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Mathematical modeling of diagnostics of residual stresses in a layered medium

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Abstract. The work is devoted to the use of mathematical modeling in the framework of the problems of non-destructive testing of structural materials. A mathematical formulation of a three-dimensional problem of diagnostics for elastic layered medium is constructed, which makes it possible to determine the spatial distribution of residual stresses from the values of displacements measured on the surface of a layered half-space, which arose as a result of a specially initiated wave process. Under the assumption of weak inhomogeneity and anisotropy of the layers, a transition is made to a linearized (which is ill-posed) problem, the procedure for solving which is constructed using the method of stationary basic processes. As an example, the case of diagnosing a one-dimensional (depending only on the distance to the surface) distribution of residual stresses is considered.

1. Introduction

Initial (residual) stresses arise at the stage of materials manufacturing (during crystallization, glass formation, polymerization [1,2]), at the stage of manufacturing and technological processing of structural elements (during pressure treatment, radiation exposure, heat treatment, etc.), as well as at the stage of assembly (during welding, soldering, gluing, etc. [3]) and operation of products (for example, [4]). The particular relevance of this problem for layered composite materials is associated with the additional possibility of residual stresses due to the difference in melting temperatures, thermal linear expansion coefficients, drying shrinkage coefficients and other (depending on the manufacturing technology) physical properties of the materials of the reinforcing layers (components of the composite material) [5]. All of these factors can also lead to distortion of the shape of the layers. Initial stresses, as a rule, play a negative role in the operation of structures, which makes it desirable to eliminate them (for example, annealing glass and ceramic products) or, at least, take them into account when assessing the quality of products. The need to study initial stresses arises when solving problems of biomechanics, rock mechanics, and other areas of solid mechanics [6,7], which makes the development of non-destructive methods for determining residual stresses relevant.

The purpose of this work is to construct a mathematical formulation and solve the problem of diagnosing residual layered media based on the use of elastic wave processes.

2. Materials and methods

To build the relationships of a mathematical model that describes diagnostic tests, the conceptual and methodological apparatus of the dynamic theory of elasticity (elastodynamics) of anisotropic inhomogeneous bodies is used [8]. The procedure for solving the problem is constructed using the methods of the theory of partial differential equations and inverse problems for differential equations [9,10].

3. Statement of the problem of diagnosing residual stresses in a layered medium

The problem of determining the residual stresses by the values of the characteristics of the elastic wave processes occurring in it, which are considered to be known at the boundary of the layered half-space, is studied.

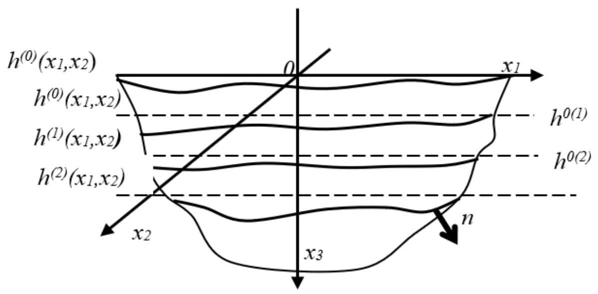

Figure 1. Layered half-space.

As a model of a massive body with a multilayer protective coating, we consider a semi-bounded body $\Omega=\{(x_1,x_2,x_3)|x_3\geq h^{(0)}\}$, where layers: $\Omega^{(s)}=\{(x_1,x_2,x_3)|h^{(s)}\geq x_3\geq h^{(s-1)}\}$, (here the function $x_3=h^{(0)}(x_1,x_2)$ describes the outer surface of the body, and the functions $x_3= h^{(s)}(x_1,x_2)$ – contact surfaces of the layers, $h^{(s)}>h^{(s-1)}, s=1,\dots,S$) lie on a semi-limited body $\Omega^{(S+1)}=\{(x_1,x_2,x_3)|x_3\geq h^{(S)}\}$, which is the basis (figure 1).

We will assume that the static residual stresses $\sigma_{ij}^{*(s)}(x)$ $(i,j=1,2,3;\ s=1,\dots,S+1;\ x=(x_1,x_2,x_3))$, may be of an arbitrary nature, including those caused by irreversible deformations (plasticity, creep, etc.), but at the same time they satisfy in the area of each layer $\Omega^{(s)}$ и в области основы $\Omega^{(S+1)}$ equilibrium equations

$$\sigma_{ij,j}^{*(s)}=0 \qquad (i,j=1,2,3;\ s=1,\dots,S+1) \tag{1}$$

homogeneous boundary conditions in the efforts homogeneous boundary conditions in the efforts

$$\{\sigma_{ij}^{*(1)}n_j^{(0)}\}(x_1,x_2,h^{(0)})=0 \qquad (i,j=1,2,3) \tag{2}$$

and the continuity conditions for the components of the force vector when passing through the contact boundaries of the layers

$$\{(\sigma_{ij}^{*(s+1)}-\sigma_{ij}^{*(s)})n_j^{(s)}\}(x_1,x_2,h^{(s)})=0 \qquad (i,j=1,2,3;\ s=1,\dots,S) \tag{3}$$

Here $n_j^{(0)}(x_1,x_2)$ are the components of the normal to the body surface, and $n_j^{(s)}(x_1,x_2)$ are the components of the normals to the contact surfaces of the layers

In formulas (1)-(3) and further the index after the decimal point means the partial derivative with respect to the corresponding coordinate, the summation is performed over the repeated index, but the summation is not performed over the superscript *(s)*, which means that the given value corresponds to the *s*-th layer.

Relations (1)-(3), not closed by the constitutive equations that establish correspondences between initial stresses and strains (strain rates), are not enough to determine six (taking into account the symmetry $\sigma_{ij}^{*(s)}=\sigma_{ji}^{*(s)}$) independent components initial stress tensor. Additional relationships will be obtained from the solution of the diagnostic problem, under the assumption that the elastic moduli (connecting the elastic components of the total stresses and strains) depend on the initial stresses.

4. Linearization of the problem of diagnostics

Consider an "ideal" layered body, in the manufacture of which it was possible to avoid residual stresses. Each layer of this body has plane parallel boundaries ($x_3 = h^{0(s)} - const,\ s=0,\dots,S$), is homogeneous isotropic and is characterized by density $\rho^{0(s)}$ and Lame moduli $\lambda^{0(s)}$, $\mu^{0(s)} - const\ (s=1,\dots,S+1)$. In this case (since without loss of generality we can assume $h^{0(s)} = 0$), the semi-bounded body Ω^0 will be the half-space $R_+^3=(x_1,x_2,x_3)|x_3\geq 0\}$.

The elastic process occurring in an "ideal" layered body is described by the equations

$$\rho^{0(s)}\ddot{u}_i^{0(s)} - \mu^{0(s)}u_{i,jj}^{0(s)} - (\mu^{0(s)} + \lambda^{0(s)})u_{j,ji}^{0(s)} = f_i^{(s)}(x,t) \quad (4)$$

initial, boundary and contact conditions

$$u_i^{0(s)}(x,0) = \varphi_i^{(s)}(x), \quad \dot{u}_i^{0(s)}(x,0) = \psi_i^{(s)}(x) \quad (5)$$

$$\{\mu^{0(1)}(u_{i,3}^{0(1)} + u_{3,i}^{0(1)}) + \delta_{i3}\lambda^{0(1)}u_{j,j}^{0(1)}\}(x_1,x_2,0,t) = p_i^{(1)}(x_1,x_2,t) \quad (6)$$

$$\{\mu^{0(s+1)}(u_{i,3}^{0(s+1)} + u_{3,i}^{0(s+1)}) + \delta_{i3}\lambda^{0(s+1)}u_{j,j}^{0(s+1)} - \mu^{0(s)}(u_{i,3}^{0(s)} + u_{3,i}^{0(s)}) - \delta_{i3}\lambda^{0(s)}u_{j,j}^{0(s)}\}(x_1,x_2,h^{0(s)},t) = 0 \quad (7)$$

$$\{u_i^{0(s+1)} - u_i^{0(s)}\}(x_1,x_2,h^{0(s)},t) = 0, \quad (i,j=1,2,3;\ s=1,\dots,S)$$

The system of relations (4)-(7) makes it possible to uniquely determine the functions $u_i^{0(s)}(x,t)$, i.e., completely describes the elastic process in an ideal layered medium. Calculation of strain and stress fields can be made using the formulas:

$$e_{ij}^{0(s)} = (u_{i,j}^{0(s)} + u_{i,j}^{0(s)})/2, \qquad \sigma_{ij}^{0(s)} = \mu^{0(s)}e_{ij}^{0(s)} + \delta_{i3}\lambda^{0(s)}e_{ij}^{0(s)}$$

When an elastic process is initiated in the medium under study, the stress field is the sum of the field of static inelastic residual stresses $\sigma_{ij}^{*(s)}(x)$ and the field of dynamic elastic stresses $\sigma_{ij}^{(s)}(x,t)$, which correspond to dynamic elastic deformations $e_{ij}^{(s)}(x,t)$ and displacements $u_i^{(s)}(x,t),\ (s=1,\dots,S+1)$.

The investigated body is considered to be made of the same materials as the "ideal" body, but with residual stresses. It will be characterized by anisotropy and spatial inhomogeneity of the elastic properties of the layers, since their elastic moduli $C_{ijkl}^{(s)}$ ($i,j,k,l=1,2,3$) are considered to be functions of the tensor components $\sigma_{mn}^{*(s)}$ ($m,n=1,2,3;\ s=1,\dots,S+1$), and these components, generally speaking, depend on the spatial coordinates $x=(x_1,x_2,x_3)$.

We will assume that the effect of the initial stresses on the elastic moduli is rather weak, i.e. quantities $||C_{ijkl}^{(s)}(\sigma_{mn}^{*(s)}) - \lambda^{0(s)}\delta_{ij}\delta_{kl} - \mu^{0(s)}(\delta_{ik}\delta_{il} + \delta_{il}\delta_{ik})||_{C^1} / \lambda^{0(s)}$ have an order of magnitude $O(\varepsilon)$, $0<\varepsilon<<1$. Neglecting values of the order of ε^2, we can consider the weak dependence of the elastic moduli on the initial stresses to be linear, i.e.

$$C_{ijkl}^{(s)}(\sigma_{mn}^{*(s)}) = \lambda^{0(s)}\delta_{ij}\delta_{kl} + \mu^{0(s)}(\delta_{ik}\delta_{il} + \delta_{il}\delta_{ik})||_{C^1} + \varepsilon C_{ijkl}^{mn(s)}\sigma_{mn}^{*(s)} \quad (i,j,k,l,m,n=1,2,3;\ s=1,\dots,S+1),$$

where the constant coefficients $C_{ijkl}^{mn(s)}$ are assumed to be obtained on the basis of test tests of material samples and are further considered to be known. We will also assume that the outer surface of the body and the contact surfaces of the layers are slightly curved, i.e.

$$h^{(s)}(x_1,x_2)=h^{0(s)}+\varepsilon h^{\varepsilon(s)}(x_1,x_2),\quad 0<\varepsilon<<1,\qquad h^{0(s)}-const\ \ (s=0,\dots,S)$$

We will assume that the elastic process in the studied layered medium $u_i^{(s)}(x,t)$, triggered by the same set of initial and boundary conditions $\{f_i^{(s)}, \varphi_i^{(s)}, \psi_i^{(s)}, p_i^{(1)}\}$ and described by the equations

$$\rho^{0(s)}\ddot{u}_i^{(s)}-(C_{ijkl}^{(s)}u_{k,l}^{(s)})_{,j}=f_i^{(s)}\qquad (i,j,k,l=1,2,3),$$

differs from the control process by an amount of the order of ε, i.e. $||u_i^{(s)}-u_i^{0(s)}||_{C^2}\sim O(\varepsilon)$.

Neglecting quantities of the order ε^2, we will get for $u_i^{\varepsilon(s)}=(u_i^{(s)}-u_i^{0(s)})/\varepsilon$

$$\rho^{0(s)}\ddot{u}_i^{\varepsilon(s)}-\mu^{0(s)}u_{i,jj}^{\varepsilon(s)}-(\mu^{0(s)}+\lambda^{0(s)})u_{j,ji}^{\varepsilon(s)}=(C_{ijkl}^{mn(s)}\sigma_{mn}^{*(s)}u_{k,l}^{0(s)})_{,j} \tag{8}$$

$$u_i^{\varepsilon(s)}(x,0)=0,\qquad \dot{u}_i^{\varepsilon(s)}(x,0)=0 \tag{9}$$

$$\{\mu^{0(1)}(u_{i,3}^{\varepsilon(1)}+u_{3,i}^{\varepsilon(1)})+\delta_{i3}\lambda^{0(1)}u_{j,j}^{\varepsilon(1)}+C_{i3kl}^{mn(1)}\sigma_{mn}^{*(1)}u_{k,l}^{0(1)}\}(x_1,x_2,0,t)=$$
$$=\{n_j^{\varepsilon(0)}\sigma_{ij}^{0(1)}-h^{\varepsilon(0)}(\mu^{0(1)}(u_{i,33}^{0(1)}+u_{3,i3}^{0(1)})+\delta_{i3}\lambda^{0(1)}u_{j,j3}^{0(1)}\}(x_1,x_2,0,t) \tag{10}$$

$$\{\mu^{0(s+1)}(u_{i,3}^{\varepsilon(s+1)}+u_{3,i}^{\varepsilon(s+1)})+\delta_{i3}\lambda^{0(s+1)}u_{j,j}^{\varepsilon(s+1)}+C_{i3kl}^{mn(s+1)}\sigma_{mn}^{*(s+1)}u_{k,l}^{0(s+1)}-\mu^{0(s)}(u_{i,3}^{\varepsilon(s)}+u_{3,i}^{\varepsilon(s)})-\delta_{i3}\lambda^{0(s)}u_{j,j}^{\varepsilon(s)}-$$
$$-C_{i3kl}^{mn(s)}\sigma_{mn}^{*(s)}u_{k,l}^{0(s)}\}(x_1,x_2,h^{0(s)},t)=\{n_j^{\varepsilon(s)}(\sigma_{ij}^{0(s+1)}-\sigma_{ij}^{0(s+1)})+h^{\varepsilon(0)}(\mu^{0(s+1)}(u_{i,33}^{0(s+1)}+$$
$$+u_{3,i3}^{0(s+1)})+\delta_{i3}\lambda^{0(s+1)}u_{j,j3}^{0(s+1)}-\mu^{0(s)}(u_{i,33}^{0(s)}+u_{3,i3}^{0(s)})-\delta_{i3}\lambda^{0(s)}u_{j,j3}^{0(s)})\}(x_1,x_2,h^{0(s)},t)$$

$$\{u_i^{\varepsilon(s+1)}-u_i^{\varepsilon(s+1)}\}(x_1,x_2,h^{0(s)},t)=\{h^{\varepsilon(s)}(u_{i,3}^{0(s+1)}-u_{i,3}^{0(s)})\}(x_1,x_2,h^{0(s)},t) \tag{11}$$

$$u_i^{\varepsilon(1)}(x_1,x_2,h^{0(s)},t)=\chi_i^{\varepsilon}(x_1,x_2,t)-\{h^{\varepsilon(s)}u_{i,3}^{0(1)}\}(x_1,x_2,h^{0(s)},t) \tag{12}$$

$$(i,j,k,l,m,n=1,2,3)$$

The system of relations (8)-(12) contains, along with unknown characteristics of the elastic process $u_i^{\varepsilon(s)}(x,t)$, also unknown components of the initial stress tensor $\sigma_{mn}^{*(s)}(x)$. However, this system contains additional (in comparison with the similar system of relations (4)-(7)) boundary conditions (12). Here $\chi_i^{\varepsilon}(x_1,x_2,t)$ – deviations from the control values of displacements on the surface of the layered half-space, which are considered to be measured in the process of diagnostic tests. When constructing relations (8)-(12), we used the expansion in a Taylor series at the "point" $x_3=h^{0(s)}$ functions included in the boundary and contact conditions on surfaces $x_3=h^{(s)}(x_1,x_2)$, $s=0,\dots,S$. Taking into account the fact that $h^{\varepsilon(s)}=(h^{(s)}-h^{0(s)})$, $n_j^{\varepsilon(s)}=(n_j^{(s)}-n_j^{0(s)})\sim O(\varepsilon)$ when deriving relations (10)-(12), terms of the order of $o(\varepsilon)$ are discarded.

5. Solution of the linearized problem of diagnostics by the method of stationary basic processes

In accordance with the method of stationary basic processes [11], we confine ourselves to considering the conditions for initiating elastic processes $\{f_i^{(s)}, \varphi_i^{(s)}, \psi_i^{(s)}, p_i^{(1)}\}$, which excite stationary oscillatory processes of the form $u_i^{0(s)}(x,t)=sin(\alpha t)g_i^{(s)}(x)$, in the «ideal» medium which is technically possible. In the case of using oscillatory basic processes, many terms in relations (8)-(12) have the form $sin(\alpha t)F(x)$ and the application of the operator $\partial_t^2+\alpha^2 I$ (where I is the unit operator) to these relations makes it possible to get rid of these term, having obtained for $v_i^{(s)}=\ddot{u}_i^{\varepsilon(s)}+\alpha^2u_i^{\varepsilon(s)}$ the system of relations:

$$\rho^{0(s)}\ddot{v}_i^{(s)}-\mu^{0(s)}v_{i,jj}^{(s)}-(\mu^{0(s)}+\lambda^{0(s)})v_{j,ji}^{(s)}=0 \tag{13}$$

$$v_i^{(s)}(x,0)=0 \tag{14}$$

$$\{\ \mu^{0(1)}(v_{i,3}^{(1)}+v_{3,i}^{(1)})+\delta_{i3}\lambda^{0(1)}v_{j,j}^{(1)}\ \}(x_1,x_2,0,t)=0 \tag{15}$$

$$\{\ \mu^{0(s+1)}(v_{i,3}^{(s+1)}+v_{3,i}^{(s+1)})+\delta_{i3}\lambda^{0(s+1)}v_{j,j}^{(s+1)}-\mu^{0(s)}(v_{i,3}^{(s)}+v_{3,i}^{(s)})+\delta_{i3}\lambda^{0(s)}v_{j,j}^{(s)}\ \}(x_1,x_2,h^{0(s)},t)=0 \tag{16}$$

$$\{v_i^{(s+1)}-v_i^{(s)}\ \}(x_1,x_2,h^{0(s)},t)=0$$

$$\{v_i^{(1)}(x_1,x_2,0,t)=\{\ \ddot{\chi}_i^{\varepsilon}+\alpha^2\chi_i^{\varepsilon}\ \}(x_1,x_2,t) \tag{17}$$

$$(i,j,k,l,m,n=1,2,3).$$

The problem of the form (13)-(17) was studied in [12]. Assuming the functions $v_i^{(s)}(x,t)$ to be known, it is easy to restore $u_i^{\varepsilon(s)}(x,t)$, by solving ordinary differential equations

$\frac{d^2}{dt^2}u_i^{\varepsilon(s)}+\alpha^2u_i^{\varepsilon(s)}=v_i^{(s)}$ with initial conditions $u_i^{\varepsilon(s)}=0$, $\frac{d}{dt}u_i^{\varepsilon(s)}=0$ when $t=0$.

Substituting the found values into relations (8),(10),(11), we obtain the following equations for $\sigma_{mn}^{*(s)}(x)$:

$$(a_{ij}^{mn(s)}\sigma_{mn}^{*(s)})_{,j}=\tilde{f}_i^{(s)} \tag{18}$$

border conditions and contact conditions:

$$\{a_{ij}^{mn(1)}\sigma_{mn}^{*(1)}\}(x_1,x_2,0)=\tilde{p}_i(x_1,x_2) \tag{19}$$

$$\{a_{i3}^{mn(s+1)}\sigma_{mn}^{*(s+1)}-a_{i3}^{mn(s)}\sigma_{mn}^{*(s)}\}(x_1,x_2,h^{0(s)})=\tilde{q}_i^{(s)}(x_1,x_2) \tag{20}$$

Here introduced (for shortening the formulas) values:

$$\{a_{ij}^{mn(s)}(x)=C_{ijkl}^{*mn(s)}g_{k,l}^{(s)}(x),$$

$$\tilde{f}_i^{(s)}(x)=-\{\ \rho^{0(s)}u_i^{\varepsilon(s)}-\mu^{0(s)}u_{i,jj}^{\varepsilon(s)}-(\mu^{0(s)}+\lambda^{0(s)})u_{j,ji}^{\varepsilon(s)}/\sin(\alpha t)\}(x,0)$$

$$\tilde{p}_1(x_1,x_1)=-\{\ \mu^{0(1)}(u_{i,3}^{\varepsilon(1)}+u_{3,i}^{\varepsilon(1)})+\delta_{i3}\lambda^{0(1)}u_{j,j}^{\varepsilon(1)}+n_j^{\varepsilon(0)}\sigma_{ij}^{0(1)}-h^{\varepsilon(0)}(\mu^{0(1)}(u_{i,33}^{0(1)}+u_{3,i}^{0(1)})+\delta_{i3}\lambda^{0(1)}u_{j,j3}^{(1)})/$$
$$/\sin(\alpha t)\}(x_1,x_2,0,0)$$

$$\tilde{q}_1(x_1,x_1)=-\{(\ \mu^{0(s+1)}(u_{i,3}^{\varepsilon(s+1)}+u_{3,i}^{\varepsilon(s+1)})+\delta_{i3}\lambda^{0(s+1)}u_{j,j}^{\varepsilon(s+1)}-\mu^{0(s)}(u_{i,3}^{\varepsilon(s)}+u_{3,i}^{\varepsilon(s)})-\delta_{i3}\lambda^{0(s)}u_{j,j}^{\varepsilon(s)}+$$
$$+n_j^{\varepsilon(s)}(\sigma_{ij}^{0(s+1)}-\sigma_{ij}^{0(s)})-h^{\varepsilon(s)}(\mu^{0(s+1)}(u_{i,33}^{0(s+1)}+u_{3,i}^{0(s+1)})+\delta_{i3}\lambda^{0(s+1)}u_{j,j3}^{0(s+1)}-$$
$$-\mu^{0(s)}(u_{i,33}^{0(s)}+u_{3,i}^{0(s)})-\delta_{i3}\lambda^{0(s)}u_{j,j3}^{0(s)}))/\sin(\alpha t)\}(x_1,x_2,h^{0(s)},t)$$

are expressed in terms of the functions found earlier, and are further considered to be known.

Thus, in order to find six unknowns of the function $\sigma_{mn}^{*(s)}$, in addition to three equilibrium equations (1) with boundary and contact conditions (2), (3), three more equations (18) with boundary and contact conditions (19),(20). Relations (18) are (like equations (1)) linear equations with partial derivatives, however (unlike (1)) they have coefficients that depend on spatial variables. Thus, finding six unknown functions $\sigma_{ij}^{*(s)}(x)$ from six equations (1),(18) is generally associated with mathematical difficulties.

Let us consider an approximate method for determining the desired components of the residual stress tensor. Let, within the framework of the diagnostic task, not one, but seven test trials were carried out under various conditions for initiating elastic processes $f_i^{(s)n}$, $\varphi_i^{(s)n}$, $\psi_i^{(s)n}$, $p_i^{(1)n}\}$, $n=1,\ldots,7$. Thus, with for $\{u_i^{\varepsilon(s)n}$, $\sigma_{ij}^{*(s)}\}$ we have seven systems of relations of the form (8)-(12), and then (after finding $u_i^{\varepsilon(s)n}$,

$n=1,\ldots,7$) seven systems of relations of the form (18) with for $\sigma_{ij}^{*(s)}$. Since the system (18) contains three equations, then with respect to the six independent components of the tensor $\sigma_{ij}^{*(s)}$ (which we denote y_m, $m=1,2,\ldots,6$) we have (taking into account (1)) an overdetermined system of 24 partial differential equations. Conditionally assuming the components y_m and their partial derivatives $y_{m,,r}$ $(r=1,2,3)$ as independent functions $U_k(x)$, $k=1+4(m-1)+r$, we can solve the resulting system of 24 algebraic equations in 24 unknowns.

The subsequent solution of six overdetermined systems of equations of the form

$$y_p^{(s)} = U_p^{(s)}, \quad y_{p,r}^{(s)} = U_{p+r}^{(s)} \qquad (p=1+4(m-1),\ m=1,2,\ldots,6,\ r=1,2,3) \tag{21}$$

can, of course, only be approximate. If we assume that the desired functions $\sigma_{ij}^{*(s)}$ and the right parts of equations (21) $U_k^{(s)}$ allow an approximate representation in the form of a polyharmonic function (finite harmonic series), then by equating the coefficients for identical harmonics in the left and right parts of equations (21), we obtain four values for each coefficient of the harmonic expansion of the desired function $\sigma_{ij}^{*(s)}$.. Ideally, since all seven diagnostic tests were carried out for the same medium and were assumed to be non-destructive (not changing its properties), these four values would have to match. However, the influence of error (measurement, model, calculation errors) will inevitably lead to some difference in these values. It is proposed to use some average (for example, by the minimum square deviation) of the values of $\sigma_{ij}^{*(s)}$ in the approximate harmonic expansion. Thus, the obtained approximate solution can be considered as the result of averaging an excess number of experimental data. This approach has both positive (for example, "smoothing random outliers" of errors of some type in a separate experiment) and negative (related mainly to the need to increase the number of tests) sides.

6. An example of solving a diagnostic problem

Consider (as an example) a special case where the residual stresses can be easily found in a single test run. Let it be known in advance that the distribution of residual stresses in the body under study depends only on one spatial variable x_3 (distance from the surface), and the outer and contact surfaces are parallel planes (although, possibly, shifted compared to the position of the layers in the “control” layered body This case, despite its obvious simplification, is quite widespread and corresponds to a uniform distribution on the surface of the body of influences that cause residual stresses (for example, uniform surface cooling).In this case, equations (1) have the form $\sigma_{i3}^{*(s)},_3 = 0$, which means (under boundary conditions (2) and contact conditions (3)), $\sigma_{i3}^{*(s)}=0$ $(i=1,2,3;\ s=1,\ldots,S+1)$.. Problem (13)-(17) becomes one-dimensional and classically correct. Relations (18)-(20) when used for diagnostics of one-dimensional basic processes $u_i^0=u_i^0(x_3,t)$ $(i=1,2,3)$ are also greatly simplified and take the form

$$(a_{i3}^{11(s)}\sigma_{11}^{*(s)} + a_{i3}^{12(s)}\sigma_{12}^{*(s)} + a_{i3}^{22(s)}\sigma_{22}^{*(s)})_{,3} = \tilde{f}_i^{(s)}, \qquad \{a_{i3}^{11(1)}\sigma_{11}^{*(1)} + a_{i3}^{12(1)}\sigma_{12}^{*(1)} + a_{i3}^{22(1)}\sigma_{22}^{*(1)}\}(0) = \tilde{p}_i$$

$$\{a_{i3}^{11(s+1)}\sigma_{11}^{*(s+1)} + a_{i3}^{12(s+1)}\sigma_{12}^{*(s+1)} + a_{i3}^{22(s+1)}\sigma_{22}^{*(s+1)} - a_{i3}^{11(s)}\sigma_{11}^{*(s)} - a_{i3}^{12(s)}\sigma_{12}^{*(s)} - a_{i3}^{22(s)}\sigma_{22}^{*(s)}\}(h^{0(s)}) = \tilde{q}_i^{(s)}$$

Finding $\sigma_{11}^{*(s)}$, $\sigma_{12}^{*(s)}$, $\sigma_{22}^{*(s)}$ is first reduced to determining their linear combinations $(a_{i3}^{11(s)}\sigma_{11}^{*(s)}+a_{i3}^{12(s)}\sigma_{12}^{*(s)}+a_{i3}^{22(s)}\sigma_{22}^{*(s)})$ by integrating the functions $\tilde{f}_i^{(s)}(x_3)$) (taking into account the existing boundary and contact) conditions and to the subsequent solution of the system of algebraic equations. Since the coefficients of the matrix of linear equations $a_{ij}^{mn(s)}$ are expressed in terms of the constants $C_{ijkl}^{mn(s)}$ considered to be known and the parameters α, $g_k^{(s)}$, depending on the conditions of diagnostic tests, the condition of the matrix non-degeneracy ($det \neq 0$) makes it possible to obtain the requirements for the characteristics of force loadings $\{f_i^{(s)},\varphi_i^{(s)},\psi_i^{(s)},p_i^{(1)}\}$ when conducting a test for diagnosing residual stresses.

7. Conclusion

The proposed mathematical formulation of the problem of diagnosing an elastic layered medium (within the framework of mathematical modeling of non-destructive testing of multilayer materials) makes it possible to determine the spatial distribution of residual stresses from the displacement values measured on the surface, which arose as a result of a specially initiated wave process. The nonlinearity of the problem is due to the fact that not only the characteristics of the process (depending on time and spatial coordinates) are unknown, but also the characteristics of the medium (depending on spatial coordinates). The linearization of the problem, performed under the assumption of weak inhomogeneity and anisotropy of the layers, greatly simplifies it, while leaving it classically correct (according to J. Hadamard). The use of the method of stationary basic processes makes it possible to solve (albeit in a rather cumbersome way) a three-dimensional linearized problem. In the one-dimensional case, the linearized problem becomes classically well-posed and is much easier to solve. Further development of research in the field of mathematical modeling of non-destructive testing of materials may be associated with solving problems of diagnosing media with complex physical properties (viscosity, thermoelasticity, electroelasticity, etc.).

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