Mathematical modeling of diagnostics of thermoelastic layered medium

Alexander V. Lomazov[1], *Vadim* A. Lomazov[2,3*], *Olga* S. Akupiyan[2], *Vladislav* L. Anichin[2], and *Elena* V. Nesterova[3]

[1]Financial University under the Government of the Russian Federation, Moscow, 125167, Russia
[2]Belgorod State Agricultural University named after V. Gorin, Mayskiy, 308503, Russia
[3]Belgorod State National Research University, Belgorod, 308015, Russia

Abstract. The article discusses the problems of non-destructive testing of multilayer composite materials. Technological influences during manufacturing and the influence of external factors during the operation of composite structures can lead to the occurrence of material defects, which manifests itself as heterogeneity and anisotropy of individual layers and, as such, affects thermomechanical processes in multilayer composites. In the work as a model of non-destructive testing, the mathematical problem of determining the thermomechanical characteristics of weakly inhomogeneous and weakly anisotropic layers of a multilayer half-space is formulated. The tests include measurements on the outer surface of the half-space of temperature and displacements arising as a result of thermoelastic processes initiated in a special way in the layered medium under study. An approach to solving the formulated problem, based on the linearization of the original nonlinear equations and the application of the method of stationary basic processes, is proposed.

1 Introduction

The study of the mechanical properties of multilayer media is relevant not only within the framework of the mechanics of composite structural materials [1] and biomechanics [2], but also in geophysics (in particular, in seismology [3]). Despite the fairly widespread development of technical methods of non-destructive testing (NDT), the features of which are discussed, for example, in review works [4-6], the problem of theoretical justification of NDT and, in particular, mathematical modeling of the physical processes used for NDT remains relevant.

The problem of thermoelastic diagnostics considered in this work is understood as the task of determining the thermomechanical characteristics of a material using experimentally obtained information about non-stationary stress and strain fields, as well as thermal fields arising in the solids under the influence of specially selected external influences [7-9].

In this paper, we study the problem of determining the characteristics of rigidity, density, specific heat capacity and thermal conductivity (which are functions of spatial variables) of weakly inhomogeneous and of weakly anisotropic thermoelastic layers, lying on a half-space.

* Corresponding author: vlomazov@yandex.ru

In this case, the values on the outer surface of the upper layer of the characteristics of thermoelastic processes are used (as the results of measurements during the testing process).

In mathematical terms, the problem under study belongs to the type of inverse problems of mathematical physics [10] or (more precisely) it can be attributed to the class of coefficient inverse problems of dynamic thermoelasticity (thermal elastic dynamics) [11].

The purpose of this article is to formulate and solve the problem of determining the thermomechanical characteristics of a layered medium based on mathematical modeling of thermoelastic processes initiated in a special way in this medium.

2 Problem statement

Let's consider inhomogeneous anisotropic layered half-space $R_+^3=\{(x_1, x_2, x_3) \mid x_3 \geq 0\}$, where are the layers: $R_1=\{(x_1,x_2,x_3) \mid h_1 \geq x_3 \geq 0\}$, $R_2=\{(x_1,x_2,x_3) \mid h_2 \geq x_3 \geq h_1\}, \ldots, R_S=\{(x_1, x_2, x_3) \mid h_S \geq x_3 \geq h_{S-1}\}$ lie on the half-space $R_{S+1}=\{(x_1,x_2,x_3) \mid x_3 \geq h_s\}$ (Figure 1)

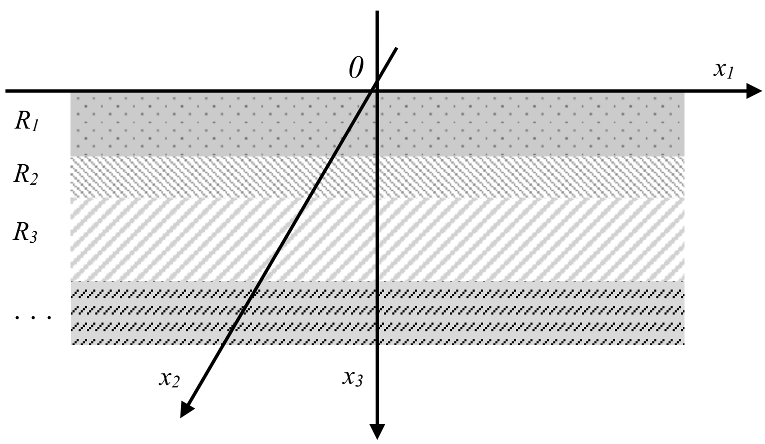

Fig. 1. Inhomogeneous anisotropic layered half-space.

Propagation of unsteady thermoelastic processes in layered half-space is described by the equations [12]:

$$C_v^{(r)}\,\partial_t\theta^{(r)}-(K_{ij}^{(r)}\theta_{,i}^{(r)})_{,j}=f_0^{(r)} \tag{1}$$

$$\rho^{(r)}\,\partial_t^2 u_i^{(r)}-(C_{ijkm}^{(r)}u_{k,m}^{(r)}-\beta_{ij}^{(r)}\theta^{(r)})_{,j}=f_i^{(r)} \qquad (i,j,k,m=1,2,3), \tag{2}$$

whith initial conditions

$$\theta^{(r)}(\mathbf{x},0)=\varphi_0^{(r)}(\mathbf{x}),\quad u_i^{(r)}(\mathbf{x},0)=\varphi_i^{(r)}(\mathbf{x}),\qquad \partial_t u_i^{(r)}(\mathbf{x},0)=\psi_i^{(r)}(\mathbf{x}) \tag{3}$$

and boundary conditions

$$K_{i3}^{(1)}\theta^{(1)}_{,i}(x_1,x_2,0,t)=p^{(1)}_0(x_1,x_2,t), \tag{4}$$

$$\{C_{i3km}^{(1)}u_{k,m}^{(1)}-\beta_{ij}^{(1)}\theta^{(1)}\}(x_1,x_2,0,t)=p_i^{(r)}(x_1,x_2,t) \tag{5}$$

as well as contact conditions of ideal thermal contact of layers

$$\{\theta^{(r+1)}-\theta^{(r)}\}(x_1,x_2,h_r,t)=0,\qquad \{K_{i3}^{(r+1)}\theta^{(r+1)}_{,i}-K_{i3}^{(r)}\theta^{(r)}_{,i}\}(x_1,x_2,h_r,t)=0, \tag{6}$$

and rigid adhesion at the boundaries of layers

$$\{u_i^{(r+1)}-u_i^{(r)}\}(x_1,x_2,h_r,t)=0 \qquad (i=1,2,3) \tag{7}$$

$$\{C_{i3km}^{(r+1)}u_{k,m}^{(r+1)}-\beta_{i3}^{(r+1)}\theta^{(r+1)}-C_{i3km}^{(r)}u_{k,m}^{(r)}+\beta_{i3}^{(r)}\theta^{(r)}\}(x_1,x_2,h_r,t)=0,$$

(for $r=1,\ldots,S$ do not sum).

Here the temperature and components of the displacement vector $\boldsymbol{u}=(u_1, u_2, u_3)$ depend on the spatial variables $\boldsymbol{x}=(x_1,x_2,x_3)$ and time t, and the specific heat capacity C_v, density ρ and components of the thermal conductivity tensors K_{ij}, volumetric temperature expansion

β_{ij} as well as the stiffness tensor C_{ijkm} are assumed to be functions of x. Operators ∂_t, ∂_t^2 mean derivatives first and second order with respect to time. The index after the decimal point means the derivative with respect to the corresponding space coordinate. For a repeating index (unless otherwise specified), summation is performed. The superscript *(r)* means that the value belongs to the layer numbered r. Curly braces delimit lists of functions (expressions) that have the same arguments (indices).

The diagnostic problem considered in this work will be to determine $\{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkm}\}(\boldsymbol{x})$ from several problems of the form (1)-(7) for N different types of thermal force loading (after substituting into (1)-(7): $u_i{}^n \rightarrow u_i$, $\{\varphi_i, \psi_s, p_s, f_s\}^n \rightarrow \{\varphi_i, \psi_s, p_s, f_s\}$ $(i=1,2,3;$ $s=0,1,2,3;$ $n=1,2,\dots,N)$) according to additional information on the outer surface of the top layer

$$\theta^{(1)n}(x_1,x_2,0,t)=\chi_0{}^n(x_1,x_2,t), \tag{8}$$
$$u_i{}^{(1)n}(x_1,x_2,0,t)=\chi_i{}^n(x_1,x_2,t),$$
$$\{K_{i3},\beta_{i3},C_{i3km}\}^{(r)}(x_1,x_2,h_r)=\{K_{i3},\beta_{i3},C_{i3km}\}^{(r)},\ (i,k,m=1,2,3).$$

This information is assumed to be obtained from measurements on the surface. The value of N, corresponding to the number of tests with different types of thermal power loads, depends on the type of anisotropy of the layers under study (the number of spatially variable characteristics to be determined).

3 Linearization of the problem of diagnostics

In what follows, in accordance with the original formulation of the problem, we will assume that the ideal multilayer coating consists of homogeneous isotropic layers, but the layers included in the coating under study (due to external influences or imperfect manufacturing technology) have acquired slightly inhomogeneous and anisotropic properties. This means that for each layer $R_1, R_2, \dots, R_S$ and half-space R_{S+1} the values $|\rho^{(r)}-\rho^{(r)0}|/\rho^{(r)0}$, $|C_v{}^{(r)} - C_v{}^{(r)0}|/C_v{}^{(r)0}$, $|K_{ij}{}^{(r)} - K_{ij}{}^{(r)0}|/K^{(r)0}$, $|C_{ijkm}{}^{(r)} - C_{ijkm}{}^{(r)0}|/\lambda^{(r)0}$ have the order of smallness $O(\varepsilon)$, $0<\varepsilon<<1$, *где* $\rho^{(r)0}$, $C_v{}^{(r)0}$, $K_{ij}{}^{(r)0}$, $C_{ijkm}{}^{(r)0}$ are characteristics corresponding to the ideal homogeneous isotropic layers, and, therefore, $K_{ij}{}^{(r)0}=K^{(r)0}\delta_{ij}$, $\beta_{ij}{}^{(r)0}=\beta^{(r)0}\delta_{ij}$, $C_{ijkm}{}^{(r)0}=\lambda^{(r)0}\delta_{ij}\delta_{km} + \mu^{(r)0}(\delta_{ik}\delta_{jm}+\delta_{im}\delta_{jk})$. Here $\lambda^{(r)0}$, $\mu^{(r)0}$ are the Lame coefficients, δ_{ij} is the Kronecker symbol, $\rho^{(r)0}, C_v{}^{(r)0}, K^{(r)0}, \lambda^{(r)0}, \mu^{(r)0} - const$.

Let us evaluate the difference between the thermoelastic process $\{\theta,u\}^{(r)n}(\boldsymbol{x},t)$ and the similarly initiated process $\{\theta,u\}^{(r)0n}(\boldsymbol{x},t)$, occurring in a ideal medium with homogeneous isotropic layers. $\{\theta,u\}^{(r)0n}(\boldsymbol{x},t)$ is described by relations (1)-(7) after replacing $\{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkm}\}^{(r)0} \rightarrow \{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkm}\}^{(r)}(\boldsymbol{x},\theta)$. We will assume that the influence of weak inhomogeneity and weak anisotropy of the layers under study on the quantitative characteristics of the processes excited in it is also quite small. Thus, on the external surface of the top layer

$$\theta^n(x_1,x_2,0,t)=\theta^{0n}(x_1,x_2,0,t)+\varepsilon\theta^{1n}(x_1,x_2,0,t)=\chi^{0n}(x_1,x_2,t)+\varepsilon\chi^{1n}(x_1,x_2,t),$$
$$u_i{}^n(x_1,x_2,0,t)=u^{0n}(x_1,x_2,0,t)+\varepsilon u_i{}^{1n}(x_1,x_2,0,t)=\chi_i{}^{0n}(x_1,x_2,t)+\chi_i{}^{1n}(x_1,x_2,t),$$
$$||\theta^{0n}(x_1,x_2,0,t)||_C{}^2 \sim O(||\theta^{1n}(x_1,x_2,0,t)||_C{}^2,$$
$$||u^{0n}(x_1,x_2,0,t)||_C{}^2 \sim O(||u_i{}^{1n}(x_1,x_2,0,t)||_C{}^2.$$

Excluding quantities of order ε^2 from consideration, we obtain from relations (1)-(8) with respect to $\{\theta, u_i\}^{1n}$ the relation

$$C_v{}^{(r)0}\,\partial_t\,\theta^{(r)1n} - K_{ij}{}^{(r)0}\theta^{(r)1n}{}_{,ij} = - C_v{}^{(r)1}\partial_t\,\theta^{(r)0n} + (K_{ij}{}^{(r)1}\theta^{(r)0n}{}_{,i})_{,j} \tag{9}$$
$$\rho^{(r)0}\partial_t{}^2 u_i{}^{(r)1n} - C_{ijkm}{}^{(r)0}u_k{}^{(r)1n}{}_{,mj} + \beta_{ij}{}^{(r)0}\theta^{(r)1n}{}_{,j} = -\rho^{(r)1}\partial_t{}^2 u_i{}^{(r)0n} + (C_{ijkm}{}^{(r)1}u_k{}^{(r)0n}{}_{,m} - \beta_{ij}{}^{(r)1}\theta^{(r)0n})_{,j},$$

closed by initial, boundary and contact conditions

$$\theta^{(r)1n}(\boldsymbol{x},0)=0, \quad u_i{}^{(r)1n}(\boldsymbol{x},0)=0, \quad \partial u_i{}^{(r)1n}(\boldsymbol{x},0)=0 \tag{10}$$
$$\{K_{i3}{}^{(1)0}\theta_{,i}{}^{(1)1n}+K_{i3}{}^{(1)1}\theta_{,i}{}^{(1)0n}\}(x_1,x_2,0,t)=0 \tag{11}$$

$$\{C_{i3km}{}^{(1)0}u_k{}^{(1)1n}{}_{,m}+C_{i3km}{}^{(1)1}u_k{}^{(1)0n}{}_{,m}-\beta_{i3}{}^{(1)0}\theta^{(1)1}-\beta_{i3}{}^{(1)1}\theta^{(1)0n}\}(x_1,x_2,0,t)=0$$
$$\{\theta^{(r+1)1n}-\theta^{(r)1n}\}(x_{1,2},h_r,t)=0, \tag{12}$$
$$\{K_{i3}{}^{(r+1)1}\theta^{(r+1)0n}{}_{,i}+K_{i3}{}^{(r+1)0}\theta^{(r+1)1n}-K_{i3}{}^{(r)1}\theta^{(r)0n}{}_{,i}-K_{i3}{}^{(r)0}\theta^{(r)1n}\}(x_1,x_2,h_r,t)=0,$$
$$\{u_i{}^{(r+1)1}-u_i{}^{(r)1}\}(x_1,x_2,h_r,t)=0 \quad (i=1,2,3)$$
$$\{C_{i3km}{}^{(r+1)1}u_k{}^{(r+1)0n}{}_{,m}-\beta_{i3}{}^{(r+1)1}\theta^{(r+1)0n}+C_{i3km}{}^{(r+1)0}u_k{}^{(r+1)1n}{}_{,m}-\beta_{i3}{}^{(r+1)0}\theta^{(r+1)1n}-$$
$$-C_{i3km}{}^{(r)1}u_k{}^{(r)0n}{}_{,m}+\beta_{i3}{}^{(r)1}\theta^{(r)0n}-C_{i3km}{}^{(r)0}u_k{}^{(r)1n}{}_{,m}+\beta_{i3}{}^{(r)0}\theta^{(r)1n}\}(x_1,x_2,h_r,t)=0,$$
$$\theta^{(1)1n}(x_1,x_2,0,t)=\chi_0{}^{1n}(x_1,x_2,t), \tag{13}$$
$$u_i{}^{(1)1n}(x_1,x_2,0,t)=\chi_i{}^{1n}(x_1,x_2,t),$$
$$\{K_{i3},\beta_{i3},C_{i3km}\}^{(r)1}(x_1,x_2,h_r)=0, \qquad (i,k,l=1,2,3)$$

(for $r=1,\ldots,S$ do not sum).

In what follows, the characteristics of the ideal layers $\{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkm}\}^{(r)0}$ will be considered known, i.e. the diagnostic problem will be to clarify the properties of the layers under study. Note that the initial problem of determining $C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkm}, \theta^n, u_i{}^n$ from N relations of the form (1)-(8) is nonlinear, because these equations contain terms that are products of unknown functions of spatial variables (characteristics of thermoelastic processes and thermomechanical characteristics of the layer material). In this sense, the transition from (1)-(8) to (9)-(13) can be considered as a linearization procedure, widely used in solving nonlinear problems [12].

4 Solution of a linearized diagnostic problem

The process of propagation of thermoelastic waves in an ideal layered medium depends on the conditions of its initiation and can be quite complex. In what follows, for simplicity, we will assume the characteristics of this process are known and have the form $\{\theta,u_i\}^{(r)0n}(\mathbf{x},t) = exp(a_nt)\{g_0,g_i\}^{(r)n}(\mathbf{x}),\ a_n<0$ (do not sum over n). This naturally imposes restrictions on the functions $\{f_0, f_i, \varphi_0, \varphi_i, \psi_i, p_0, p_i\}^n$, i.e. on the conditions for the initiation of thermoelastic processes in the medium under study. Note that the specific form $\{f_0, f_i, \varphi_0, \varphi_i, \psi_i, p_0, p_i\}^n$ can be obtained by directly substituting $\{\theta,u_i\}^{(r)0n}(\mathbf{x},t)$ into (1)-(7) after replacing $\{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkl}\}^{(r)0} \to \{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkl}\}^{(r)}(\mathbf{x},\theta)$.

A feature of the problem of diagnosing a multilayer half-space is the ability to sequentially consider one layer after another and for each layer solve the diagnostic problem (determine the characteristics of the process occurring in a given layer and the characteristics of the layer material). We will show this using the example of diagnosing the first coating layer and describe the transition to diagnosing the second layer.

The procedure of diagnosing the first layer within the accepted assumption of the stationarity of thermoelastic processes in the ideal layered medium is divided into two stages:

stage 1 – determination of $\{\theta, u_i\}^{(1)1n}(\mathbf{x},t),\ n=1,N$;

stage 2 – recovery $\{C_v, \rho, K_{ij}, \beta_{ij}, C_{ijkl}\}^{(1)1}(x)$ from the equations (9).

Stage 1. After applying to equations (9), considered for $r=1$ in the layer $R_1=\{(x_1, x_2, x_3)\ |h_1 \geq x_3 \geq 0\}$ (i.e., describing thermoelastic processes in the first layer), the operator $L=\partial_t+a_nI$ (here ∂_t – the operator of partial differentiation with respect to the t and I – the unit operator), we obtain for each fixed n for new unknowns $T=L\ \theta^{(1)1n}$, $v_i=L\ u_i{}^{(1)1n}$

$$C_v{}^{(1)0}\partial_t T-(K_{ij}{}^{(1)0}T_{,i})_{,j}=0, \quad \rho^{(1)0}v_i-C_{ijkm}{}^{(1)0}v_{k,mj}+\beta_{ij}{}^{(1)0}T_{,j}=0 \tag{14}$$
$$v_i(\mathbf{x},0)=0 \tag{15}$$
$$K_{i3}{}^{(r)0}T_{,i}(x_1,x_2,0,t)=0, \quad T(x_1,x_2,0,t)=L\chi_0{}^{1n}(x_1,x_2,t), \tag{16}$$
$$C_{i3km}{}^{(r)0}v_{k,m}(x_1,x_2,0,t)=0, \quad v_i(x_1,x_2,0,t)=L\chi_i{}^{1n}(x_1,x_2,t), \quad (i=1,2,3), \tag{17}$$

The first equation (14) and boundary conditions (16) represent, with respect to $T(x,t)$, the Cauchy problem for the heat equation with data on a non-spatial manifold [13]. This problem has a unique solution in the entire half-space $R_+{}^3=\{(x_1, x_2,x_3)|x_3 \geq 0\}$. Methods for its construction are described, for example, in [8]. Thus, the desired solution $T(\boldsymbol{x},t)$ in the first layer can be considered as a restriction of the function found in the half-space R_+^3 to the layer $R_1=\{(x_1, x_2, x_3)|h_1 \geq x_3 \geq 0\}$.

Boundary conditions (17) allow us to find $\{U,U_{,3},\boldsymbol{W},\boldsymbol{W}_{,3}\}$ for $x_3=0$, where $U=div\ \boldsymbol{v}$, $\boldsymbol{W}=(W_1,W_2,W_3)= rot\ \boldsymbol{v}$. From condition (15) it follows that the initial conditions for these functions will be homogeneous: $U(\boldsymbol{x},0)=0$, $\boldsymbol{W}(\boldsymbol{x},0)=0$. Application of the operators *div* and *rot* to the second equation (14) allows us to obtain

$$\rho^0\partial_t^2U-(\lambda^0+2\mu^0)\Delta U=-\beta^0\Delta T, \tag{18}$$

$$\rho^0\partial_t^2\boldsymbol{W}-\mu^0\Delta\boldsymbol{W}=0 \tag{19}$$

Thus, two independent Cauchy problems were obtained for wave equations with data on a non-spatial manifold. Methods for solving such problems in the R_+^3 domain are also known [14]. The required functions $v_i(\boldsymbol{x},t)$ can be restored from the relations $U=div\ \boldsymbol{v}$, $W=rot\ \boldsymbol{v}$, considered as a system of three independent linear differential equations closed by boundary conditions $v_i(x_1,x_2,0,t)=L\chi_i^{1n}(x_1,x_2,t)$, $(i=1,2,3)$. Then, (as in the case of defining the function $T(\boldsymbol{x},t)$) the restriction of the functions $(\boldsymbol{x},t)$ from the domain R_+^3 to the domain $R_1=\{(x_1, x_2, x_3)|h_1 \geq x_3 \geq 0\}$ is considered.

Having determined $\{T,v_i\}(\boldsymbol{x},t)$, it is easy is from the relations $T=L\theta^{(1)1n}$, $v_i=Lu_i^{\ (1)1n}$, considered as ordinary differential equations for t, with homogeneous boundary conditions to $\{\theta,u_i\}^{(1)1n}(\boldsymbol{x},t)$ in the layer $R_1= \{(x_1,x_2,x_3)|h_1\geq x_3\geq 0\}$. Having completed all the above steps for $n=1,2,...,N$, we will complete the first stage of the problem.

Stage 2. Knowing $\theta^{ln},u_i^{\ 1n}$, you can determine the right-hand sides of equations (9), which will have the form $exp(-a_nt)\{F_0^{\ 1n},F_1^{\ 1n},F_2^{\ 1n},F_3^{\ 1n}\}(\boldsymbol{x})$ (do not sum over n) . Thus, the task of the second stage will be to determine $\{C_v,\rho,K_{ij},\beta_{ij},C_{ijkm}\}^1(\boldsymbol{x})$ from the equations

anCv1 g0n + (Kij1 g0n,i),j= F01n (20)

–an2ρ1gin +(Cijkm1 gkn,m – βij1 g0n),j = Fi1n (i=1,2,3) (21)

(do not sum over n) and homogeneous boundary conditions with respect to $\{K_{i3}, \beta_{i3}, C_{i3km}\}^1$ from (13). The value N (the number of different test modes) is chosen so that the number of scalar equations (20), (21) corresponds to the number of unknown functions $\{C_v,\rho, K_{ij}, \beta_{ij}, C_{ijkl}\}^1(\boldsymbol{x})$.

In the case of general anisotropy, the stiffness coefficient tensor C_{ijkl} contains 21 independent components, and the tensors of thermal conductivity coefficients K_{ij} and volumetric thermal expansion coefficients β_{ij} contain 6 independent components each. Thus, in this case there are 35 unknowns. A high order causes difficulties in solving a system of the form (20), (21). However, in a number of cases, consideration of special types of anisotropy and additional functional connections between individual thermomechanical characteristics of the material, as well as a special selection of functions $\{g_0, g_1, g_2 ,g_3\}^n$ (test conditions) can significantly reduce the order and simplify the solution.

As part of solving the problem of diagnosing the first layer, contact conditions at the boundary of the layers were not actually used. The influence of contact conditions is taken into account when considering the problem of diagnosing the second and subsequent coating layers. The conditions at the boundary of the first and second layers are a special case of relations (12) and can be presented in the form

$$\theta^{(2)1n}(x_1,x_2,h_1,t)=\theta^{(1)1n}(x_1,x_2,h_1,t) \tag{22}$$

$$\{K_{i3}^{(2)1}\theta^{(2)0n}{}_{,i}+K_{i3}^{(2)0}\theta^{(2)1n}\}(x_1,x_2,h_1,t)=\{K_{i3}^{(1)1}\theta^{(1)0n}{}_{,i}+K_{i3}^{(1)0}\theta^{(1)1n}\}(x_1,x_2,h_1,t)\ ,$$

$$u_i^{(2)1}(x_1,x_2,h_1,t)=u_i^{(1)1}(x_1,x_2,h_1,t) \quad (i=1,2,3)$$

$$\{C_{i3km}^{(2)1}u_k^{(2)0n}{}_{,m}-\beta_{i3}^{(2)1}\theta^{(2)0n}+C_{i3km}^{(2)0}u_k^{(2)1n}{}_{,m}-\beta_{i3}^{(2)0}\theta^{(2)1n}\}(x_1,x_2,h_1,t)=$$

$$=\{C_{i3km}^{\ (1)1}u_k^{(1)0n}{}_{,m}+\beta_{i3}^{(1)1}\theta^{(1)0n}-C_{i3km}^{\ (1)0}u_k^{(1)1n}{}_{,m}+\beta_{i3}^{(1)0}\theta^{(r)1n}\}(x_1,x_2,h_1,t)$$

It is easy to see that with this form of writing, relations (22) have the form of boundary conditions (11) and (13), since their right-hand sides can be considered known (the right-hand sides of (22) include quantities related to the first layer and considered to be found when solving the diagnostic problem of the first layer). Thus, by conditionally discarding the first layer, we get the task of diagnosing a layered coating completely similar to the original one, but consisting in the study of a multilayer coating containing one less layer. Diagnostics of the second layer and transition to the third layer are carried out in a completely similar way. Thus, a procedure is constructed for sequentially solving diagnostic problems one layer after another.

4 Conclusion

The problem of diagnostics for a thermoelastic layered medium makes it possible to determine in each layer specific heat capacity, density and components of the thermal conductivity tensors, volumetric temperature expansion as well as the stiffness tensor which are assumed to be functions of spatial coordinates. As the data used to find these thermomechanical characteristics the values of temperature and displacement on the outer surface of the first layer (which arose as a result of a specially initiated stationary thermoelastic process in the layered medium) are taken. These values are assumed to be obtained as a result of measurements during testing.

The problem of diagnostics for a thermoelastic layered medium studied in this work is nonlinear, since not only the relative temperature and displacements (which are functions of spatial variables and time), but also the characteristics of the material of the layers (coefficients of the equations) depending on spatial variables are unknown. The assumption of weak heterogeneity and weak anisotropy of layers (valid for a fairly wide class of materials) made it possible to linearize the diagnostic problem. The solution to the linearized problem is constructed using the method of stationary basic processes.

Continuation of research (the main results of which are presented in the article) can be aimed at studying the possibility of NDT modeling of multilayer materials with different types of layer reinforcement (fine reinforcement, fiber reinforcement, etc.), with different types of microdamage (pores, solid inclusions, microcracks) and also taking into account complex physical effects that arise during dynamic thermal force loading of materials (thermo-visco-elasticity [15], thermo-electro-elasticity [16], etc.). In addition, the issue of organizing the NDT procedure is of undoubted practical importance, for the implementation of which it is advisable to use the mathematical apparatus of planning and control theory (for example, [17]).

References

1. T.W. Clyne, D. Hull, *An Introduction to Composite Materials* (Cambridge University Press, 2019)
2. K.A. Athanasiou, R.M. Natoli, *Introduction to Continuum Biomechanics* (Morgan & Claypool Publishers, 2008)
3. V.I. Keilis-Borok., T.B. Yanovskaya, *Inverse Problems of Seismology (Structural Review)*. In: Keilis-Borok V I, Flinn EA. Computational Seismology (Boston, Springer, 1972)
4. M.S. Hirmaz, IJSER **6(8)**, 5-9 (2018)
5. J. Preethikaharshini et al, Journal of Materials Science **57(62)**, 16091-16146 (2022)
6. N. Gandhi et al, Journal of Manufacturing and Materials Processing **6(4)**, 71 (2022)
7. V.A. Lomazov, Problem of diagnostics of elastic semi-bounded bodies Prikl. Mat. Mekh. **53(5)**, 766-772 (1989)

8. V.A. Lomazov, Yu.V. Nemirovskii, Journal of Applied Mechanics and Technical Physics **44**, 146-153 (2003)
9. V.A. Lomazov et al., Journal of Physics: Conference Series **2573(1)**, 012049 (2023)
10. A.G. Ramm, *Inverse Problems. Mathematical and Analytical Techniques with Applications to Engineering* (NY, Springer, 2010)
11. A.O. Vatulyan, S.A. Nesterov, Mathematical Forum **15**, 26-27 (2023)
12. W. Nowacki, *Thermoelasticity* (Warsaw, Pergamon Press, 2013)
13. A.A. Samarsky, P.N. Vabishchevich, *Numerical methods for solving inverse problems of mathematical physics* (Moscow, URSS, 2015)
14. R.M. Garipov, V.B. Kardakov, Dokl. Academy of Sciences of the USSR **213(5)**, 1047-1050 (1973)
15. G.A. Maugin, *Viscoelasticity of solids (old and new)* In book: Continuum Mechanics through the Ages - From the Renaissance to the Twentieth Century (Cham, Springer, 2016)
16. Z.B. Kuang, *Theory of Electroelasticity* (Cham, Springer, 2014)
17. D.A. Petrosov et al., Journal of Advanced Research in Dynamical and Control Systems **10(10)**, 1840-1846 (2018)