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HEAT AND MASS TRANSFER AND PHYSICAL GASDYNAMICS

On the Influence of Heat Transfer on the Photophoresis of a Heated Large Aerosol Particle

N. V. Malai[a], *, E. R. Shchukin[b], **, and Yu. I. Shostak[a]

[a] *Belgorod State National Research University, Belgorod, Russia*

[b] *Joint Institute for High Temperatures, Russian Academy of Sciences, Moscow, Russia*

**e-mail: malay@bsu.edu.ru*

***e-mail: evgrom@yandex.ru*

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Abstract—For the first time, in the quasi-stationary Stokes approximation at low Reynolds and Peclet numbers, a theory has been constructed that takes into account the effect of convective heat transfer on the photophoresis of a heated large spherical aerosol particle using the method of matched asymptotic expansions. When solving gasdynamics equations, the power-law form of the dependences of the molecular transfer coefficients (viscosity, thermal conductivity) and density of the gaseous medium on temperature is taken into account.

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INTRODUCTION

In gaseous media thermodynamically nonequilibrium in temperature, an ordered motion of aerosol particles suspended in them arises, due to molecular forces, in particular, photophoretic motion [1, 2]. The photophoresis mechanism can be briefly described as follows. When electromagnetic radiation interacts with a particle inside it, thermal energy is released with a certain volume density q_i, which heats its surface inhomogeneously. The gas molecules surrounding the particle, after colliding with its surface, are reflected from the heated side with a greater momentum than from the opposite side. As a result, the particle acquires uncompensated momentum directed from the hot side of the surface to the colder side. Depending on the size and shape of the particle surface, the optical properties of its material, and the radiation wavelength, both the illuminated and shadow sides of the surface can be heated. Therefore, both positive (movement of the particle in the direction of radiation propagation) and negative (movement in the opposite direction) photophoresis is observed. The photophoresis phenomenon almost always accompanies aerodisperse systems that are thermodynamically nonequilibrium in temperature.

The problem of the behavior of a light-absorbing particle in a viscous gaseous medium is therefore divided into two interrelated parts: determination of the distribution of electromagnetic energy in the volume of the particle, based on the theory of light scattering, e.g., the Mie problem [3], and calculation of the photophoretic force and velocity of the particle in an inhomogeneous surrounding gas heated by it.

The photophoretic force can have a significant effect on the deposition of particles in the channels of heat and mass exchangers and on the movement of particles in the zones of enlightenment of disperse systems and in the vicinity of leaching particles. It can be used for fine purification of gases with small volumes, sampling of aerosol samples, application of special coatings of a given thickness from particles, etc.

When describing the behavior of particles suspended in thermodynamically nonequilibrium, in terms of temperature, viscous gaseous media, the dimensionless parameter θ is introduced, characterizing the difference between the average particle surface temperature T_S and temperature of the gaseous medium far from it T_∞, in relation to the latter, i.e., $\theta = (T_S - T_\infty)/T_\infty$. The relative temperature difference is considered small if the inequality $\theta \ll 1$, and significant if $\theta \sim O(1)$. When the first condition is met, the coefficients of molecular transfer (viscosity, thermal conductivity) and density of a viscous gaseous medium can be considered constant, and the viscous medium itself can be considered isothermal. This condition greatly simplifies the procedure of finding expressions for the force and speed of photophoresis. The main results for such a description were obtained in [2, 4, 5]. If $\theta \sim O(1)$, a particle is called heated (heating of the surface of the particle can be due, e.g., to a volumetric chemical reaction, the radioactive decay of the particle's substance, absorption of electromagnetic radiation, etc.) and a viscous medium is considered nonisothermal. When finding the force and speed of photophoresis in this case, it is necessary to take into account the temperature dependence of the molecular transfer coefficients and density of a viscous gaseous medium, while the system of gas-dynamic equations that describes such a medium becomes essentially nonlinear. There are few papers in the scientific literature that study this case; in particular, they consider, e.g., the gravitational motion of heated particles [6, 7], photophoresis of heated large particles [8], thermophoresis of large heated particles [9], and diffusion evaporation (sublimation) [10]. These studies showed that the heating of particle surfaces significantly affects their behavior in a gaseous medium.

It should be noted that the effect of heat transfer on the behavior of a particle in a viscous nonisothermal gaseous medium was not studied in [6–10]. The stationary equation of convective heat transfer has the form [11, 12]

$$\rho_e c_{pe}(\mathbf{U}_e\nabla)T_e = \operatorname{div}(\lambda_e \nabla T_e).$$

Here, the left-hand side of the equation is responsible for convective heat transfer, while the right-hand side is responsible for molecular heat transfer. For small Reynolds numbers and relative temperature fluctuations in the gas, convective heat transfer can be neglected. This article studies the case for small Reynolds numbers, but significant temperature fluctuations in the gas; here, convective heat transfer is comparable in order of magnitude to molecular heat transfer.

FORMULATION OF THE PROBLEM

We consider is a solid spherical particle with radius R suspended in a gas with density ρ_e, thermal conductivity λ_e, and dynamic viscosity μ_e; the surface is heated nonuniformly by electromagnetic radiation. Nonuniform heating leads to a nonuniform temperature distribution along the surface of the particle.

The gas interacting with the inhomogeneously heated surface begins to move along the surface in the direction of increasing temperature. This phenomenon is called thermal gas slip, and it causes the photophoretic force. Under the action of the photophoretic force, the particle begins to move. In addition, the viscous drag force of the medium acts on the particle. When both these forces are balanced, the particle begins to move uniformly at a constant speed, which is called the photophoretic speed $\mathbf{U}_{\text{ph}}$.

When describing the properties of a gaseous medium and a particle, the power-law form of the dependences of the molecular transfer coefficients and density on temperature is taken into account [13]: $\mu_e = \mu_\infty t_e^\beta$, $\lambda_e = \lambda_\infty t_e^\alpha$, $\rho_e = \rho_\infty / t_e$, $\lambda_i = \lambda_{i0} t_i^\gamma$, where $\mu_\infty = \mu_e(T_\infty)$, $\lambda_\infty = \lambda_e(T_\infty)$, $\lambda_{i0} = \lambda_i(T_\infty)$, $\rho_\infty = \rho_e(T_\infty)$, $t_k = T_k/T_\infty$ $(k = e, i)$, $0.5 \le \alpha, \beta \le 1$, $-1 \le \gamma \le 1$. The exponents of the molecular transport coefficients, e.g., for air are $\alpha = 0.81$, $\beta = 0.72$ at $273 \le T_e \ge 900$ K. The relative error does not exceed 5% [13]. Subscripts e and i refer to the gas and particle, respectively; S are the values of the physical quantities taken at the average particle surface temperature, and ∞ characterize the gaseous medium far from the particle.

The theoretical description of photophoresis assumes that, due to the short thermal relaxation time, the heat transfer process in the particle–gas system proceeds as quasi-stationary. The motion of a large [1] particle is considered at low Peclet and Reynolds numbers, neglecting free convection (the Grashof number is much less than unity). The problem is solved by the hydrodynamic method; i.e., the gas-dynamic equations are solved with the corresponding boundary conditions.

Photophoresis is conveniently described in a spherical coordinate system (r, θ, φ), which is related to the center of mass of the aerosol particle; the Oz axis is directed towards the propagation of a homogeneous radiation flux with intensity I_0. The problem is reduced to analyzing an infinite plane-parallel gas flow around a particle, the velocity $\mathbf{U}_\infty$ of which is to be determined ($\mathbf{U}_\infty \parallel Oz$). With this choice of the origin, the particle can be considered stationary and the gaseous medium can be considered as moving in the opposite direction of the actual particle motion ($\mathbf{U}_\infty = -\mathbf{U}_{\text{ph}}$). The velocity, pressure, and temperature distributions are axially symmetric with respect to the Oz axis; i.e., they are functions of two variables y, θ $(y = r/R)$.

Under the formulated assumptions, the following system of gas dynamic equations is solved [11, 12]:

$$\frac{\partial}{\partial x_k} P_e = \frac{\partial}{\partial x_j}\left\{\mu_e\left[\frac{\partial U_k^{(e)}}{\partial x_j} + \frac{\partial U_j^{(e)}}{\partial x_k} - \frac{2}{3}\delta_k^j \frac{\partial U_n^{(e)}}{\partial x_n}\right]\right\},$$
$$\frac{\partial}{\partial x_k}\left(\rho_e U_k^e\right) = 0,\quad \rho_e c_{pe}(\mathbf{U}_e\nabla)T_e = \operatorname{div}(\lambda_e\nabla T_e), \qquad (1)$$
$$\operatorname{div}(\lambda_i \nabla T_i) = q_i.$$

Here x_k are the Cartesian coordinates, $U_k^{(e)}$ are the mass velocity components $\mathbf{U}_e$, P_e is pressure, and c_{pe} is the heat capacity at constant pressure.

The determining parameters in the problem are the material constants μ_∞, ρ_∞, λ_∞ and the particles that remain in motion R, T_∞ and U_∞ ($U_\infty = |\mathbf{U}_\infty|$). From these parameters, the dimensionless Reynolds number ($\text{Re}_\infty = RU_\infty\rho_\infty/\mu_\infty$) can be determined, which plays the role of a small parameter in the problem being solved.

For $\varepsilon \ll 1$ ($\varepsilon = \text{Re}_\infty$), we seek the solution to the hydrodynamics equations in the form

$$\mathbf{V}_e = \mathbf{V}_e^{(0)} + \varepsilon\mathbf{V}_e^{(1)} + \ldots, \quad P_e = P_e^{(0)} + \varepsilon P_e^{(1)} + \ldots,$$

where $\mathbf{V}_e = \mathbf{U}_e/U_\infty$.

GENERAL SOLUTION OF A VELOCITY-LINEARIZED SYSTEM OF NAVIER–STOKES EQUATIONS

Based on the given formulation, the expressions for the dimensionless mass velocity components $V_r^{(e)}$ and $V_\theta^{(e)}$ should be sought as expansions in Legendre and Gegenbauer polynomials [12]. To determine the total force acting on a particle, we confine ourselves to the first terms of these expansions [12]. Given this expression, the mass velocity components can be sought in the form

$$V_r^{(e)}(y,\theta) = \cos\theta G(y), \quad V_\theta^{(e)}(y,\theta) = -\sin\theta g(y)$$

with boundary conditions

$$\lim_{y\to\infty}\left(U_r^{(e)}(y,\theta) - U_\infty\cos\theta\right) = 0,$$
$$\lim_{y\to\infty}\left(U_\theta^{(e)}(y,\theta) + U_\infty\sin\theta\right) = 0.$$

Here $G(y)$ and $g(y)$ are arbitrary functions depending on the coordinate y.

The study of the velocity-linearized system of Navier–Stokes equations in spherical coordinates (1) showed that if the thermal conductivity of a particle is much larger than the thermal conductivity of a gas (weak angular asymmetry of the temperature distribution, which occurs for most gases), then this equation can be reduced to an inhomogeneous third-order differential equation with an isolated singularity. We seek the solution of the resulting equation in the form of generalized power series (a detailed analysis was carried out in [14]).

Thus, the general expressions for the mass velocity components that satisfy the condition of boundedness of the solution at $y \to \infty$, and the pressures have the form

$$U_r^{(e)} = U_\infty\cos\theta G(y), \quad G(y) = A_1G_1 + A_2G_2 + G_3,$$
$$U_\theta^{(e)} = -U_\infty\sin\theta g(y), \quad g(y) = A_1G_4 + A_2G_5 + G_6,$$
$$P_e = P_\infty + \frac{\mu_\infty U}{R}t_{e0}^{\beta}\left\{\frac{y^2}{2}\frac{d^3G}{dy^3} + y\left[3 + \frac{\beta-1}{2}yf\right]\frac{d^2G}{dy^2} + \left[2 - y^2f' - \frac{\beta}{2}y^2f^2 + (\beta-2)yf\right]\frac{dG}{dy} + \left[\frac{f}{3} - \frac{y^2f''}{2} - yf'\left(2 + \frac{y\beta f}{2}\right)\right]G\right\}.$$

Here

$$G_1(y) = \frac{1}{y^3}\sum_{n=0}^{\infty}C_n^{(1)}\ell^n,$$

$$G_2(y) = \frac{1}{y}\sum_{n=0}^{\infty}C_n^{(2)}\ell^n + \omega_2\ln(y)G_1(y),$$

$$G_3(y) = \sum_{n=0}^{\infty}C_n^{(3)}\ell^n + \omega_3\ln(y)G_1(y),$$

$$f(y) = \frac{1}{t_{e0}(y)}\frac{dt_{e0}(y)}{dy}.$$

The values of the coefficients $C_n^{(1)}\,(n \ge 1)$, $C_n^{(2)}\,(n \ge 3)$, and $C_n^{(3)}\,(n \ge 4)$ are determined by the following recurrence relations:

$$C_n^{(1)} = \frac{1}{n(n+3)(n+5)}\Big\{\Big[(n-1)\left(3n^2+13n+8\right) + \gamma_1(n+2)(n+3) + \gamma_2(n+2)\Big]C_{n-1}^{(1)} - \Big[(n-1)(n-2)(3n+5) + 2\gamma_1\left(n^2-4\right) + \gamma_2(n-2) + \gamma_3(n+3)\Big]C_{n-2}^{(1)} + (n-2)\left[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3\right]C_{n-3}^{(1)}\Big\},$$

$$C_n^{(2)} = \frac{1}{(n+1)(n+3)(n-2)} \times \Big\{\Big[(n-1)\left(3n^2+n-6\right) + \gamma_1 n(n+1) + n\gamma_2\Big] \times C_{n-1}^{(2)} - \left[\gamma_3(n+1) + +(n-1)(n-2)(3n-1) + 2\gamma_1 n(n-2) + \gamma_2(n-2)\right]C_{n-2}^{(2)} + (n-2) \times \left[(n-1)(n-3) + \gamma_3 + \gamma_1(n-3)C_{n-3}^{(2)}\right] + \frac{\omega_2}{\Gamma_0^2}\sum_{k=0}^{n-2}(n-k-1)S_k^{(1)} - 6(-1)^n\frac{\gamma_4!}{(\gamma_4-n)!\,n!}\Big\},$$

$$C_n^{(3)} = \frac{1}{n(n+2)(n-3)} \times \Big\{(n-1)\left[3n^2-5n-4+\gamma_1 n+\gamma_2\right]C_{n-1}^{(3)} - \left[(n-1)(n-2)(3n-4) + 2\gamma_1(n-1)(n-2) + \gamma_2(n-2) + n\gamma_3\right]C_{n-2}^{(3)} + (n-2)\left[(n-1)(n-3) + \gamma_1(n-3) + \gamma_3\right]C_{n-3}^{(3)} + \frac{\omega_3}{2\Gamma_0^3}\sum_{k=0}^{n-3}(n-k-2)(n-k-1)S_k^{(1)}\Big\},$$

where

$$S_k^{(1)} = \left(3k^2 + 16k + 15\right)C_k^{(1)} - ((k-1)(6k+13) + \gamma_1(2k+5) + \gamma_2)C_{kk-1}^{(1)} + (3(k-1)(k-2) + 2\gamma_1(k-2) + \gamma_3)C_{k-2}^{(1)}\ell(y) = \Gamma_0/(y+\Gamma_0)\Gamma_0 = \text{const},$$

$$G_k = \left(1 + \frac{\ell}{2(1+\alpha)}\right)G_{k-3} + \frac{1}{2}yG_{k-3}'\,(k = 4,5,6),$$

f', f'', G_1', G_2', G_3' are the first and second derivatives with respect to y from the corresponding functions.

When calculating coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ by the recursive formulas, it is necessary to take into account that

$$C_0^{(1)} = 1,\quad C_0^{(2)} = 1,\quad C_0^{(3)} = 1,\quad C_1^{(3)} = 0,\quad C_2^{(2)} = 1,$$

$$C_1^{(2)} = -\frac{1}{8}(2\gamma_1 + \gamma_2 + 6\gamma_4),$$

$$\frac{\omega_3}{2\Gamma_0^3} = -\frac{\gamma_3}{60}(10 + 3\gamma_1 + \gamma_2),$$

$$\gamma_4 = \beta/(1+\alpha),\ \gamma_1 = \frac{1-\beta}{1+\alpha},$$

$$\frac{\omega_2}{\Gamma_0^2} = \frac{1}{15}\left[3\gamma_3 - (8 + 6\gamma_1 + 2\gamma_2)C_1^{(2)} + 3\gamma_4(\gamma_4 - 1)\right],$$

$$C_2^{(3)} = \frac{1}{4}\gamma_3,\quad C_3^{(3)} = 1,$$

$$\gamma_2 = 2\frac{1+\beta}{1+\alpha},\quad \gamma_3 = \frac{2 + 2\alpha - \beta}{(1+\alpha)^2},$$

and the coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ for $n < 0$ are 0.

The integration constants A_1, A_2, Γ_0 are determined from the boundary conditions of the problem.

TEMPERATURE FIELDS OUTSIDE AND INSIDE A PARTICLE

To find the photophoretic force and speed, we need to know the temperature fields. For this, it is necessary to solve Eqs. (1) with the following boundary conditions: on the particle surface $(y = 1)$, the equality of temperatures and the continuity of radial heat fluxes are taken into account, including the heat associated with radiation

$$T_e = T_i,\quad -\lambda_e\frac{\partial T_e}{\partial y} = -\lambda_i\frac{\partial T_i}{\partial y} - \sigma_0\sigma_1 R\left(T_i^4 - T_\infty^4\right).$$

The temperature far from the particle and the finiteness of the temperature at its center are taken into account in the boundary conditions

$$T_e\big|_{y\to\infty} \to T_\infty,\quad T_i\big|_{y\to 0} \neq \infty.$$

Here σ_0 is the Stefan–Boltzmann constant and σ_1 is the integral degree of blackness.

It was shown in [15] for the thermal problem that, near the sphere, the inertial and convective terms assume the same order as the terms describing molecular transfer; therefore, the usual method of expansion in a small parameter yields a known error; i.e., it can be used to rigorously satisfy the boundary conditions at infinity and obtain an exact unified solution that is uniformly valid for the entire flow region. Therefore, the solution to the equation describing the temperature field outside the particle is found by the method of matched asymptotic expansions [16], in which the temperature field is represented as two asymptotic expansions. In this case, the internal and external asymptotic expansions of the dimensionless temperature are written as

$$t_e(y,\theta) = \sum_{n=0}^{\infty} f_n(\varepsilon)t_{en}(y,\theta),\quad f_0(\varepsilon) = 1,$$
$$\varepsilon = \text{Re}_\infty,\quad t_e^*(\xi,\theta) = \sum_{n=0}^{\infty} f_n^*(\varepsilon)t_{en}^*(\xi,\theta), \tag{2}$$

where $\xi = \varepsilon y$ is the "compressed" radial coordinate [15].

At the same time, it is necessary that

$$\frac{f_{n+1}}{f_n} \to 0,\quad \frac{f_{n+1}^*}{f_n^*} \to 0,\quad \varepsilon \to 0.$$

The missing boundary conditions for the internal and external expansions follow from the condition that the asymptotic continuations of both are identical to some intermediate region, i.e.,

$$t_e(y\to\infty,\theta) = t_e^*(\xi\to 0,\theta).$$

The asymptotic expansion for the temperature field inside the particle, as shown by the boundary conditions on its surface, should be sought in a form similar to (2):

$$t_i(y,\theta) = \sum_{n=0}^{\infty} f_n(\varepsilon)t_{in}(y,\theta).$$

As for functions $f_n(\varepsilon)$ and $f_n^*(\varepsilon)$, it is only assumed that their order of smallness in ε increases with increasing n.

In dimensionless variables, convective heat equation (1) has the form

$$\varepsilon\frac{\text{Pr}_\infty}{t_e}\left(V_r^e\frac{\partial t_e}{\partial y} + \frac{V_\theta^e}{y}\frac{\partial t_e}{\partial\theta}\right) = \text{div}\left(t_e^\alpha\nabla t_e\right),$$

and taking into account the compressed radial coordinate, we obtain the following equation for the temperature t_e^*:

$$\frac{\Pr_\infty}{t_e^*}\left(V_r^{e*}\frac{\partial t_e^*}{\partial\xi}+\frac{V_\theta^{e*}}{\xi}\frac{\partial t_e^*}{\partial\theta}\right)=div\left(t_e^{*\alpha}\nabla t_e^*\right),$$

$$t_e^*\to 1 \quad \text{at } \xi\to\infty,$$

$$\mathbf{V}_e^*(\xi,\theta)=\mathbf{n}_z+\varepsilon\mathbf{V}_e^{(1)*}(\xi,\theta)+\ldots,$$

$$P_e^*(\xi,\theta)\to P_\infty \quad \text{at } \xi\to\infty.$$

Here $V_r^{e*}=V_r^{e*}(\xi,\theta)$, $V_\theta^{e*}=V_\theta^{e*}(\xi,\theta)$ are the components of the vector $\mathbf{V}_e^*$; $t_e^*=t_e^*(\xi,\theta)$; $\mathbf{n}_z$ is the unit vector in the direction of the Oz axis.

When finding expressions for the photophoretic force and speed, we confine ourselves to the zero and first asymptotic expansions for the temperature fields:

$$t_e^*(\xi,\theta)=t_{eo}^*(\xi)+\varepsilon t_{e1}^*(\xi,\theta),$$
$$t_e(y,\theta)=t_{eo}(y)+\varepsilon t_{e1}(y,\theta),$$
$$t_i(y,\theta)=t_{io}(y)+\varepsilon t_{i1}(y,\theta),$$

where

$$t_{eo}^*(\xi)=1,\quad t_{e0}(y)=\left(1+\frac{\Gamma_0}{y}\right)^{\frac{1}{1+\alpha}},$$

$$t_{i0}(y)=\left(B_0+\frac{H_0}{y}-\frac{1}{y}\int_y^1\psi_0dy+\int_y^1\frac{\psi_0}{y}dy\right)^{\frac{1}{1+\gamma}},$$

$$z=r\cos\theta,$$

$$t_{e1}^*(\xi,\theta)=\frac{\Gamma_0}{(1+\alpha)\xi}\exp\left\{-\frac{\Pr_\infty}{2}\xi(1-\cos\theta)\right\},$$

$$H_0=\frac{(1+\gamma)R^2}{3\lambda_{i0}T_\infty}J_0,\quad V=\frac{4}{3}\pi R^3,\quad x=\cos\theta,$$

$$t_{i1}(y)=\frac{D_0}{t_{i0}^\gamma}+\frac{\cos\theta}{t_{i0}^\gamma}\left[B_1y+\frac{H_1}{y^2}\right.$$
$$\left.+\frac{1}{3}\left(y\int_1^y\frac{\psi_1}{y^2}dy-\frac{1}{y^2}\int_1^y\psi_1ydy\right)\right],$$

$$H_1=\frac{R}{3\lambda_{i0}T_\infty}J_1,\quad t_{e1}(y,\theta)=\frac{1}{t_{e0}^\alpha}\left\{\frac{\omega_0}{2}\left(\frac{D_1}{y}-1\right)\right.$$
$$\left.+\cos\theta\left[\frac{\Gamma}{y^2}+\frac{\omega_0}{3}\left(\tau_3+A_2\frac{\tau_2}{y}-A_1\frac{\tau_1}{y^3}\right)\right]\right\},$$

$$J_0=\frac{1}{V}\int_V q_idV,\quad J_1=\frac{1}{V}\int_V q_izdV,$$

$$\psi_0(y)=-\frac{R^2(1+\gamma)}{2\lambda_{i0}T_\infty}y^2\int_{-1}^{+1}q_1dx,$$

$$\psi_1(y)=-\frac{3R^2}{2\lambda_{i0}T_\infty}y^2\int_{-1}^{+1}q_1xdx,\quad V=\frac{4}{3}\pi R^3,$$

$$\tau_1(y)=(1-\ell)\sum_{n=0}^\infty\frac{\ell^n}{n+1}\left[\Omega_n^{(1)}-\frac{(1-\ell)^3}{6}\frac{n+1}{n+4}\Omega_n^{(3)}\right],$$

$$\Omega_n^{(1)}=\sum_{k=0}^nC_k^{(1)},\quad \omega_0=\frac{\Gamma_0\Pr_\infty}{1+\alpha},$$

$$\tau_2(y)=\frac{1}{1-\ell}\left[1+\ell\ln\ell+C_1^{(2)}\ell(\ell-\ln\ell)\right.$$
$$\left.-\sum_{n=2}^\infty\frac{C_n^{(2)}\ell^n}{n-1}\left(1-\frac{n-1}{n}\ell\right)\right]$$
$$+\frac{\omega_2}{y^2}(1-\ell)S_n^{(2)}+(1-\ell)^2\sum_{n=0}^\infty\frac{\Omega_n^{(4)}\ell^n}{n+2},\quad \tau_3(y)=\frac{1}{(1-\ell)^2}$$
$$\times\left[\frac{1}{2}-2\ell-\ell^2\ln\ell+C_2^{(3)}\left(2\ell^3-\ell^2\ln\ell-\frac{1}{2}\right)\right.$$
$$\left.-\sum_{n=3}^\infty\frac{C_n^{(3)}\ell^n}{n-2}\left(1-2\frac{n-2}{n-1}\ell+\frac{n-2}{n}\ell^2\right)\right]+\frac{\omega_3}{y^3}(1-\ell)S_n^{(2)}$$
$$+(1-\ell)\sum_{n=0}^\infty\frac{\Omega_n^{(6)}\ell^n}{n+1},\quad \Omega_n^{(2)}=\sum_{k=0}^n\frac{\Omega_k^{(1)}}{k+1},$$

$$\Omega_n^{(3)}=\sum_{k=0}^n(n-k+1)(n-k+2)(n-k+3)C_k^{(1)},$$

$$\Omega_n^{(4)}=\sum_{k=0}^n(n-k+1)C_k^{(2)},\quad \Omega_n^{(5)}=\sum_{k=0}^n\frac{\Omega_k^{(3)}}{k+4},$$

$$\theta_n^{(1)}=\Omega_n^{(2)}+\ln y\Omega_n^{(1)},\quad \Omega_n^{(6)}=\sum_{k=0}^nC_k^{(3)},$$

$$\theta_n^{(2)}=\Omega_n^{(5)}+\ln y\Omega_n^{(3)},$$

$$S_n^{(2)}=\sum_{n=0}^\infty\frac{\ell^n}{6(n+4)}\left[(1-\ell)^3\theta_n^{(2)}-6\frac{n+4}{n+1}\theta_n^{(1)}\right],$$

$\Pr_\infty=(\mu_\infty c_p)/\lambda_\infty$ is the Prandtl number and $\int_V q_izdV$ is the dipole moment of the density of heat sources [2, 4, 8], $dV=r^2\sin\theta drd\theta d\varphi$. Integration here is carried out over the entire particle volume.

The average particle surface temperature $T_S=t_{iS}T_\infty$ is determined by solving the following system of equations:

$$\begin{cases} t_{iS} = t_{eS}, \quad \Gamma_0 = t_{eS}^{1+\alpha} - 1, \\ \dfrac{\ell^{(S)}}{1+\alpha} t_{eS} = \dfrac{R^2}{3\lambda_{eS} T_\infty} J_0 - \sigma_0 \sigma_1 \dfrac{R T_\infty^3}{\lambda_{eS}} \left(t_{eS}^4 - 1 \right), \end{cases}$$
$$\ell^{(S)} = \frac{t_{eS}^{1+\alpha} - 1}{t_{eS}^{1+\alpha}}, \quad t_{iS} = t_{i0}(y=1), \tag{3}$$
$$t_{eS} = t_{e0}(y=1), \quad \lambda_{eS} = \lambda_\infty t_{eS}^{\alpha}, \quad \lambda_{iS} = \lambda_{i0} t_{iS}^{\gamma}.$$

PHOTOPHORETIC FORCE AND SPEED: ANALYSIS OF THE RESULTS

To find the force and speed of photophoresis, the boundary conditions for the mass velocity components on the particle surface are required:

$$U_r^{(e)}\Big|_{y=1} = 0, \quad U_\theta^{(e)}\Big|_{y=1} = K_{TS}^{(0)} \frac{\nu_{eS}}{R t_{eS}} \frac{\partial t_e}{\partial \theta}, \tag{4}$$

where K_{TS} is the thermal slip coefficient.

Numerical estimates require the values of thermal slip coefficients. The thermal slip coefficient is determined from the solution the Boltzmann equation in the Knudsen layer and, in the general case, depends on the type of intermolecular interaction model used and average particle surface temperature [1, 17, 18]. Since in this problem we confine ourselves to calculating the force and speed of photophoresis up to the first order of smallness in ε, it is necessary to expand the slip coefficient in a series in the small parameter, and taking into account the boundary condition for the tangential mass velocity component (4), as well as [17, 18], we can take $K_{TS}^{(0)} = 1.161$ as the zero approximation for numerical estimates of photophoretic force and speed [1, 17, 18].

The resulting force acting on a particle is determined by integrating the stress tensor over the surface [11]:

$$F_z = \int_{(S)} \left(-P_e \cos\theta + \sigma_{rr} \cos\theta - \sigma_{r\theta} \sin\theta \right) \times r^2 \sin\theta d\theta d\varphi \Big|_{r=R}. \tag{5}$$

Here σ_{rr}, $\sigma_{r\theta}$ are the stress tensor components [11].

After substituting the expressions obtained above into (5) and integrating, we find that the resulting force is the sum of the force of the viscous resistance of the medium $\mathbf{F}_\mu$, photophoretic force $\mathbf{F}_{\text{ph}}$ (“pure photophoresis”), and force $\mathbf{F}_{mh}$ due to convective heat transfer:

$$\mathbf{F}_\mu = -6\pi R \mu_\infty U f_\mu \mathbf{n}_z,$$
$$\mathbf{F}_{\text{ph}} = 6\pi R \mu_\infty f_{\text{ph}} J_1 \mathbf{n}_z, \tag{6}$$
$$\mathbf{F}_{\text{ph}} = 6\pi R \mu_\infty f_{mh} \omega_0 \mathbf{n}_z.$$

The values of coefficients f_m and f_{qh}, f_{mh} can be estimated by the following formulas:

$$f_m = \frac{2}{3} \frac{N_2}{N_1}, \quad f_{\text{ph}} = \frac{4}{3} K_{TS}^{(0)} \frac{\nu_{eS}}{\delta T_\infty t_{eS}} \frac{G_1}{N_1 \lambda_{iS}},$$
$$f_{mh} = \frac{4}{9} K_{TS}^{(0)} \frac{\nu_{eS} G_1}{R t_{eS}^{1+\alpha} N_1 \delta} \frac{\lambda_{eS}}{\lambda_{iS}}$$
$$\times \left[\tau_3' + 2\tau_3 + \frac{G_3}{G_1} (\tau_1' - \tau_1) \left(1 - \frac{N_2 G_2}{N_1 G_1} \right) - \frac{N_2}{N_1} (\tau_2' + \tau_2) \right],$$
$$N_1\big|_{y=1} = G_1 G_2' - G_2 G_1', \quad N_2\big|_{y=1} = G_1 G_3' - G_3 G_1',$$
$$G_1'(y), \quad G_2'(y)$$

etc., are the first derivatives of the corresponding functions and $\nu_{eS} = \nu_\infty t_{eS}^{1+\beta}$ is the kinematic viscosity of the gaseous medium. Functions G_1, G_2, τ_2, N_1, etc., are taken for $y = 1$.

The speed of uniform particle motion $\mathbf{U}_p$ is determined from the condition of equality to zero of the total force acting on it. It can be seen from (6) that the speed also consists of two terms: the photophoretic speed (pure photophoresis) and the speed caused by movement of the medium:

$$\mathbf{U}_p = -\left(h_{hp} J_1 + h_{mh} \omega_0 \right) \mathbf{n}_z,$$
$$h_{\text{ph}} = f_{\text{ph}} / f_\mu, \quad h_{mh} = f_{mh} / f_\mu.$$

CONCLUSIONS

In this study, for the first time, expressions were obtained that take into account the contribution of convective heat transfer to the pure photophoresis (force and speed) of a large heated solid spherical particle.

The resulting formulas for the force and speed of photophoresis can also be used for small relative temperature fluctuations, i.e., when heating of the particle surface is low. In this case, the average particle surface temperature differs slightly from the temperature of the surrounding gaseous medium far from it, and for $\Gamma_0 \to 0 \, (\ell \to 0)$ we have

$$G_1 = 1, \quad G_1' = -3, \quad G_1'' = 12, \quad G_2 = 1,$$
$$G_2' = -1, \quad G_2'' = 2, \quad G_3 = 1, \quad G_3' = 0,$$
$$N_1 = 2, \quad N_2 = 3, \quad \tau_1 = \frac{3}{4}, \quad \tau_1' = 0,$$
$$\tau_2 = \frac{3}{2}, \quad \tau_2' = 0, \quad \tau_3 = \frac{3}{2}, \quad \tau_3' = 0.$$

Numerical estimates for the α, $\beta = 0.5$, 0.7, 1 values in the temperature range from 273 to 1000 K showed that heating of the particle surface significantly affects the functions $G_i(y), N_i(y), \tau_i(y)$, etc., and their derivatives compared with the values of the functions for small relative temperature fluctuations. This indicates a nonlinear nature of the dependence of the photophoretic force and speed on the average heating temperature of the particle surface.

Convective heat transfer is proportional to the coefficient $\omega_0 = \frac{\Gamma_0 \mathrm{Pr}_\infty}{1+\alpha}$. For most gases, the Prandtl number is on the order of unity and the coefficient $\Gamma_0 = \left(\frac{T_S}{T_\infty}\right)^{1+\alpha} - 1$ depends on the average relative heating temperature of the particle surface T_S, determined by formula (3). For example, when $T_\infty = 273$, $T_S = 1000$ K and $\alpha = 1$, $\omega_0 \approx 6$. Consequently, this contribution to the force and speed of pure photophoresis is greater, the more strongly the particle is heated. Thus, when describing photophoresis with significant temperature drops, it is necessary to take into account the convective term in the heat transfer equation.

This method for solving the convective heat transfer equation can also be applied to solve other similar physical problems, e.g., the convective diffusion equation, the influence of convective heat and mass transfer on the process of evaporation of a heated drop, etc.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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