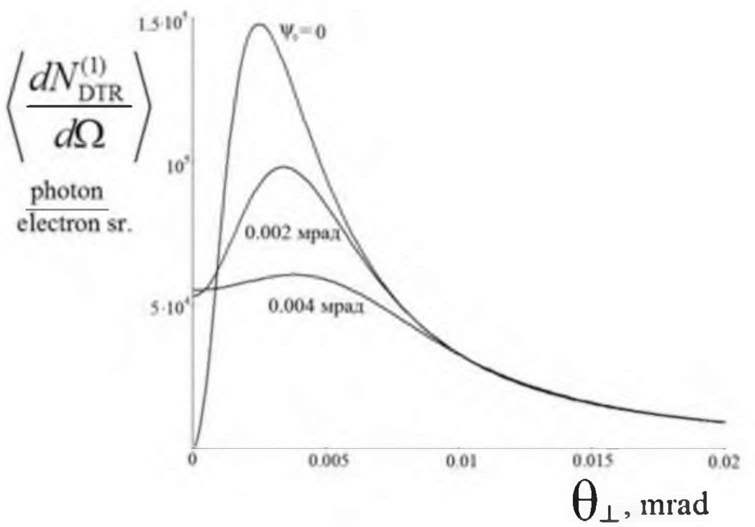

Fig. 6 The same, that in Fig. 4 under value of relativistic electron energy $E = 200\, ГэВ$.

For electrons of super high energies the angular density of DTR becomes very sensitive to angular divergence of electron beam. That can be used for analyze of the electron beams of super high energies.

It need be noted that under super high energies of radiating electrons the measurement of DTR characteristics is difficult task because of its sharp directionality. But in this case one can enough easy calculate the dependence on divergence of electron beam of number of the photons radiated in collimator of large lateral dimensions. So, the angular density or intensity of DTR can be effectively used for estimation of divergence of high energy electron beam, for example on the international linear electron collider ILC being under construction now.

As appropriate, the obtained expressions (31) and (32) for spectral-angular densities of PXR and DTR can be averaged by analogy to averaging of the angular densities (34) and (35).

In the present work the averaging of the radiation from electrons in the beam was carried out, as an example, with the assumption of Gauss angular distribution of electrons in the beam. But the methodology proposed in the present work allows the use for calculation of spectral-angular characteristics of radiations by the beams with arbitrary distribution of electrons. So, the expressions (31), (32), (34) and (35) allow calculate for each electron in the beam the spectral-angular characteristics of PXR and DTR with taking into account the angle between its trajectory and axis of the electron beam, with the assumption that electron rectilinearly crosses the monocrystalline plate and then find the integral spectral-angular characteristics of radiation by the beam as a whole

Conclusion

The dynamic theory of coherent X-Ray radiation produced by divergent relativistic electron beam crossing a monocrystalline plate in Bragg scattering geometry has been developed. The expressions describing spectral-angular characteristic of PXR and DTR have been derived based on the two-wave approximation of diffraction theory taking into account the deviation of electron velocity vector from the electron beam axis direction. The expressions for the general case of asymmetric reflection of the electron coulomb field relative to target surface have been obtained. The DTR and PXR are considered in the case of a thin target when the multiple scattering of the electrons on the target atoms is negligible which is important for measuring electron beam divergence in a real time regime without change under influence of measurement process. The calculations of the radiation spectral-angular distribution have been made using the averaging by the Gauss two-dimensional angular distribution of the electrons in the beam. It was shown that the angular density of DTR depends on electron beam divergence more strongly than the angular density of PXR. The amplification of DTR angular density dependence on electron beam divergence with increase of the radiating electron energy has been identified. It is shown that under ultrahigh energies the DTR angular density becomes sensitive to very small angular divergence of the electron beam, which can be used for analysis of the dimension and divergence of such an electron beam as in the planning international linear electron collider ILC.

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