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ON STUDYING THE SPREAD MODEL OF THE HIV/AIDS EPIDEMIC

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Abstract. The aim of this work is to study sufficient conditions for the asymptotic stability of the stationary solution of the initial-boundary value problem for a system of nonlinear partial differential equations describing the growth and spread of the HIV/AIDS epidemic. The above-mentioned model takes into account not only the factors taken into account by classical models, but also includes migration processes.

Keywords: system of nonlinear partial differential equations, initial-boundary value problem, stationary solution, mathematical modeling, spread model of the HIV/AIDS epidemic, migration processes.

1. Introduction. Description of the Result

Let $\Omega \subset \mathbb{R}^2$ be a bounded domain with piecewise smooth boundary. Consider the initial-boundary value problem

$$\frac{\partial S}{\partial t} = \mu k - c\beta\left(\beta_1 - \frac{\beta_2 I}{m+I}\right) SI - c\beta bJS - \mu S + \vartheta_1 \Delta S, \tag{1.1}$$

$$\frac{\partial I}{\partial t} = c\beta\left(\beta_1 - \frac{\beta_2 I}{m+I}\right) SI + c\beta bJS - (\mu + k_1)I + \delta J + \vartheta_2 \Delta I, \tag{1.2}$$

$$\frac{\partial J}{\partial t} = k_1 I - (\mu + k_2 + \delta)J + \vartheta_3 \Delta J, \quad x = (x_1, x_2) \in \Omega, \quad t > 0, \tag{1.3}$$

$$\left(\mu_1 S + \eta_1 \frac{\partial S}{\partial \overrightarrow{\nu}}\right)\Big|_{\partial\Omega} = B_1,$$

$$\left(\mu_2 I + \eta_2 \frac{\partial I}{\partial \overrightarrow{\nu}}\right)\Big|_{\partial\Omega} = B_2,$$

$$\left(\mu_3 J + \eta_3 \frac{\partial J}{\partial \overrightarrow{\nu}}\right)\Big|_{\partial\Omega} = B_3, \quad t \geqslant 0; \tag{1.4}$$

$$S\,|_{t=0} = S_0, \quad I\,|_{t=0} = I_0, \quad J\,|_{t=0} = J_0, \quad x \in \overline{\Omega}, \tag{1.5}$$

where $\Delta = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2$ is the Laplace operator in $\mathbb{R}^2$, $\overrightarrow{\nu}$ is the unit exterior normal vector to the boundary $\partial\Omega$ of the domain Ω, in the boundary condition (1.4) we assume that $\mu_\kappa^2 + \eta_\kappa^2 > 0$, $\mu_\kappa \eta_\kappa \geqslant 0$, $\kappa = 1, 2, 3$.

We use model (1.1)–(1.3) in [30], to which we added the condition of territorial spread (diffusion). In this regard, we use the notations, terms, and descriptions presented in [30]. The (1.1)–(1.3) model is the result of several stages of modification of the classical Kermack–McKendrick model of epidemic spread [12] (see [4, 30, 35, 37]).

The functions and parameters included into system (1.1)–(1.5) have the following meaning:

$S = S(x_1, x_2, t)$ is the number of susceptible individuals in a given population;

$I = I(x_1, x_2, t)$ is the number of asymptomatic carriers of infection;

$J = J(x_1, x_2, t)$ is the size of the symptomatic group;

k is the total population;

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μ is the mortality rate of the population;

c is the frequency of contacts;

β is the probability of transmission of a disease through contact with an infectious disease in the asymptomatic stage;

b is the probability of disease transmission through contact with an infectious disease in the symptomatic stage;

k_1 is the rate of transition from the asymptomatic stage to the symptomatic one;

k_2 is the rate of transition from the symptomatic state to AIDS;

δ is the speed of treatment from the symptomatic stage to the asymptomatic one;

d is the AIDS mortality rate;

$m > 0$ is the half-saturation constant reflecting the effect of media coverage on contact transmission;

β_1 is the frequency of contacts before media notification with $\beta_1 \geqslant \beta_2 > 0$.

The term $\beta_2 I/(m + I)$ estimates the effect of reducing the contact rate when infected individuals are informed through the media. The function $I/(m + I)$ takes into account the disease saturation or psychological effects [5, 30]. The author of paper [30] says that the main reason we include the media effect in step I is that it extends to infected individuals without symptoms. Therefore, the media should warn these people about the possibility of infection and raise their awareness.

Human immunodeficiency virus (HIV) is a chronic, infectious, contact-transmitted, slowly progressive disease characterized by damage to the immune system and the development of acquired immunodeficiency syndrome (AIDS). The number of infected people in most countries is growing every year. Estimates of the prevalence of HIV worldwide have been given by the World Health Organization (WHO) and the United Nations Programme on HIV/AIDS (UNAIDS) since the late 1980s [39, 40]. The first AIDS cases were registered in 1981, and since then it has become one of the most devastating diseases that humanity has ever faced. According to UNAIDS, the number of people who have died from AIDS-related diseases is 36 million.

Although the factors responsible for the spread of HIV are recognized, the lack of health services and the reluctance of the population to take preventive measures make it very difficult to combat this disease. Another serious problem is that in many countries people do not even suspect that they have HIV [22].

Mathematical modeling has long been used to study the spread of many serious diseases: HIV/AIDS, tuberculosis, malaria, and many others. Mathematical models can provide a deep understanding of how patients respond to infectious agents and can predict the dynamics of infection spread in the population. Thus, studying the dynamic characteristics represented by these models can play a significant role in understanding infectious diseases. In this regard, many mathematical models have been formulated and investigated to understand the long-term dynamic behavior of HIV, as well as to predict the incidence of HIV/AIDS [1–4, 9, 10, 15, 17, 18, 21, 23, 24, 31, 32, 36].

Our contribution to the modification of the model under consideration is the addition of "diffusion" terms obtained by the action of the operator $\vartheta_\kappa \Delta$ to the functions S, I, J, $\kappa = 1, 2, 3$, in order to take into account migration processes, which, as we believe, obey the Fourier law.

Similar modifications of models are given in works devoted to mathematical models describing not only the growth but also the spread of various types of populations. In particular, the problems of ecosystem stability are considered in [34]. The model of tumor cell growth and spread is described in [38].

In this paper we study the diffusion model, namely the sufficient conditions under which the Lyapunov stability of its stationary solution implies its asymptotic stability ("stability relative to small deviations" in the terms of systems theory). In [34] it is shown that adding diffusion terms can change the stability of the stationary solution for both better and worse.

Let H be the diameter of Ω. Next, we let

$$A_{11} = -c\beta\left(\beta_1 - \frac{\beta_2}{m+I}\right)I - c\beta bJ - \mu - \frac{\vartheta_1}{H^2}, \tag{1.6}$$

$$A_{22} = c\beta\beta_1 S - \frac{c\beta\beta_2 mS}{(m+I)^2} - \mu - k_1 - \frac{\vartheta_2}{H^2}, \tag{1.7}$$

$$A_{33} = -\mu - k_2 - \delta - \frac{\vartheta_3}{H^2}, \tag{1.8}$$

$$A_{12} = A_{21} = \frac{1}{2}\left(-c\beta\beta_1 S + \frac{c\beta\beta_2 mS}{(m+I)^2} + c\beta\beta_1 I - \frac{c\beta\beta_2 I}{m+I} + c\beta bJ\right), \tag{1.9}$$

$$A_{23} = A_{32} = \frac{1}{2}(c\beta bS + \delta + k_1), \tag{1.10}$$

$$A_{13} = A_{31} = -\frac{1}{2}c\beta bS. \tag{1.11}$$

Proposition. *Let $w(x) = (w_1(x_1,x_2), w_2(x_1,x_2), w_3(x_1,x_2))$ be a regular stationary solution of the system* (1.1)–(1.3) *satisfying the boundary conditions* (1.4). *If for $S = w_1$, $I = w_2$, $J = w_3$ the quadratic form*

$$Q(w_1,w_2,w_3;z_1,z_2,z_3) = \sum_{\kappa=1}^{3}\sum_{\iota=1}^{3} A_{\kappa\iota} z_\kappa z_\iota \tag{1.12}$$

is negatively defined, then the stationary solution w is asymptotically stable at small deviations (*i.e., from Lyapunov stability there follows asymptotic stability*).

2. Materials and Methods

The research methods used to prove the main result in this paper were used earlier in [6, 8, 19, 25, 27–29]. In [26] the research methodology has been adjusted due to the fact that the equation contains the Bessel operator (see [11, 13, 16, 33] in that regard).

Consider also the initial-boundary value problem (see also [27, 28])

$$\frac{\partial u_s}{\partial t} = \vartheta_s \Delta u_s + F_s(u), \quad x = (x_1,\dots,x_n) \in \Omega \subset \mathbb{R}^n, \quad t > 0, \tag{2.1}$$

$$\left(\mu_s u_s + \eta_s \frac{\partial u_s}{\partial \nu}\right)\bigg|_{x\in\partial\Omega} = B_s(x), \tag{2.2}$$

$$u_s(x,0) = u_s^0(x), \quad s = 1,\dots,m, \tag{2.3}$$

where Ω is the domain bounded by piecewise smooth boundary $\Gamma = \partial\Omega$, $\nu = \overrightarrow{\nu}$ is the unit exterior normal vector $\partial\Omega$, $u = (u_1(x,t),\dots,u_m(x,t))$, $\vartheta_s \geqslant 0$, $B_s(x) \in C(\partial\Omega)$, $u_s^0(x) \in C(\overline{\Omega})$, $s = 1,\dots,m$, $\overline{\Omega} = \Omega \cup \partial\Omega$, Δ is the Laplace operator given by formula

$$\Delta v = \sum_{j=1}^{n} \frac{\partial^2 v}{\partial x_j^2}.$$

Of course, we must require the compatibility conditions of the initial and boundary data to be met. However, in the context of this article, we will deviate from this issue. We will assume that all conditions for the existence of classical (regular) solutions to the problem under consideration are met, and, in addition, all initial functions have the necessary properties that allow us to perform all the operations that we perform below.

If

$$\vartheta_s = 0, \quad s = 1,\dots,m, \tag{2.4}$$

then we obtain a lumped-parameter model without diffusion terms. In this case, the variables $x_1, \dots, x_n$ are included in Eqs. (2.1) as parameters and the derivatives with respect to these variables are not contained in Eqs. (2.1). If

$$\sum_{s=1}^{m} \vartheta_s^2 > 0, \tag{2.5}$$

then we obtain a system with distributed parameters.

Let $w = (w_1(x), \dots, w_m(x))$ be the stationary solution of problem (2.1)–(2.3), i.e., the solution of the problem

$$\vartheta_s \Delta w_s + F_s(w) = 0, \quad x \in \Omega, \tag{2.6}$$

$$\left(\mu_s w_s + \eta_s \frac{\partial w_s}{\partial \nu}\right)\bigg|_{x \in \partial\Omega} = B_s(x), \quad s = 1, \dots, m. \tag{2.7}$$

Let functions $F_s(u)$, $s = 1, \dots, m$, be differentiable at w. Then, for sufficiently small deviations $z_s = z_s(x_1, \dots, x_n, t) = u_s - w_s$, $s = 1, \dots, m$, we have

$$F_s(u) = F_s(w + z) = F_s(w) + \sum_{k=1}^{m} b_{sk}\, z_k + \sum_{k=1}^{m} \epsilon_{sk}(z)\, z_k, \tag{2.8}$$

where

$$b_{sk} = \frac{\partial F_s(w)}{\partial z_k}, \quad \lim_{z \to 0} \epsilon_{sk}(z) = 0, \quad s, k = 1, \dots, m.$$

Substituting the representation $u_s = w_s + z_s$ into (2.1) with (2.8), we obtain:

$$\frac{\partial z_s}{\partial t} = \vartheta_s \Delta w_s + F_s(w) + \vartheta_s \Delta z_s + \sum_{k=1}^{m} b_{sk} z_k + \sum_{k=1}^{m} \epsilon_{sk}(z)\, z_k, \quad s = 1, \dots, m. \tag{2.9}$$

Since w is the stationary solution, from (2.9) we obtain:

$$\frac{\partial z_s}{\partial t} = \vartheta_s \Delta z_s + \sum_{k=1}^{m} b_{sk} z_k + \sum_{k=1}^{m} \epsilon_{sk}(z)\, z_k, \quad s = 1, \dots, m. \tag{2.10}$$

We multiply each sth equation of system (2.1) by z_s and integrate the resulting equality over Ω. Taking into account (2.8), we obtain:

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega} z_s^2\, dx = \vartheta_s \int_{\Omega} z_s\, \Delta z_s\, dx + \int_{\Omega} \sum_{k=1}^{m} b_{sk}\, z_s\, z_k\, dx + \int_{\Omega} \sum_{k=1}^{m} \epsilon_{sk}(z)\, z_s\, z_k\, dx, \quad s = 1, \dots, m. \tag{2.11}$$

We can discard the third term on the right-hand side of (2.11), since it does not affect the sign of the resulting sum when the deviations of z are small. Next, we apply Green's formula (see [7]) to the first term on the right-hand side of (2.11). As a result, we obtain:

$$\frac{1}{2}\frac{\partial}{\partial t}\int_{\Omega} z_s^2\, dx = -\vartheta_s \int_{\Omega} |\nabla z_s|^2\, dx + \vartheta_s \int_{\partial\Omega} z_s \frac{\partial z_s}{\partial \nu}\, d\Gamma + \int_{\Omega} \sum_{k=1}^{m} b_{sk}\, z_s\, z_k\, dx, \quad s = 1, \dots, m, \tag{2.12}$$

where $d\Gamma$ is an element of the boundary $\partial\Omega$, i.e., the second term on the right-hand side of (2.12) is a line integral of the first kind over $\partial\Omega$ (for $n = 2$), or a surface integral of the first kind over $\partial\Omega$ (for $n \geqslant 3$), or the sum of the endpoints of the interval Ω (for $n = 1$). When $\mu_s = 0$ or $\eta_s = 0$, the integrand in the integral over $\partial\Omega$ equals to zero due to the boundary condition (2.2). From the same boundary condition, where $\mu_s \eta_s > 0$, we obtain:

$$\frac{\partial z_s}{\partial \nu}\bigg|_{\partial\Omega} = -\frac{\mu_s}{\eta_s} z_s\bigg|_{\partial\Omega}.$$

Therefore, equality (2.12) can be rewritten as

$$\frac{1}{2}\frac{\partial}{\partial t}\int\limits_{\Omega} z_s^2\,dx = -\vartheta_s\int\limits_{\Omega}|\nabla z_s|^2\,dx - \vartheta_s\int\limits_{\partial\Omega}\sigma_s\,z_s^2\,d\Gamma + \int\limits_{\Omega}\sum_{k=1}^{m} b_{sk}\,z_s\,z_k\,dx, \quad s=1,\dots,m, \tag{2.13}$$

where

$$\sigma_s = \frac{\mu_s}{\eta_s},$$

for $\mu_s\eta_s > 0$, or $\sigma_s = 0$, for $\mu_s\eta_s = 0$. Summing (2.13) in s, we obtain:

$$\frac{1}{2}\frac{\partial}{\partial t}\int\limits_{\Omega}|z|^2\,dx = \sum_{s=1}^{m}\left(-\vartheta_s\int\limits_{\Omega}|\nabla z_s|^2\,dx - \vartheta_s\int\limits_{\partial\Omega}\sigma_s\,z_s^2\,d\Gamma\right) + \int\limits_{\Omega}\sum_{s=1}^{m}\sum_{k=1}^{m}\Theta_{sk}\,z_s\,z_k\,dx, \tag{2.14}$$

where

$$\Theta_{sk} = (b_{sk} + b_{ks})/2.$$

The sign of the left-hand side of (2.14) is considered as an indicator of the stability of the trivial solution. Therefore, it is important to find the ratio of terms on the right-hand side that leads to the negativeness of this expression. In the parentheses on the right-hand side, both the first term and the second term are not greater than zero. Next, we need to take into account the sign of the last term on the right-hand side. If the quadratic form

$$\sum_{s=1}^{m}\sum_{k=1}^{m}\Theta_{sk}\,z_k z_s\,dx \tag{2.15}$$

is negatively defined, then the left-hand side of (2.14) will be negative and the stationary solution will be asymptotically stable.

In the case of a lumped parameter model (a system of ordinary differential equations), i.e., if conditions (2.4) are satisfied, the negativeness of the quadratic form (2.15) is also a necessary condition for the trivial solution to be stable.

Consider a diffusion model with distributed parameters. In this case, we can weaken the sufficient condition for asymptotic stability of the stationary solution. For this purpose, we use the Steklov–Poincaré–Friedrichs inequality (see [14, 20])

$$\int\limits_{\Omega}|\nabla z_s|^2 dx \geqslant \frac{1}{H^2}\int\limits_{\Omega} z_s^2 dx,$$

where H is the diameter of Ω. Hence,

$$\frac{1}{2}\frac{\partial}{\partial t}\int\limits_{\Omega}|\,z\,|^2\;dx \leqslant -\sum_{s=1}^{m}\frac{\vartheta_s}{H^2}\int\limits_{\Omega} z_s^2\,dx - \sum_{s=1}^{m}\vartheta_s\int\limits_{\partial\Omega}\frac{\mu_s}{\eta_s}\,z_s^2 d\Gamma + \int\limits_{\Omega}\sum_{s=1}^{m}\sum_{k=1}^{m}\Theta_{sk}\,z_k z_s\,dx. \tag{2.16}$$

Finally, we can say that a sufficient condition for the stability of a stationary solution is the negativity of the quadratic form

$$\sum_{s=1}^{m}\sum_{k=1}^{m}A_{sk}\,z_k z_s, \tag{2.17}$$

where

$$A_{sk} = \Theta_{sk} - \delta_{ks}\vartheta_s/H^2. \tag{2.18}$$

3. Proof of the Main Result

Now, to prove the main result, we set $m = 3$, $n = 2$ in (2.1):

$$u_1 = S(x_1, x_2, t), \quad u_2 = I(x_1, x_2, t), \quad u_3 = J(x_1, x_2, t), \tag{3.1}$$

$$F_1 = F_1(S, I, J) = \mu\rho - \left(\beta_1 - \frac{\beta_2 I}{m + I}\right) SI - c\beta bJS - \mu S, \tag{3.2}$$

$$F_2 = F_2(S, I, J) = \left(\beta_1 - \frac{\beta_2 I}{m + I}\right) SI + c\beta bJS - (\mu + k_1)I, \tag{3.3}$$

$$F_3 = F_3(S, I, J) = k_1 I - (\mu + k_2 + \delta)J. \tag{3.4}$$

The partial derivatives of F_s have the form

$$\frac{\partial F_1}{\partial S} = \left(\frac{\beta_2 I}{m + I} - \beta_1\right) I - c\beta bJ - \mu,$$

$$\frac{\partial F_1}{\partial I} = c\beta \left(\frac{\beta_2 I}{m + I} - \beta_1\right) S - \frac{c\beta\beta_2 SI}{(m + I)^2},$$

$$\frac{\partial F_1}{\partial J} = -c\beta bS;$$

$$\frac{\partial F_2}{\partial S} = \left(\beta_1 - \frac{\beta_2 I}{m + I}\right) I + c\beta bJ,$$

$$\frac{\partial F_2}{\partial I} = c\beta\beta_1 S - \frac{c\beta\beta_2 Sm}{(m + I)^2} - \mu - k_1,$$

$$\frac{\partial F_2}{\partial J} = c\beta bS + \delta;$$

$$\frac{\partial F_3}{\partial S} = 0, \quad \frac{\partial F_3}{\partial I} = k_1, \quad \frac{\partial F_3}{\partial J} = -(\mu + k_2 + \delta).$$

Substituting the partial derivatives into (2.18), we obtain representations (1.6)–(1.11) for coefficients $A_{s\kappa}$, which concludes the proof.

4. Results and Discussion

Note that if the domain diameter is small enough and $w_\kappa = const \geqslant 0$, then the quadratic form (1.12) is negatively defined, so the constant stationary solution is obviously stable in a small domain. It should also be noted that any stationary (constant) solution of system (1.1)–(1.3) without migrations (i.e., when $\vartheta_1 = \vartheta_2 = \vartheta_3 = 0$, and we are dealing with an ODE system) is a stationary solution of this system (PDE) with migrations (i.e., when $\vartheta_1^2 + \vartheta_2^2 + \vartheta_3^2 > 0$), of course, with the appropriate boundary conditions. At the same time, it may turn out that this constant solution is not asymptotically stable in the model without migration, but is asymptotically stable in a small domain if migration processes are taken into account. This can be demonstrated with a simple example. The constant point (stationary state) $(k, 0, 0)$ is a solution of system (1.1)–(1.3) for any set of diffusion coefficients. In the case of $\vartheta_1 = \vartheta_2 = \vartheta_3 = 0$, this solution is asymptotically stable as

$$\mu < k_1 - \frac{(k_1 + \delta)^2}{4(k_2 + \mu)}$$

and only under this condition. If $\vartheta_1^2 + \vartheta_2^2 + \vartheta_3^2 > 0$, then the solution $(k, 0, 0)$ with sufficiently small H will be asymptotically stable without this condition.

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