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Effect of Multiple Scattering on the Spectral-Angular Density of Diffracted Transition Radiation

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Abstract—The work investigates the diffracted transition radiation of a relativistic electron crossing a single-crystal plate in the Bragg scattering geometry. Expressions are obtained that describe the spectral-angular density of diffracted transition radiation with and without the multiple scattering of the relativistic electron in a single-crystal plate taken into account. The influence of multiple scattering on the spectrum of diffracted transition radiation of the relativistic electron is shown.

Keywords: multiple scattering, diffracted transition radiation, relativistic electron

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1. INTRODUCTION

When a charged particle crosses the interface between two media, transition radiation (TR) arises [1, 2]. The influence of multiple scattering of a charged particle by the atoms of a medium on the spectral-angular density of TR in an amorphous medium was discussed in [3–5]. In [3, 4], the influence of multiple scattering, respectively, on the TR spectrum and angular density was considered at a qualitative level, and in [5], using a rigorous kinetic approach to averaging the TR spectral-angular density over all possible trajectories of electron motion in substance.

The transition radiation arising at the front boundary of a single-crystal target undergoes diffraction at a system of parallel atomic planes of the crystal, forming diffracted transition radiation (DTR) in a direction close to the direction of Bragg scattering in a narrow spectral range [6–8]. It is important to note that, in the X-ray frequency range, DTR can be experimentally detected and studied from one target boundary, in contrast to TR, which can be observed only from two boundaries, i.e., under conditions of a significant influence on the spectral-angular density of radiation of the interference of TR waves from different boundaries and the photoabsorption of waves by the target material. At the same time, TR also has a very wide spectrum, which significantly reduces the possibility of studying the effect of multiple scattering of relativistic electrons on the spectral-angular density of TR. DTR has a very narrow frequency range and is essentially transition radiation from only one boundary; therefore, in our opinion, it is an important and convenient object for studying the effect of multiple scattering of radiating relativistic electrons on the TR angular density.

In [9–11], the influence of the electron-beam divergence on the spectral-angular characteristics of DTR and parametric X-ray radiation (PXR) was studied in the Laue scattering geometry.

This work is devoted to studying the DTR excited by a beam of relativistic electrons crossing a single-crystal plate of arbitrary thickness in the Bragg scattering geometry, taking into account multiple scattering of the beam by relativistic electrons at target atoms. To take into account multiple scattering, the traditional method of averaging the spectral-angular and angular densities of radiation over an expanding beam of rectilinear electron trajectories is used.

2. GEOMETRY OF THE RADIATION PROCESS

We consider a beam of relativistic electrons crossing a single crystal in the Bragg scattering geometry (Fig. 1). Let us introduce the angular variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$ and $\boldsymbol{\theta}_0$ in accordance with definitions of the speed of a relativistic electron isolated in a beam $\mathbf{V}$ and unit vectors: $\mathbf{n}$ in the direction of the momentum of a photon emitted close to the direction of the electron velocity vector, and $\mathbf{n}_\mathbf{g}$ in the direction of Bragg scattering:

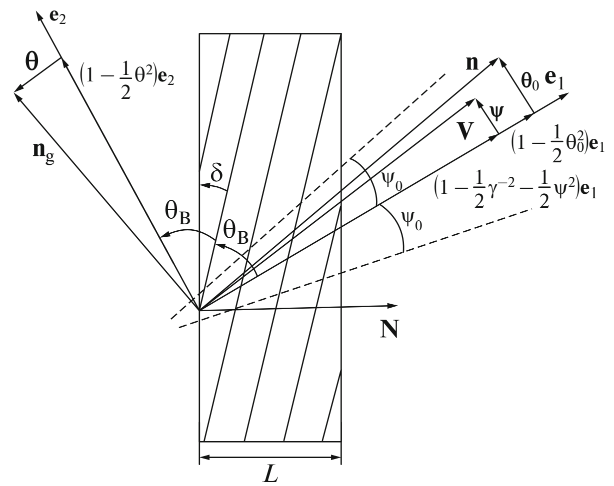

Fig. 1. Geometry of the radiation process.

$$\mathbf{V} = \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \quad \mathbf{e}_1\boldsymbol{\psi} = 0,$$
$$\mathbf{n} = \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \quad \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \tag{1}$$
$$\mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B, \quad \mathbf{n}_\mathbf{g} = \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \quad \mathbf{e}_2\boldsymbol{\theta} = 0,$$

where $\boldsymbol{\theta}$ is the radiation angle measured from the axis of the radiation detector $\mathbf{e}_2$, $\boldsymbol{\psi}$ is the deflection angle of the considered electron in the beam, calculated from the axis of the electron beam $\mathbf{e}_1$, $\boldsymbol{\theta}_0$ is the angle between the direction of propagation of the incident photon and the axis $\mathbf{e}_1$, and $\gamma = 1/\sqrt{1-V^2}$ is the Lorentz factor of the electron. The angular variables are considered as the sum of components parallel and perpendicular to the drawing plane: $\boldsymbol{\theta} = \boldsymbol{\theta}_\parallel + \boldsymbol{\theta}_\perp$, $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0\parallel} + \boldsymbol{\theta}_{0\perp}$, $\boldsymbol{\psi} = \boldsymbol{\psi}_\parallel + \boldsymbol{\psi}_\perp$. ψ_0 is the initial divergence of the electron beam, $\mathbf{N}$ is the normal to the target surface, δ is the angle between the crystallographic diffracting plane and the target surface, and θ_B is the Bragg angle.

3. SPECTRAL-ANGULAR AND ANGULAR RADIATION DENSITIES

In [12], within the framework of the two-wave approximation of the dynamic theory of diffraction, the theory of coherent X-ray radiation excited in a single crystal in the Bragg scattering geometry by a beam of relativistic electrons in the direction $\mathbf{n}$ (Fig. 1), close to direction $\mathbf{e}_1$ beam axis, was developed. Expressions were obtained that describe the amplitudes of the electric-field strengths of parametric X-ray waves near the direction of the velocity of a relativistic electron (VRE) and transition radiation (TR). Based on them, expressions were obtained and studied that describe the spectral-angular characteristics of VRE, TR and their interference term.

Using notation and reasoning similar to those carried out in [11], in this work we have obtained an expression for the intensity amplitude of the coherent X-ray radiation $E_{\mathrm{Rad}}^{(s)}$, excited in a thin nonabsorbing single-crystal target by a relativistic electron moving in a beam at an angle of $\boldsymbol{\psi}(\psi_\parallel, \psi_\perp)$ to the beam axis $\mathbf{e}_1$. The radiation-field amplitude was presented as the sum of fields, one of which corresponds to the contribution of the PXR mechanism $\left(E_{\mathrm{PXR}}^{(s)}\right)$, and another contribution to DTR $\left(E_{\mathrm{DTR}}^{(s)}\right)$. The DTR field amplitude has the form:

$$E_{\mathrm{DTR}}^{(s)} = \frac{8\pi^2 ieV\Omega^{(s)}}{\omega}\frac{\omega^2\chi_\mathbf{g}C^{(s,\tau)}}{2\omega\Delta}$$
$$\times\left[\frac{1}{\frac{\gamma_0}{|\gamma_\mathbf{g}|}\left(-\chi_0(\omega) - \frac{2}{\omega}\frac{\gamma_0}{\gamma_\mathbf{g}}\lambda_\mathbf{g}^* + \beta\frac{\gamma_0}{\gamma_\mathbf{g}}\right)} + \frac{\omega}{2\frac{\gamma_0}{|\gamma_\mathbf{g}|}\lambda_0^*}\right] \tag{2}$$
$$\times\left(\Delta^{(2)} - \Delta^{(1)}\right),$$

where

$$\Delta = \lambda_\mathbf{g}^{(2)}\exp\left(i\frac{\lambda_\mathbf{g}^* - \lambda_\mathbf{g}^{(2)}}{\gamma_\mathbf{g}}L\right) - \lambda_\mathbf{g}^{(1)}\exp\left(i\frac{\lambda_\mathbf{g}^* - \lambda_\mathbf{g}^{(1)}}{\gamma_\mathbf{g}}L\right),$$
$$\Delta^{(2)} = \exp\left(i\frac{\lambda_\mathbf{g}^* - \lambda_\mathbf{g}^{(2)}}{\gamma_\mathbf{g}}L\right), \quad \Delta^{(1)} = \exp\left(i\frac{\lambda_\mathbf{g}^* - \lambda_\mathbf{g}^{(1)}}{\gamma_\mathbf{g}}L\right),$$
$$C^{(s,\tau)} = (-1)^\tau C^{(s)}, \quad C^{(1)} = 1, \quad C^{(2)} = |\cos 2\theta_B|,$$
$$\lambda_\mathbf{g}^{(1,2)} = \frac{\omega\left|\chi'_\mathbf{g}C^{(s)}\right|}{2}\left(\xi^{(s)} \pm \sqrt{\xi^{(s)2} - \varepsilon}\right),$$
$$\lambda_\mathbf{g}^* = \frac{\omega\left|\chi'_\mathbf{g}C^{(s)}\right|}{2}\left(2\xi^{(s)} - \varepsilon\sigma^{(s)}\right),$$
$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1+\varepsilon}{2\nu^{(s)}},$$
$$\sigma^{(s)} = \frac{1}{\left|\chi'_\mathbf{g}\right|C^{(s)}}\left(\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2 - \chi'_0\right),$$
$$\eta^{(s)}(\omega) = \frac{2\sin^2\theta_B}{V^2\left|\chi'_\mathbf{g}\right|C^{(s)}}\left(1 - \frac{\omega(1 - \theta_\parallel \mathrm{ctg}\theta_B)}{\omega_B}\right),$$
$$\nu^{(s)} = \frac{\chi'_\mathbf{g}C^{(s)}}{\chi'_0}, \quad \varepsilon = \frac{\sin(\theta_B - \delta)}{\sin(\theta_B + \delta)}, \tag{3}$$

χ'_0 is the real part of the average dielectric susceptibility of a single crystal, $\chi'_\mathbf{g}$ is the real part of the Fourier coefficient of expansion of the dielectric susceptibility of a single crystal in reciprocal lattice vectors g:

$$\chi(\omega,\mathbf{r}) = \sum_{\mathbf{g}}\left(\chi'_{\mathbf{g}}(\omega) + i\chi''_{\mathbf{g}}(\omega)\right)\exp(i\mathbf{gr}),$$

$$\chi'_{\mathbf{g}} = \chi'_0\left(\mathrm{F}(\mathrm{g})/Z\right)\left(\mathrm{S}(\mathbf{g})/N_0\right)\exp\left(-\frac{1}{2}g^2u_\tau^2\right),$$

where F(g) is the form factor of an atom containing Z electrons; S(**g**) is the structural factor of an elementary cell containing N_0 atoms; and u_τ is the root-mean-square amplitude of thermal vibrations of crystal atoms. The paper considers the X-ray frequency range $(\chi'_{\mathbf{g}} < 0, \chi'_0 < 0)$.

Since the inequality holds in the X-ray frequency range $\frac{2\sin^2\theta_\mathrm{B}}{V^2\left|\chi'_{\mathbf{g}}\right|C^{(s)}} \gg 1$, then $\eta^{(s)}(\omega)$ is a fast function of the frequency ω. For further analysis of the PXR and DTR spectra, it is convenient to consider $\eta^{(s)}(\omega)$ (or $\xi^{(s)}(\omega)$) as a spectral variable characterizing the frequency ω.

Parameter $\nu^{(s)}$, taking values in the interval $0 \le \nu^{(s)} \le 1$, determines the degree of reflection of the radiation-wave field from the considered system of parallel atomic planes of a single crystal, which is determined by the nature of the interference of waves reflected from different planes. Parameter ε for a fixed value θ_B determines the orientation of the input surface of the target relative to the reflecting system of parallel atomic planes of the single crystal.

Substituting (2) into the known expression for the spectral-angular density of X-ray radiation:

$$\omega\frac{d^2N^{(s)}_\mathrm{DTR}}{d\omega d\Omega} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E^{(s)}_\mathrm{DTR}\right|^2, \tag{4}$$

we obtain an expression for the spectral-angular density of the DTR of a relativistic electron, taking into account the deviation of the direction of its velocity **V** relative to the electron-beam axis $\mathbf{e}_1$ (angle $\boldsymbol{\psi}(\psi_\perp,\psi_\parallel)$):

$$\omega\frac{d^2N^{(s)}_\mathrm{DTR}}{d\omega d\Omega} = \frac{e^2}{\pi^2}\left(\frac{\Omega^{(s)}}{\Delta(\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel)} - \frac{\Omega^{(s)}}{\Delta(\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel)-\chi'_0}\right)^2 R^{(s)}_\mathrm{DTR}, \tag{5a}$$

$$R^{(s)}_\mathrm{DTR} = \frac{\varepsilon^2}{\xi^{(s)^2} - (\xi^{(s)^2}-\varepsilon)\,\mathrm{cth}^2\left(\frac{b^{(s)}\sqrt{\varepsilon-\xi^{(s)2}}}{\varepsilon}\right)}, \tag{5b}$$

where

$$\begin{gathered}\Delta(\theta_\perp,\theta_\parallel,\psi_\perp,\psi_\parallel) = \gamma^{-2} + (\theta_\perp-\psi_\perp)^2 + (\theta_\parallel+\psi_\parallel)^2,\\ \Omega^{(1)} = \theta_\perp-\psi_\perp,\quad \Omega^{(2)} = \theta_\parallel+\psi_\parallel,\\ b^{(s)} = \frac{1}{2\sin(\theta_\mathrm{B}+\delta)}\frac{L}{L^{(s)}_\mathrm{ext}},\quad L^{(s)}_\mathrm{ext} = 1\Big/\omega\left|\chi'_{\mathbf{g}}\right|C^{(s)}.\end{gathered} \tag{6}$$

4. ACCOUNTING FOR THE MULTIPLE SCATTERING OF RELATIVISTIC ELECTRONS IN A SINGLE CRYSTAL

Let us consider one electron moving as part of an electron beam incident on a single-crystal target at an angle of $\boldsymbol{\psi}$ relative to the beam axis $\mathbf{e}_1$. As a result of multiple scattering at target atoms, the direction of its motion with respect to the beam axis will change as a function of the path traveled in the target t: $\boldsymbol{\psi} \to \boldsymbol{\psi} + \Delta\boldsymbol{\psi}(t)$.

Components $\Delta\psi_\perp$, $\Delta\psi_\parallel$ of the scattering angle $\Delta\boldsymbol{\psi}$ will be described by the Gaussian function, which varies with the length of the path traveled in the target t:

$$f(\Delta\psi_\perp,\Delta\psi_\parallel,t) = \frac{1}{\pi\left(\psi_0^2+\psi_s^2t\right)}e^{-\frac{\Delta\psi_\perp^2+\Delta\psi_\parallel^2}{\psi_0^2+\psi_s^2t}}, \tag{7}$$

ψ_0 is the initial divergence of the electron beam,

$\psi_s^2 = \frac{E_s^2}{m^2\gamma^2}\frac{1}{L_R}\left(1+0.038\ln\left(\frac{t}{L_R}\right)\right)^2$ is the mean square of the angle of multiple electron scattering per unit length as a function of the path t, passed by an electron in a single-crystal target [13], $E_s \approx 21$ MeV.

As a result of averaging expression (5) for the DTR spectral-angular density of one electron, assuming $\psi_0 = 0$ in (7), we get:

$$\begin{gathered}\left\langle\omega\frac{d^2N^{(s)}_\mathrm{DTR}}{d\omega d\Omega}\right\rangle_{\Delta\psi}\\ = \frac{e^2}{\pi^2}\frac{1}{L_e}\int_0^{L_e}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{\Omega^{(s)}}{\Delta} - \frac{\Omega^{(s)}}{\Delta^*-\chi'_0}\right)^2 R^{(s)}_\mathrm{DTR}\frac{1}{\pi\psi_s^2t}\\ \times e^{-\frac{\Delta\psi_\perp^2+\Delta\psi_\parallel^2}{\psi_s^2t}}d\Delta\psi_\perp d\Delta\psi_\parallel dt,\end{gathered} \tag{8}$$

where $\Delta^*(\theta_\perp,\theta_\parallel,\psi_\perp+\Delta\psi_\perp,\psi_\parallel+\Delta\psi_\parallel) = \gamma^{-2} + (\theta_\perp-(\psi_\perp+\Delta\psi_\perp))^2 + (\theta_\parallel+(\psi_\parallel+\Delta\psi_\parallel))^2$.

By value t, which represents the path traveled by the electron in the target, we integrate from zero to the total length of the path of the electron in the target L_e.

Let us analyze the effect of multiple scattering of electrons at atoms of the medium on the spectral-angular density of diffracted transition radiation for various values of the electron energy, which is determined by the Lorentz factor γ. The angular density of the TR at the front boundary of the target, according to (5a), is proportional to the following expression:

$$\left(\frac{\Omega^{(s)}}{\Delta}-\frac{\Omega^{(s)}}{\Delta^{*}-\chi_0'}\right)^2 = \left(\frac{\Omega^{(s)}}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_\parallel+\psi_\parallel)^2} - \frac{\Omega^{(s)}}{\gamma^{-2}+(\theta_\perp-\psi_\perp)^2+(\theta_\parallel+\psi_\parallel)^2-\chi_0'}\right)^2. \quad (9)$$

The first term in brackets corresponds to the field of an electron in free space, the second to the field of an electron in a single crystal. It can be seen that in the case of a high electron energy $\left(\gamma^{-2}\ll\left|\chi_0'\right|\right)$, an overwhelming contribution to the angular density of the DTR, and hence the TR, is given by the first term corresponding to the electron field in free space. Since the maximum of the DTR angular density corresponds to the angle $\theta\approx\gamma^{-1}$, then multiple scattering in a substance under these conditions will in no way affect the angular density of the DTR (or TR), since the second term is negligibly small compared to the first. Since experiments to study the properties of TR and DTR are carried out mainly at high energies $\left(\gamma^{-2}\ll\left|\chi_0'\right|\right)$, then the effect of multiple scattering on the DTR (or TR) is not significant in this case and is considered to be absent.

In the case of low electron energies $\left(\gamma^{-2}\geq\left|\chi_0'\right|\right)$, in expression (9), the second term will be comparable in value to the first, which will lead to the absence of DTR (and TR). Multiple scattering can significantly affect the DTR (and TR) angular density, since it will lead to a decrease in the second term in (9) and a weakening of the compensation of the first term by the second. Thus, multiple scattering at low energies of a relativistic electron $\left(\gamma^{-2}\gg\left|\chi_0'\right|\right)$ can increase the DTR angular density by orders of magnitude. This effect of multiple scattering in the angular density of DTR (and TR) has not been previously declared by anyone and has not been studied either theoretically or experimentally.

Transition radiation is formed as the difference between the field of an electron that it had when it entered the target and the screened field of the electron in the medium. Multiple scattering can affect only the field component of the transition-radiation wave, which is formed during the motion of an electron in the target material. In this regard, in the expression for the spectral-angular density of DTR (8), which takes into account multiple electron scattering, we introduced the replacement $\Delta\to\Delta^*$ only in the second term describing the electron field in the target material.

Averaging expression (8) over all possible initial rectilinear trajectories of an electron in a beam, we obtain an expression describing the spectral-angular density of the DTR of a beam of relativistic electrons, taking into account multiple scattering in a single crystal, normalized to one electron:

$$\left\langle\omega\frac{d^2N_{\mathrm{DTR}}^{(s)}}{d\omega d\Omega}\right\rangle_{\Delta\psi}=\frac{e^2}{\pi^2}\frac{1}{L_e}\frac{1}{\pi\psi_0^2}\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^{L_e}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{\Omega^{(s)}}{\Delta}-\frac{\Omega^{(s)}}{\Delta^*-\chi_0'}\right)^2R_{\mathrm{DTR}}^{(s)}\times\frac{1}{\pi\left(\psi_0^2+\psi_s^2t\right)}e^{-\frac{\Delta\psi_\perp^2+\Delta\psi_\parallel^2}{\psi_0^2+\psi_s^2t}}d\Delta\psi_\perp d\Delta\psi_\parallel dt\right]\times e^{-\frac{\psi_\perp^2+\psi_\parallel^2}{\psi_0^2}}d\psi_\perp d\psi_\parallel. \quad (10)$$

In expression (10), radiation is considered as the sum of the contributions of beam electrons with different angles of incidence with respect to the beam axis $\boldsymbol{\psi}$, which are given by the normalized function of the angular distribution of the electron beam. The contribution of multiple scattering is represented by the addition to the distribution width $\psi_0^2\Rightarrow\psi_0^2+\psi_s^2t$. ψ_0 is the initial divergence of the electron beam. In fact, in this way we are averaging the angular density of the DTR over an expanding beam of rectilinear trajectories of radiating electrons. Expressions (8) and (10), which describe the spectral-angular density of the DTR, taking into account the multiple scattering of relativistic electrons in a single crystal, is the main result of this work.

Since the electrons in the beam excite radiation in a coherent manner, we can consider the effect of multiple scattering on the angular density of the DTR using the example of a single emitting electron crossing the front boundary of a single-crystal target along the axis $\mathbf{e}_1$ ($\boldsymbol{\psi}=0$). Numerical calculations of the spectral-angular density of the DTR without taking into account multiple scattering will be carried out according to formula (5), and taking multiple scattering into account according to formula (8). We will consider a relativistic electron with energy $\gamma=100$, crossing a carbon signle crystal $C(111)$, with the process parameters: $\theta_{\mathrm{B}}=16.2°$, $\omega_{\mathrm{B}}=10900$ eV, $\varepsilon=1$.

Figure 2 shows the curves constructed according to formula (5), which describe the spectral-angular density of the DTR at a fixed observation angle without taking into account multiple scattering. The figure shows an increase in the amplitude of the DTR spectrum with an increase in the target thickness and satu-

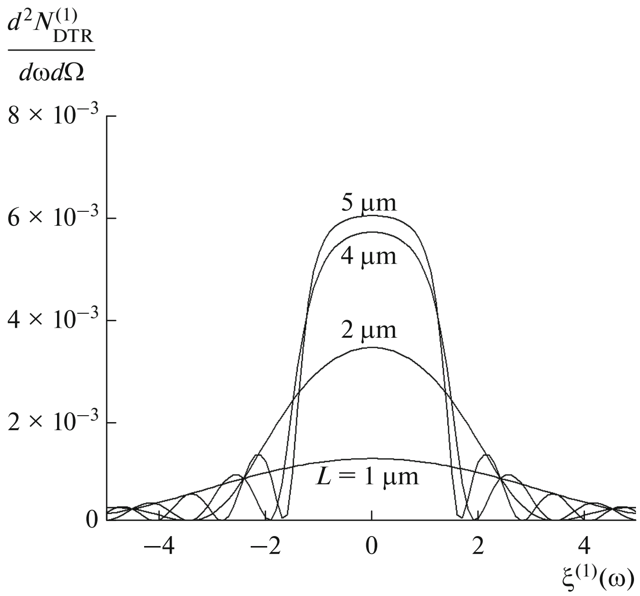

Fig. 2. Spectral-angular densities of the DTR for different thicknesses of a single crystal at fixed viewing angles: $\theta_\perp = 10$, $\theta_\| = 0$. $\gamma = 100$.

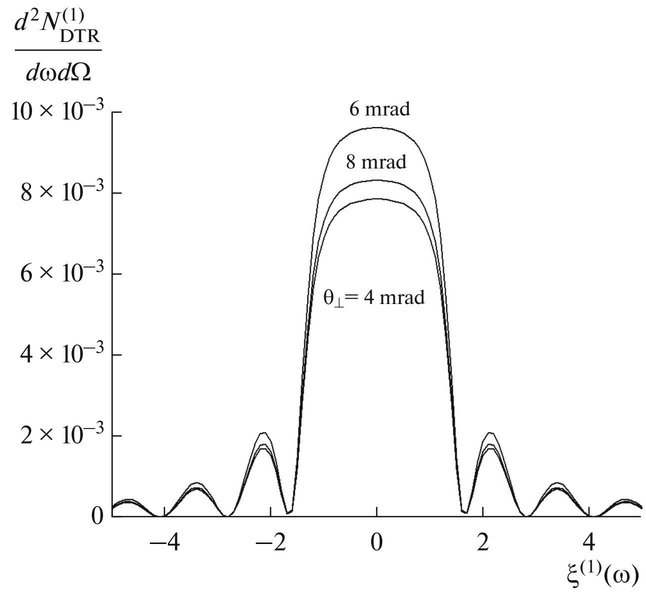

Fig. 3. Spectral-angular densities of the DTR for different viewing angles $\theta_\perp$, $\theta_\| = 0$. $\gamma = 100$.

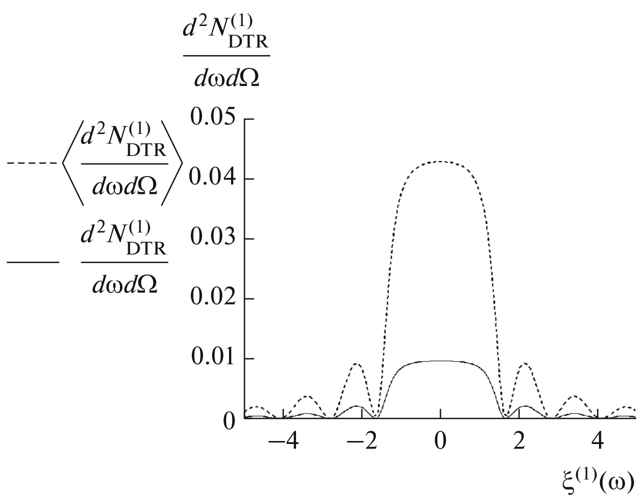

Fig. 4. Spectral-angular densities of the DTR with (dashed curve) and without (solid curve) multiple scattering taken into account: $\gamma = 100$, $L = 5$ μm, $\theta_\perp = 6$ mrad.

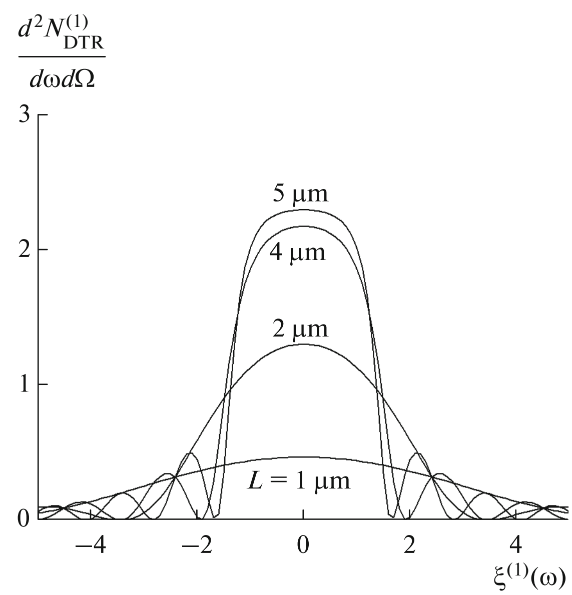

Fig. 5. Spectral-angular densities of the DTR for different thicknesses of a single crystal at fixed viewing angles: $\theta_\perp = 3$ mrad, $\theta_\| = 0$. $\gamma = 300$.

ration of the growth at approximately $L = 5$ μm. The conditions under consideration showed that such a thickness of a single crystal is the limiting one for the generation of DTR.

Figure 3 shows the curves describing the spectral-angular densities of the DTR for different viewing angles $\theta_\perp$, at $\theta_\| = 0$. It can be seen from the figure that the spectral-angular density is maximum at an angle approximately $\theta_\perp = 6$ mrad.

Figure 4 shows the curves plotted by formulas (5) and (8), which describe the DTR spectrum with (dashed curve) and without (solid curve) taking into account multiple electron scattering at atoms of a target of thickness $L = 5$ μm and viewing angle $\theta_\perp = 6$ mrad, $\theta_\| = 0$. It follows from the figure that,

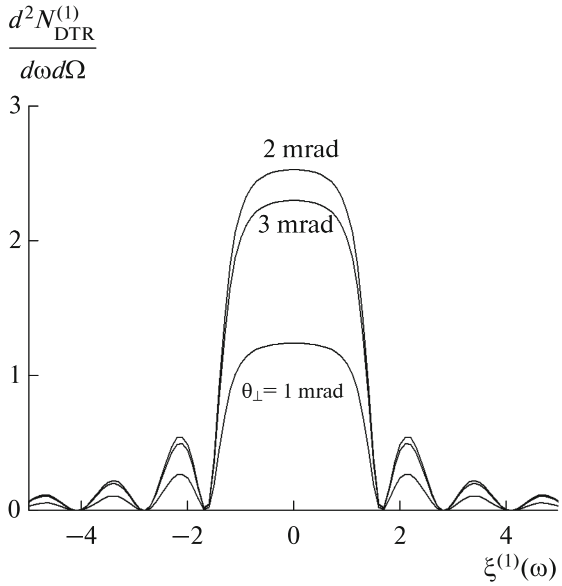

Fig. 6. Spectral-angular densities of the DTR for different viewing angles $\theta_\perp$, $\theta_\| = 0$. $\gamma = 300$, $L = 5$ μm.

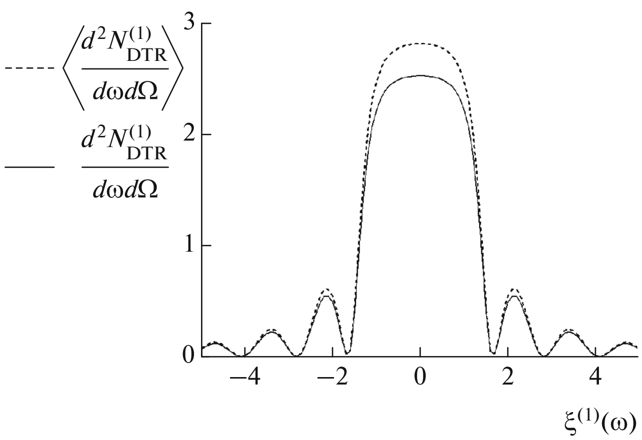

Fig. 7. Spectral-angular densities of the DTR with (dashed curve) and without (solid curve) multiple scattering taken into account: $\gamma = 300$, $L = 5$ μm, $\theta_\perp = 2$ mrad.

under the conditions under consideration, taking into account multiple scattering gives an approximately fivefold increase in the amplitude of the DTR spectral-angular density compared to the calculation without taking into account multiple scattering. Curves in Figs. 2–4 are plotted for an electron energy corresponding to the Lorentz factor ($\gamma = 100$). Similar curves in Figs. 5–7 are constructed for a higher electron energy ($\gamma = 300$). With such an electron energy (Fig. 7) the effect of the multiple scattering of electrons on atoms of the medium on the spectral-angular density of radiation becomes much weaker. In the case of an increase in the energy of electrons $\gamma > 300$ the effect of multiple scattering on the spectral-angular density of the DTR will be negligible.

CONCLUSIONS

The dynamic theory of diffracted transition radiation generated by a beam of relativistic electrons in a single-crystal plate in the Bragg scattering geometry under conditions of multiple scattering of incident particles is developed in this work. Expressions are obtained that describe the spectral-angular density of the DTR both with and without taking into account the multiple scattering of beam electrons at target atoms. The influence of multiple electron scattering on the spectral-angular density of the DTR is studied. Calculations of the spectral-angular density of the DTR demonstrate an increase in the angular density of the DTR with an increase in the target thickness. The main new result of the work is establishment of the fact of a significant increase in the spectral-angular density of diffracted transition radiation under conditions of the multiple scattering of emitting electrons by atoms of a single-crystal target.

The results of this work can be useful in setting up new experiments to study the properties of PXR and DTR and interpreting the results of experiments at low electron energies ($\gamma \le 100$), in which it is fundamentally important to correctly take into account the contribution of the DTR.

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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