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Coherent X-ray Radiation of Relativistic Electrons in a Composite Target

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Abstract—A dynamic theory of coherent X-ray radiation generated by a beam of relativistic electrons in a composite target "amorphous layer-vacuum-periodic layered medium" has been developed. A periodic layered medium consists of three different layers arranged periodically, with the layers located at an arbitrary angle to the target surface. Coherent X-ray radiation exits through the rear surface of the target; that is, radiation in the periodic layered medium occurs in the Laue scattering geometry. Within the framework of the two-wave approximation of the dynamic theory of diffraction, expressions describing the spectral-angular densities of parametric X-ray radiation (PXR) in the periodic layered medium and diffracted transition radiation (DTR) are obtained and studied.

Keywords: coherent X-ray radiation, dynamic diffraction, periodic layered structure

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INTRODUCTION

Traditionally, the radiation of a relativistic particle in a periodic layered medium was considered resonant transition radiation (RTR) [1]. Since the intensity of transition radiation when crossing one plate is low, it was proposed to increase it using media consisting of many plates [2] and artificial periodic media [3]. Further experimental and theoretical studies of RTR are presented in [4–9]. Moreover, experimental works [8, 9], in which RTR spectra were measured with high accuracy, completely confirmed the theoretical calculations.

The radiation of a relativistic electron crossing a target made of a periodic layered medium was first considered in [10] within the framework of the dynamic theory of X-ray diffraction in a periodic layered medium. Coherent X-ray radiation (CXR) include parametric X-ray radiation (PXR) and diffracted transition radiation (DTR). PXR arises as a result of diffraction of pseudo-photons of the Coulomb field of a relativistic electron by layers of a periodic medium, similar to PXR in a single crystal on atomic planes [11–15], and DTR arises as a result of diffraction by target layers of transition radiation generated near the front surface of the target, similar to how DTR is generated in a single crystal [16, 17]. The CXR of a relativistic electron in a periodic layered medium for the general case of asymmetric reflection of the electron field relative to the target surface in the Laue scattering geometry was first examined in [18]; in the Bragg scattering geometry, in [19]. In [18, 19] the possibility of increasing the intensity of PXR and DTR was shown due to the manifestation of dynamic diffraction effects in a periodic layered medium. It should be noted that layered structures are also interesting from the point of view of generating soft X-ray radiation, which is currently being actively studied by many scientific groups [20–25]. In all the works cited above, coherent X-ray radiation of relativistic electrons in a periodic layered medium was considered in the case of two different layers per period. In [26] coherent X-ray radiation from a relativistic electron crossing a periodic layered medium with three different layers in a period in the Bragg scattering geometry was considered for the first time. The possibility of a significant increase in the intensity of PXR and DTR in a three-layer structure compared to a two-layer structure is shown. Modern technologies make it possible to produce periodic layered atomic structures with layer thicknesses of about 2 nm, although the structure itself can have a thickness of 1–5 μm. In such structures, X-ray radiation will have a short extinction length, many times shorter than the photoabsorption length. This fact indicates the possibility of manifestation of dynamic diffraction effects in coherent X-ray radiation of relativistic electrons, well known for single-crystal media. The manifestation of dynamic diffraction effects in coherent X-ray radiation of relativistic electrons in a three-layer structure was considered in [27].

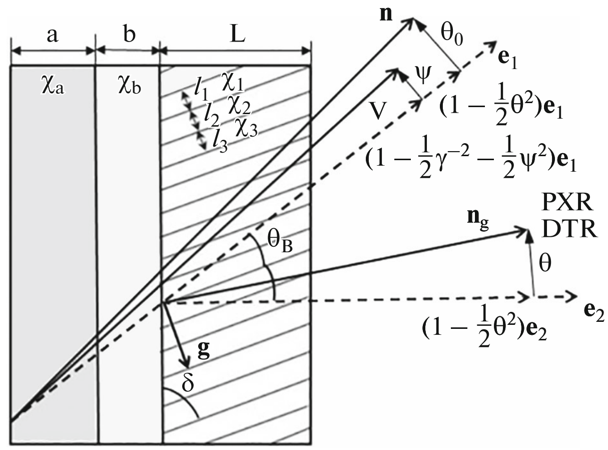

Fig. 1. Geometry of relativistic electron radiation in a composite structure.

This paper addresses coherent X-ray radiation of relativistic electrons crossing a composite target "amorphous layer-amorphous layer-periodic layered medium." Coherent X-ray radiation in a periodic layered medium with three layers in the period is examined in the Laue scattering geometry, when the radiation escapes through the rear boundary of the target. Expressions describing the amplitudes of PXR and DTR from such a structure are obtained. Next, we consider the case where the second layer is a vacuum (air). Expressions describing the spectral-angular densities of PXR and DTR and their interference were derived and investigated.

GEOMETRY OF THE RADIATION PROCESS IN COMPOSITE TARGET

Let us consider coherent X-ray radiation (CXR) of relativistic electrons crossing a three-layer composite structure consisting of two amorphous layers and a layer of periodic layered medium with three layers per period (Fig. 1). Amorphous layers have corresponding layer thicknesses a and b and dielectric susceptibilities χ_a and χ_b. The third layer consists of periodically arranged layers with thicknesses l_1, l_2, and l_3 and dielectric susceptibilities χ_1, χ_2, and χ_3 respectively. The period of the layered structure is $T = l_1 + l_2 + l_3$. The reflective layers of the periodic layered structure are located at a certain angle δ to the target surface (Fig. 1), which corresponds to the case of asymmetric reflection of the radiation field; at an angle of $\delta = \frac{\pi}{2}$ to a special case of symmetric reflection. Coherent X-ray emission from a relativistic electron in a periodic layered medium and X-ray scattering occurs in the Laue scattering geometry; that is, the X-rays emerge through the rear boundary and the target and propagate in the Bragg scattering direction.

Angular variables $\boldsymbol{\psi}$, $\boldsymbol{\theta}$, and $\boldsymbol{\theta}_0$ are introduced in expressions for the velocity of a relativistic electron $\mathbf{V}$ and unit vectors $\mathbf{n}$ and $\mathbf{n}_\mathbf{g}$:

$$\mathbf{V} = \left(1 - \frac{1}{2}\gamma^{-2} - \frac{1}{2}\psi^2\right)\mathbf{e}_1 + \boldsymbol{\psi}, \quad \mathbf{e}_1\boldsymbol{\psi} = 0,$$

$$\mathbf{n} = \left(1 - \frac{1}{2}\theta_0^2\right)\mathbf{e}_1 + \boldsymbol{\theta}_0, \quad \mathbf{e}_1\boldsymbol{\theta}_0 = 0, \quad \mathbf{e}_1\mathbf{e}_2 = \cos 2\theta_B, \quad (1)$$

$$\mathbf{n}_\mathbf{g} = \left(1 - \frac{1}{2}\theta^2\right)\mathbf{e}_2 + \boldsymbol{\theta}, \quad \mathbf{e}_2\boldsymbol{\theta} = 0,$$

where $\mathbf{n}$ is the unit vector in the direction of the photon momentum emitted near the direction of the electron velocity vector; it determines the directions of PXR radiation along the velocity of the relativistic electron (PXRV) and transition radiation (TR); $\mathbf{n}_\mathbf{g}$ is the unit vector in the direction of Bragg scattering; it determines the direction of the emitted PXR and DTR photons; $\boldsymbol{\theta}$ is the radiation angle measured from the axis of the radiation detector $\mathbf{e}_2$; $\boldsymbol{\psi}$ is the angle of deviation of the electron in question in the beam, measured from the axis of the electron beam $\mathbf{e}_1$; θ_0 is the angle between the direction of propagation of the incident photon and the axis $\mathbf{e}_1$; and $\gamma = 1/\sqrt{1 - V^2}$ is the Lorentz factor of the electron. Angular variables are considered the sum of components parallel and perpendicular to the plane of the drawing $\boldsymbol{\theta} = \boldsymbol{\theta}_\parallel + \boldsymbol{\theta}_\perp$, $\boldsymbol{\theta}_0 = \boldsymbol{\theta}_{0\parallel} + \boldsymbol{\theta}_{0\perp}$, and $\boldsymbol{\psi} = \boldsymbol{\psi}_\parallel + \boldsymbol{\psi}_\perp$. Vector $\mathbf{g}$ is similar to the reciprocal lattice vector in a crystal; it is perpendicular to the target layers and its length is equal to $g = \frac{2\pi}{T}$.

DYNAMIC SCATTERING OF CXR IN PERIODIC LAYERED MEDIUM

Let us consider the scattering of X-ray radiation in a periodic layered medium within the framework of the dynamic theory of diffraction. Let us write the equation for the Fourier transform of the electric field strength $\mathbf{E}_{\omega,\mathbf{k}}$, which follows from the system of Maxwell's equations

$$\left(k^2-\omega^2\left(1+\chi_0\right)\right)\mathbf{E}_{\omega,\mathbf{k}}-\mathbf{k}(\mathbf{k}\mathbf{E}_{\omega,\mathbf{k}}) \\ -\omega^2\sum_{\mathbf{g}}\chi'_{-\mathbf{g}}\mathbf{E}_{\omega,\mathbf{k}+\mathbf{g}}=4\pi i\omega\mathbf{j}_{\omega,\mathbf{k}}. \quad (2)$$

The Fourier transform of the electric field strength and the current density of the emitting electron have the following form: $\mathbf{E}_{\omega,\mathbf{k}}=\int dt\, d^3r\ \mathbf{E}(\mathbf{r},t)\exp\left(i\omega t-i\mathbf{k}\mathbf{r}\right)$ and $\mathbf{j}_{\omega,\mathbf{k}}=2\pi e\mathbf{V}\delta\left(\omega-\mathbf{k}\mathbf{V}\right)$.

$\chi_0(\omega)$ is the average dielectric susceptibility of a periodic layered medium, $\chi_\mathbf{g}$ and $\chi_{-\mathbf{g}}$ are Fourier coefficients of the expansion of the dielectric susceptibility in vectors $\mathbf{g}$: and $\chi(\omega,\mathbf{r})=\sum_\mathbf{g}\chi_\mathbf{g}(\omega)\exp(i\mathbf{g}\mathbf{r})$ = $\sum_g\chi_g(\omega)\exp(-igx)$, where $\chi_\mathbf{g}=\chi'_\mathbf{g}(\omega)+i\chi''_{\mathbf{g}\mathbf{g}}(\omega)$, and $g=\frac{2\pi}{T}n$ $(n=0,\pm1,\pm2,\ldots)$. Average dielectric susceptibility χ_0 in the periodic structure under consideration has the form $\chi_0(\omega)=\frac{l_1}{T}\chi_1+\frac{l_2}{T}\chi_2+\frac{l_3}{T}\chi_3$. Coefficient $\chi_\mathbf{g}$ for the layered structure under consideration takes the form

$$\chi_\mathbf{g}=\frac{1}{igT} \\ \times\left(\chi_{l_3}-\chi_{l_1}+\left(\chi_{l_1}-\chi_{l_2}\right)e^{igl_1}+\left(\chi_{l_2}-\chi_{l_3}\right)e^{-igl_3}\right). \quad (3)$$

The electromagnetic field emitted by a relativistic electron in the X-ray frequency range is practically transverse, which means that the Fourier transforms of the electric field strengths of the incident radiation $\mathbf{E}_{\omega,\mathbf{k}}$ and diffracted radiation $\mathbf{E}_{\omega,\mathbf{k}+\mathbf{g}}$ in a periodic layered medium can be represented in the form:

$$\mathbf{E}_{\omega,\mathbf{k}}=E^{(1)}_{\omega,\mathbf{k}}\mathbf{e}^{(1)}+E^{(2)}_{\omega,\mathbf{k}}\mathbf{e}^{(2)}, \\ \mathbf{E}_{\omega,\mathbf{k}+\mathbf{g}}=E^{(1)}_{\omega,\mathbf{k}+\mathbf{g}}\mathbf{e}^{(1)}_\mathbf{g}+E^{(2)}_{\omega,\mathbf{k}+\mathbf{g}}\mathbf{e}^{(2)}_\mathbf{g}, \quad (4)$$

where vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(2)}$ are perpendicular to the vector $\mathbf{k}$ and vectors $\mathbf{e}^{(1)}_\mathbf{g}$ and $\mathbf{e}^{(2)}_\mathbf{g}$ are perpendicular to the vector $\mathbf{k}_\mathbf{g}=\mathbf{k}+\mathbf{g}$. Vectors $\mathbf{e}^{(2)}$ and $\mathbf{e}^{(2)}_\mathbf{g}$ lie in the plane of vectors $\mathbf{k}$ and $\mathbf{k}_\mathbf{g}$ (π-polarization), and vectors $\mathbf{e}^{(1)}$ and $\mathbf{e}^{(1)}_\mathbf{g}$ are perpendicular to it (σ-polarization). Polarization vectors have the form:

$$\mathbf{e}^{(1)}=\mathbf{e}^{(1)}_\mathbf{g}=\frac{\left[\mathbf{k},\mathbf{g}\right]}{\left|\left[\mathbf{k},\mathbf{g}\right]\right|},\quad \mathbf{e}^{(2)}=\frac{\left[\mathbf{k},\mathbf{e}^{(1)}\right]}{k}, \\ \text{and } \mathbf{e}^{(2)}_\mathbf{g}=\frac{\left[\mathbf{k}_\mathbf{g},\mathbf{e}^{(1)}\right]}{k_\mathbf{g}}.$$

Substituting expressions (4) into Eq. (2), we obtain a system of equations:

$$\left(k^2-\omega^2\left(1+\chi_0(\omega)\right)\right)E^{(s)}_{\omega,\mathbf{k}}-\omega^2\chi_{-\mathbf{g}}(\omega)E^{(s)}_{\omega,\mathbf{k}+\mathbf{g}}C^{(s,\tau)} \\ =8\pi^2 i\omega e\,\Omega^{(s)}\delta(\omega-\mathbf{k}\mathbf{V}), \\ \left((\mathbf{k}+\mathbf{g})^2-\omega^2\left(1+\chi_0(\omega)\right)\right) \\ \times E^{(s)}_{\omega,\mathbf{k}+\mathbf{g}}-\omega^2\chi_\mathbf{g}(\omega)E^{(s)}_{\omega,\mathbf{k}}C^{(s,\tau)}=0. \quad (5)$$

In (5) the following notations were introduced

$$C^{(s,\tau)}=\mathbf{e}^{(s)}_\mathbf{g}\mathbf{e}^{(s)}=(-1)^\tau C^{(s)},\quad C^{(1)}=1,\quad C^{(2)}=\left|\cos 2\theta_\mathrm{B}\right|, \\ \Omega^{(1)}=\mathbf{e}^{(1)}\mathbf{V}=\theta_\perp-\psi_\perp,\quad \Omega^{(2)}=\mathbf{e}^{(2)}\mathbf{V}=\theta_\parallel+\psi_\parallel.$$

The dispersion equation of free X-ray waves, following from (5), has the form:

$$(k_\mathbf{g}^2-\omega^2(1+\chi_0))(k^2-\omega^2(1+\chi_0)) \\ -\omega^4\chi_\mathbf{g}\chi_{-\mathbf{g}}C^{(s)2}=0. \quad (6)$$

We will look for a solution to Eq. (6) in the form $k=\omega\sqrt{1+\chi_0}+\lambda_0$ and $k_\mathbf{g}=\omega\sqrt{1+\chi_0}+\lambda_\mathbf{g}$, where λ_0 and $\lambda_\mathbf{g}$ are dynamic additions to the lengths of wave vectors, related by the equation $\lambda_\mathbf{g}=\lambda_0\frac{\gamma_\mathbf{g}}{\gamma_0}+\frac{\omega}{2}\beta$, where $\beta=\alpha-\chi_0\left(1-\frac{\gamma_\mathbf{g}}{\gamma_0}\right)$, $\gamma_0=\cos\varphi_0$, $\gamma_\mathbf{g}=\cos\varphi_\mathbf{g}$, φ_0 and $\varphi_\mathbf{g}$ are angles between the wave vectors of the incident and diffracted waves $\mathbf{k}=k\cdot\mathbf{n}$ and $\mathbf{k}_\mathbf{g}=k_\mathbf{g}\cdot\mathbf{n}_\mathbf{g}$ and the normal vector to the surface of the plate $\mathbf{N}$, and $\alpha=\left(k_\mathbf{g}^2-k^2\right)/\omega^2$ is the Bragg resonance detuning. In the X-ray region of radiation frequencies, the quantities λ_0 and $\lambda_\mathbf{g}$ are significantly less than the frequency of emitted photons ω and the following inequalities $\lambda_0^2\ll 2\omega\lambda_0$ and $\lambda_\mathbf{g}^2\ll 2\omega\lambda_\mathbf{g}$ are satisfied. In this case, the dynamic additions to the lengths of the wave vectors take the form:

$$\lambda_0^{(1,2)}=\frac{1}{2\varepsilon L^{(s)}_\mathrm{ext}}\left(-\xi^{(s)}+\frac{i\rho^{(s)}(1-\varepsilon)}{2}\pm K^{(s)}(\xi^{(s)})\right), \quad (7)$$

$$\lambda_\mathbf{g}^{(1,2)}=\frac{1}{2L^{(s)}_\mathrm{ext}}\left(\xi^{(s)}-\frac{i\rho^{(s)}(1-\varepsilon)}{2}\pm K^{(s)}(\xi^{(s)})\right). \quad (8)$$

In expressions (7) and (8) we introduced the following notations:

$$L_{\mathrm{ext}}^{(\mathrm{s})}=\frac{\pi}{C^{(\mathrm{s})}\omega\chi'_{l_2}\sqrt{\left(1-\delta'_{l_1l_2}\right)\delta'_1\sin^2(\mathrm{I}_1\pi)+\left(\delta'_{l_1l_2}-1\right)\delta'_2\sin^2(\mathrm{I}_2\pi)+\delta'_1\delta'_2\sin^2(\mathrm{I}_3\pi)}},$$

$$K^{(\mathrm{s})}=\sqrt{\xi^{(\mathrm{s})2}+\varepsilon-i\rho^{(\mathrm{s})}((1-\varepsilon)\xi^{(\mathrm{s})}+2\kappa^{(\mathrm{s})}\varepsilon)-\rho^{(\mathrm{s})2}\left(\frac{(1-\varepsilon)^2}{4}+\kappa^{(\mathrm{s})2}\varepsilon\right)},\quad \varepsilon=\frac{\sin(\delta+\theta_{\mathrm{B}})}{\sin(\delta-\theta_{\mathrm{B}})},$$

$$\xi^{(\mathrm{s})}(\omega)=\eta^{(\mathrm{s})}(\omega)+\frac{1-\varepsilon}{2\nu^{(\mathrm{s})}},\quad \eta^{(\mathrm{s})}(\omega)=\frac{2\pi^2L_{\mathrm{ext}}^{(\mathrm{s})}}{V^2T^2\omega_{\mathrm{B}}}\left(1-\frac{\omega}{\omega_{\mathrm{B}}}\left(1-\theta_{\|}\sqrt{\frac{T^2\omega_{\mathrm{B}}^2}{\pi^2}-1}\right)\right), \tag{9}$$

$$\delta'_1=\frac{\delta'_{l_1l_2}}{\delta'_{l_1l_3}}-\delta'_{l_1l_2},\quad \delta'_2=\frac{\delta'_{l_1l_2}}{\delta'_{l_1l_3}}-1,\quad \delta'_{l_1l_2}=\frac{\chi'_{l_1}}{\chi'_{l_2}},\quad \delta'_{l_1l_3}=\frac{\chi'_{l_1}}{\chi'_{l_3}},$$

$$\mathrm{I}_1=\left(1+\left(\frac{l_1}{l_3}\right)^{-1}+\left(\frac{l_1}{l_2}\right)^{-1}\right)^{-1},\quad \mathrm{I}_2=\left(1+\frac{l_1}{l_2}+\frac{l_1}{l_2}\left(\frac{l_1}{l_3}\right)^{-1}\right)^{-1},\ \text{and}\ \mathrm{I}_3=\left(1+\frac{l_1}{l_3}+\frac{l_1}{l_3}\left(\frac{l_1}{l_2}\right)^{-1}\right)^{-1},$$

where $\xi^{(\mathrm{s})}(\omega)$ and $\eta^{(\mathrm{s})}(\omega)$ is a spectral function that changes rapidly with changing radiation frequency ω in the vicinity of the Bragg frequency ω_{B} and $L_{\mathrm{ext}}^{(\mathrm{s})}$ is the extinction length of X-ray waves in the considered periodic layered medium with three layers per period. In order not to specify the materials of the layers and their thickness, the extinction length $L_{\mathrm{ext}}^{(\mathrm{s})}$ and dynamic scattering parameters $\nu^{(\mathrm{s})}$, $\rho^{(\mathrm{s})}$, and $\kappa^{(\mathrm{s})}$ of X-ray radiation in the periodic structure under consideration with three layers per period are presented as functions of variables, which are the ratios of the layer thicknesses $\frac{l_1}{l_2}$ and $\frac{l_1}{l_3}$, as well as the ratio of the real parts of the dielectric susceptibilities of the layer substances $\delta'_{l_1l_2}=\frac{\chi'_{l_1}}{\chi'_{l_2}}$ and $\delta'_{l_1l_3}=\frac{\chi'_{l_1}}{\chi'_{l_3}}$. To perform calculations and analyze the dependence of the spectral-angular characteristics of radiation on the parameters of the layers, it will be sufficient to specify the period of the layered structure T and the real part of the dielectric susceptibility χ'_{l_2} of the second layer. The obtained parameters of dynamic scattering are as follows:

$$\nu^{(\mathrm{s})}=\frac{C^{(\mathrm{s})}}{\pi}\frac{\sqrt{\left(1-\delta'_{l_1l_2}\right)\delta'_1\sin^2(\mathrm{I}_1\pi)+\left(\delta'_{l_1l_2}-1\right)\delta'_2\sin^2(\mathrm{I}_2\pi)+\delta'_1\delta'_2\sin^2(\mathrm{I}_3\pi)}}{I_2\left|\frac{l_1}{l_2}\delta'_{l_1l_2}+1+\frac{l_1}{l_2}\left(\frac{l_1}{l_3}\right)^{-1}\frac{\delta'_{l_1l_2}}{\delta'_{l_1l_3}}\right|}, \tag{10}$$

$$\rho^{(\mathrm{s})}=\frac{\pi}{C^{(\mathrm{s})}}\frac{\mathrm{I}_2\left|\frac{l_1}{l_2}\rho_{l_1}+\rho_{l_2}+\frac{l_1}{l_2}\left(\frac{l_1}{l_3}\right)^{-1}\rho_{l_3}\right|}{\sqrt{\left(1-\delta'_{l_1l_2}\right)\delta'_1\sin^2(\mathrm{I}_1\pi)+\left(\delta'_{l_1l_2}-1\right)\delta'_2\sin^2(\mathrm{I}_2\pi)+\delta'_1\delta'_2\sin^2(\mathrm{I}_3\pi)}}, \tag{11}$$

$$\kappa^{(\mathrm{s})}=\frac{C^{(\mathrm{s})}}{\pi}\frac{\sqrt{\left(\rho_1\sin^2(\mathrm{I}_1\pi)+\rho_2\sin^2(\mathrm{I}_2\pi)+\rho_3\sin^2(\mathrm{I}_3\pi)\right)}}{\mathrm{I}_2\left|\frac{l_1}{l_2}\rho_{l_1}+\rho_{l_2}+\frac{l_1}{l_2}\left(\frac{l_1}{l_3}\right)^{-1}\rho_{l_3}\right|}, \tag{12}$$

where we introduced the following notations:

$$\rho_{l_1} = \frac{\chi''_{l_1}}{\left|\chi'_{l_2}\right|},\ \ \rho_{l_2} = \frac{\chi''_{l_2}}{\left|\chi'_{l_2}\right|},\ \ \rho_{l_3} = \frac{\chi''_{l_3}}{\left|\chi'_{l_2}\right|},$$
$$\rho_1 = \left(\rho_{l_1} - \rho_{l_2}\right)\left(\rho_{l_1} - \rho_{l_3}\right),\ \rho_2 = \left(\rho_{l_2} - \rho_{l_1}\right)\left(\rho_{l_2} - \rho_{l_3}\right),$$
$$\text{and } \rho_3 = \left(\rho_{l_3} - \rho_{l_1}\right)\left(\rho_{l_3} - \rho_{l_2}\right). \quad (13)$$

Let us consider the physical meaning of the parameters in (11), which determine the process of propagation of X-ray radiation in a periodic layered medium. $\xi^{(s)}(\omega)$ and $\eta^{(s)}(\omega)$ are spectral functions that change rapidly with changing radiation frequency ω in the vicinity of the Bragg frequency ω_B. Parameter $\nu^{(s)}$, which characterizes the reflection of electromagnetic waves from the layered structure of the target, can take values from the interval $0 \le \nu^{(s)} \le 1$. The value of parameter $\nu^{(s)}$ shows the degree of interference of X-ray waves reflected from different layers over the period of the target under consideration. If $\nu^{(s)} \approx 1$, then interference is most constructive, and when $\nu^{(s)} \approx 0$ interference is most destructive. Parameter $\rho^{(s)}$ determines the degree of photoabsorption of X-ray radiation in a layered medium. Photoabsorption of radiation in the target layers is determined by the relations ρ_{l_1}, ρ_{l_2}, and ρ_{l_3}. The lower the value of the parameter $\rho^{(s)}$, the smaller the photoabsorption of X-ray radiation. This parameter can be represented as the ratio $\rho^{(s)} = \frac{L^{(s)}_{\text{ext}}}{L_{\text{abs}}}$ of X-ray extinction lengths $L^{(s)}_{\text{ext}}$ in a layered medium to the average length of its photoabsorption $L_{\text{abs}} = T/\omega\left(l_1\chi''_{l_1} + l_2\chi''_{l_2} + l_3\chi''_{l_3}\right)$.

The value of parameter $\kappa^{(s)}$ determines the location in a layered medium of the antinodes of a standing wave, which is formed as a result of the interference of incident and diffracted waves. Parameter $\kappa^{(s)}$ takes values from the interval $0 \le \kappa^{(s)} \le 1$. If the maxima of the antinodes lie on a layer with a higher electron density, then the value of the parameter $\kappa^{(s)}$ is closer to zero. If the antinode maxima lie on a layer with a lower electron density (less photoabsorption of X-ray radiation), then the value of the parameter $\kappa^{(s)}$ is closer to one.

Parameter ε determines the asymmetry of the reflection of the electron field and X-ray radiation relative to the target surface. At fixed θ_B parameter ε determines the angle between the target surface and the reflective layers δ. In the case of symmetrical reflection of the electron field and X-ray waves relative to target surface when the reflective layers and the target surface are perpendicular $\left(\delta = \frac{\pi}{2}\right)$, the asymmetry parameter is equal to one $\varepsilon = 1$.

AMPLITUDE OF COHERENT RADIATION IN COMPOSITE STRUCTURE

The solution of the first Eq. (5) for the electric Coulomb field strength of an electron in a vacuum ($\chi_{-\mathbf{g}} = \chi_0 = 0$) in front of the target looks like

$$E^{(s)\,\text{VAC}}_{\omega,\mathbf{k}} = \frac{8\pi^2 i\omega e\Omega^{(s)}\delta(\omega - \mathbf{kV})}{\left(k^2 - \omega^2\right)}. \quad (14)$$

Since the lengths of the wave vectors of the incident and diffracted photons in a periodic layered medium are equal to $k = \omega\sqrt{1+\chi_0} + \lambda_0$ and $k_{\mathbf{g}} = \omega\sqrt{1+\chi_0} + \lambda_{\mathbf{g}}$, for further analysis we will consider $\lambda_{\mathbf{g}}$ to be a variable, according to which we will further integrate when applying boundary conditions. In this case, expression (14) takes the form:

$$E^{(s)\,\text{VAC}}_{\omega,\mathbf{k}} = \frac{8\pi^2 ie\Omega^{(s)}}{\omega} \times \frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\chi_0(\omega) + \frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}} - \beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)}\delta\left(\lambda^*_{\mathbf{g}} - \lambda_{\mathbf{g}}\right), \quad (15)$$

where

$$\delta(\omega - \mathbf{kV}) = \delta(\lambda^*_0 - \lambda_0) = \frac{\gamma_{\mathbf{g}}}{\gamma_0}\delta(\lambda^*_{\mathbf{g}} - \lambda_{\mathbf{g}}),$$
$$\lambda^*_0 = \omega\left(\frac{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2 - \chi_0}{2}\right), \quad (16)$$
$$\lambda^*_{\mathbf{g}} = \frac{\omega\beta}{2} + \frac{\gamma_{\mathbf{g}}}{\gamma_0}\lambda^*_0, \text{ and } \lambda_{\mathbf{g}} = \lambda_0\frac{\gamma_{\mathbf{g}}}{\gamma_0} + \frac{\omega}{2}\beta.$$

In amorphous media, the field consists of the Coulomb field of the electron and the field of emitted free photons of transition radiation with intensities $E^{(s)}_a$ and $E^{(s)}_b$

$$E^{(s)\,\text{m}}_{a0} = \frac{8\pi^2 ie\Omega^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\chi_0 - \chi_a + \frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}} - \beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)} \times \delta\left(\lambda_{\mathbf{g}} - \lambda^*_{\mathbf{g}}\right) + E^{(s)}_a\delta\left(\lambda_{\mathbf{g}} - \lambda'_{\mathbf{g}a}\right), \quad (17)$$

$$E^{(s)\,\text{m}}_{b0} = \frac{8\pi^2 ie\Omega^{(s)}}{\omega}\frac{1}{\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\chi_0 - \chi_b + \frac{2}{\omega}\frac{\gamma_0}{\gamma_{\mathbf{g}}}\lambda_{\mathbf{g}} - \beta\frac{\gamma_0}{\gamma_{\mathbf{g}}}\right)} \times \delta\left(\lambda_{\mathbf{g}} - \lambda^*_{\mathbf{g}}\right) + E^{(s)}_a\delta\left(\lambda_{\mathbf{g}} - \lambda'_{\mathbf{g}b}\right), \quad (18)$$

where

$$\lambda'_{\mathbf{g}a} = \lambda^*_{\mathbf{g}} - \frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\| + \psi_\|)^2 - \chi_a}{2}\right),$$

$$\lambda'_{\mathbf{g}b} = \lambda^*_{\mathbf{g}} - \frac{\gamma_{\mathbf{g}}}{\gamma_0}\omega\left(\frac{\gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\| + \psi_\|)^2 - \chi_b}{2}\right).$$

In the third layer consisting of a periodic layered medium, the electromagnetic field consists of the Coulomb field of a relativistic electron and the fields of two free X-ray waves propagating in the periodic structure. For the incident and diffracted waves, we can write the Fourier transform of the electric field strength:

$$E_0^{(s)\,\mathrm{plm}} = \frac{8\pi^2 ie\Omega^{(s)}}{\omega}\frac{\omega^2\beta + 2\omega\frac{\gamma_{\mathrm{g}}}{\gamma_0}\lambda_0}{4\frac{\gamma_{\mathrm{g}}}{\gamma_0}\left(\lambda_0 - \lambda_0^{(1)}\right)\left(\lambda_0 - \lambda_0^{(2)}\right)} \times \delta\left(\lambda_0 - \lambda_0^*\right) + E_0^{(s)^{(1)}}\delta(\lambda_0 - \lambda_0^{(1)}) + E_0^{(s)^{(2)}}\delta(\lambda_0 - \lambda_0^{(2)}), \quad (19)$$

$$E_{\omega,\mathbf{k}_{\mathbf{g}}}^{(s)\,\mathrm{plm}} = -\frac{8\pi^2 ie\Omega^{(s)}}{\omega}\frac{\omega^2\chi_{\mathbf{g}}(\omega)C^{(s,\tau)}\delta(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}})}{4\frac{\gamma_0^2}{\gamma_{\mathbf{g}}^2}\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(1)}\right)\left(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(2)}\right)} + E_{\mathbf{g}}^{(s)(1)}\delta(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(1)}) + E_{\mathbf{g}}^{(s)(2)}\delta(\lambda_{\mathbf{g}} - \lambda_{\mathbf{g}}^{(2)}). \quad (20)$$

The radiated field in the vacuum behind the target in the direction of Bragg scattering will have the following form: $E_{\mathbf{g}}^{(s)\,\mathrm{VAC}\,II} = E_{\mathbf{g}}^{(s)\mathrm{Rad}}\delta\left(\lambda_{\mathbf{g}} + \omega\frac{\chi_0}{2}\right)$.

To determine the amplitude of the radiation field $E_{\mathbf{g}}^{(s)\mathrm{Rad}}$ we will use the boundary conditions on the four boundaries of the three-layer target under consideration

$$\begin{gathered}
\int E_{\omega,\mathbf{k}}^{(s)\,\mathrm{VAC}} d\lambda_{\mathbf{g}} = \int E_{a0}^{(s)\,\mathrm{m}} d\lambda_{\mathbf{g}}, \\
\int E_{a0}^{(s)\,\mathrm{m}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}a} d\lambda_{\mathbf{g}} = \int E_{b0}^{(s)\,\mathrm{m}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}a} d\lambda_{\mathbf{g}}, \\
\int E_{b0}^{(s)\,\mathrm{m}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+b)} d\lambda_{\mathbf{g}} = \int E_0^{(s)\,\mathrm{plm}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+b)} d\lambda_{\mathbf{g}}, \quad (21) \\
\int E_{\mathbf{g}}^{(s)\,\mathrm{plm}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+b)} d\lambda_{\mathbf{g}} = 0, \\
\int E_{\mathbf{g}}^{(s)\,\mathrm{plm}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+b+L)} d\lambda_{\mathbf{g}} = \int E_{\mathbf{g}}^{(s)\,\mathrm{VAC\,II}} e^{i\frac{\lambda_{\mathbf{g}}}{\gamma_{\mathbf{g}}}(a+b+L)} d\lambda_{\mathbf{g}}.
\end{gathered}$$

The resulting expression, describing the radiation amplitude $E_{\mathbf{g}}^{(s)\mathrm{Rad}}$ in the direction of Bragg scattering of an electron crossing a three-layer structure at a constant speed, we will represent as the sum of the amplitudes of the diffracted transition radiation and parametric X-ray radiation

$$E_{\mathbf{g}}^{(s)\mathrm{Rad}} = E_{\mathrm{DTR}}^{(s)} + E_{\mathrm{PXR}}^{(s)}, \quad (22)$$

$$\begin{aligned}
E_{\mathrm{DTR}}^{(s)} = {} & \frac{8\pi^2 ie\Omega^{(s)}}{\omega} e^{i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{(a+b+L)}{\gamma_g}} \times \frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)}\right)}\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L} - e^{i\frac{\lambda_{\mathbf{g}}^{(2)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L}\right) \\
& \times \left[\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_a}\right)e^{-i\frac{\omega a}{2\gamma_0}(\Gamma - \chi_a) - i\frac{\omega b}{2\gamma_0}(\Gamma - \chi_b)} + \left(\frac{1}{\Gamma - \chi_a} - \frac{1}{\Gamma - \chi_b}\right)e^{-i\frac{\omega b}{2\gamma_0}(\Gamma - \chi_b)} + \frac{1}{\Gamma - \chi_b} - \frac{1}{\Gamma - \chi_0}\right],
\end{aligned} \quad (23)$$

$$\begin{aligned}
E_{\mathrm{PXR}}^{(s)} = {} & \frac{8\pi^2 ie\Omega^{(s)}}{\omega} e^{i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{(a+b+L)}{\gamma_g}} \frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)}\right)} \\
& \times \left[\left(\frac{1}{\Gamma - \chi_0} - \frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(1)}\right)}\right)\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L} - 1\right) - \left(\frac{1}{\Gamma - \chi_0} - \frac{\omega}{2\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^* - \lambda_{\mathbf{g}}^{(2)}\right)}\right)\left(e^{i\frac{\lambda_{\mathbf{g}}^{(2)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L} - 1\right)\right],
\end{aligned} \quad (24)$$

where $\Gamma = \gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\| + \psi_\|)^2$.

Since PXR is generated only in the third layer, its amplitude (24) does not depend on the characteristics of amorphous media and on the thicknesses of the first two layers; however, the interference of PXR and DTR significantly depends on the characteristics of the first two layers.

We will consider a special case when the second layer is a vacuum ($\chi_b = 0$). In this case, expression (23) takes the following form:

$$\begin{aligned}
E_{\mathrm{DTR}}^{(s)} = {} & \frac{8\pi^2 ie\Omega^{(s)}}{\omega} e^{i\left(\frac{\omega\chi_0}{2} + \lambda_g^*\right)\frac{(a+b+L)}{\gamma_g}} \times \frac{\omega^2\chi_{\mathbf{g}}C^{(s)}}{2\omega\frac{\gamma_0}{\gamma_{\mathbf{g}}}\left(\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^{(2)}\right)}\left(e^{i\frac{\lambda_{\mathbf{g}}^{(1)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L} - e^{i\frac{\lambda_{\mathbf{g}}^{(2)} - \lambda_{\mathbf{g}}^*}{\gamma_{\mathbf{g}}}L}\right) \\
& \times \left[\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_a}\right)e^{-i\frac{\omega b}{2\gamma_0}\Gamma}\left(e^{-i\frac{\omega a}{2\gamma_0}(\Gamma - \chi_a)} - 1\right) + \frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_0}\right].
\end{aligned} \quad (25)$$

In expression (25), which describes the diffracted transition radiation in the direction of Bragg scattering, the first term in square brackets corresponds to the transition radiation generated in the amorphous layer and the second corresponds to the PI generated on the input surface in the third layer of the periodic layered medium. The explicit separation of these terms makes it possible to study the interference of these transition radiations.

SPECTRAL-ANGULAR DENSITIES OF DTR AND PXR

Let us consider coherent X-ray radiation in the case of the absence of photoabsorption of X-ray radiation by the medium. Let us substitute the amplitude (25) into the well-known expression for the spectral-angular density of X-ray radiation

$$\omega\frac{d^3N}{d\omega d\theta_\perp d\theta_\parallel} = \omega^2(2\pi)^{-6}\sum_{s=1}^{2}\left|E_{\mathbf{g}}^{(s)\mathrm{Rad}}\right|^2, \tag{26}$$

we obtain an expression describing the spectral-angular density of DTR in the considered three-layer target:

$$\omega\frac{d^3N_{\mathrm{DRT}}^{(s)}}{d\omega d\theta_\perp d\theta_\parallel} = F_{\mathrm{DRT}}^{(s)} = F_1^{(s)} + F_2^{(s)} + F_{\mathrm{INT}}^{(s)}, \tag{27}$$

$$F_1^{(s)} = \frac{e^2}{\pi^2}\Omega^{(s)2}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_a'}\right)^2 \times \sin^2\left(\frac{\omega a}{4\sin(\delta - \theta_B)}\left(\Gamma - \chi_a'\right)\right)R_{\mathrm{DRT}}^{(s)}, \tag{28}$$

$$F_2^{(s)} = \frac{e^2}{4\pi^2}\Omega^{(s)2}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_0'}\right)^2 R_{\mathrm{DRT}}^{(s)}, \tag{29}$$

$$F_{\mathrm{INT}}^{(s)} = -\frac{e^2}{2\pi^2}\Omega^{(s)2}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_0'}\right)\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_a'}\right) \times\left[\cos\left(\frac{\omega b}{2\sin(\delta - \theta_B)}\Gamma\right) - \cos\frac{\omega b}{2\sin(\delta - \theta_B)}\Gamma + \frac{\omega a}{2\sin(\delta - \theta_B)}\left(\Gamma - \chi_a'\right)\right)\right]R_{\mathrm{DRT}}^{(s)}, \tag{30}$$

$$R_{\mathrm{DRT}}^{(s)} = \frac{4\varepsilon^2}{\xi^{(s)}(\omega)^2 + \varepsilon}\sin^2\left(\frac{B^{(s)}\sqrt{\xi^{(s)}(\omega)^2 + \varepsilon}}{\varepsilon}\right), \tag{31}$$

$$\omega\frac{d^3N_{\mathrm{PXR}}^{(s)}}{d\omega d\theta_\perp d\theta_\parallel} = \frac{e^2}{\pi^2}\frac{\Omega^{(s)2}}{\left(\Gamma - \chi_0'\right)^2}\left(1 - \frac{\xi^{(s)}(\omega)}{\sqrt{\xi^{(s)}(\omega)^2 + \varepsilon}}\right)^2 \times\frac{\sin^2\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)} + \frac{\xi^{(s)}(\omega) - \sqrt{\xi^{(s)}(\omega)^2 + \varepsilon}}{\varepsilon}\right)\right)}{\left(\sigma^{(s)} + \frac{\xi^{(s)}(\omega) - \sqrt{\xi^{(s)}(\omega)^2 + \varepsilon}}{\varepsilon}\right)^2},$$

$$\Gamma = \gamma^{-2} + (\theta_\perp - \psi_\perp)^2 + (\theta_\parallel + \psi_\parallel)^2, \tag{32}$$

$$B^{(s)} = \frac{1}{2\sin(\delta - \theta_B)}\frac{L}{L_{\mathrm{ext}}^{(s)}},\quad \sigma^{(s)} = \omega L_{\mathrm{ext}}^{(s)}\left(\Gamma - \chi_0'\right),$$

$$\xi^{(s)}(\omega) = \eta^{(s)}(\omega) + \frac{1 - \varepsilon}{2\nu^{(s)}}.$$

Functions $F_1^{(s)}$ and $F_2^{(s)}$ describe the spectral-angular densities of DTR corresponding to the waves of transition radiation generated in the amorphous layer and at the front boundary of the periodic medium layer, respectively, and the function $F_{\mathrm{INT}}^{(s)}$ describes the interference of these waves.

Constructive interference of transition radiation from different boundaries of the first amorphous layer can significantly increase the spectral-angular density of DTR; its condition following from (28) is the ratio

$$\frac{\omega a}{4\sin(\delta - \theta_B)}\left(\Gamma - \chi_a'\right) = (2n + 1)\frac{\pi}{2},\ (n = 0, 1, 2, \ldots). \tag{33}$$

Additionally, the spectral-angular density of DTR can be increased due to constructive interference of transition radiation waves from the amorphous layer and the front boundary of the crystalline layer, the condition for which following from (2.11d) has the form:

$$\frac{\omega b}{2\sin(\delta - \theta_B)}\Gamma = (2m + 1)\pi,\ (m = 0, 1, 2, \ldots). \tag{34}$$

It can be shown that when $\left|\chi_0'\right| > \left|\chi_a'\right|$ interference term $F_{\mathrm{INT}}^{(s)}$ may exceed the contribution of each TR to the total DTR output. In the special case when $\chi_0' = \chi_a'$ from (27) under conditions (21) and (22) we arrive at the expression

$$\omega\frac{d^3N_{\mathrm{DRT}}^{(s)}}{d\omega d\theta_\perp d\theta_\parallel} = 9\frac{e^2}{4\pi^2}\Omega^{(s)2}\left(\frac{1}{\Gamma} - \frac{1}{\Gamma - \chi_0'}\right)^2 R_{\mathrm{DRT}}^{(s)}, \tag{35}$$

that is, under the conditions under consideration, the DTR from the three-layer structure under consideration is 9 times higher than the DTR from one plate of a periodic layered medium.

Using (24) and (25), we obtain an expression describing the interference of the DTR and PXR radiation mechanisms.

$$\omega\frac{d^3N_{\text{INT}}^{(s)}}{d\omega d\theta_\perp d\theta_\parallel}=\frac{e^2}{4\pi^2}\frac{\Omega^{(s)2}}{\Gamma-\chi_0'}\times\left[\left(\frac{1}{\Gamma-\chi_a'}-\frac{1}{\Gamma}\right)R_{\text{INT}}^{(s)(1)}+\left(\frac{1}{\Gamma-\chi_0'}-\frac{1}{\Gamma}\right)R_{\text{INT}}^{(s)(2)}\right],\quad(36)$$

$$R_{\text{INT}}^{(s)(1)}=-8\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\xi^{(s)2}+\varepsilon}\times\sin\left(\frac{\omega a}{4\sin(\delta-\theta_B)}\left(\Gamma-\chi_a'\right)\right)\times\sin\left(\frac{B^{(s)}\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\sin\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right)\times\sin\left(\frac{\omega b}{2\sin(\delta-\theta_B)}\Gamma+\frac{\omega a}{4\sin(\delta-\theta_B)}\left(\Gamma-\chi_a'\right)+\frac{B^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi+\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right),\quad(37)$$

$$R_{\text{INT}}^{(s)(2)}=4\varepsilon\frac{\xi^{(s)}-\sqrt{\xi^{(s)2}+\varepsilon}}{\xi^{(s)2}+\varepsilon}\times\sin\left(B^{(s)}\frac{\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\sin\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi-\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right)\times\cos\left(\frac{B^{(s)}}{2}\left(\sigma^{(s)}+\frac{\xi+\sqrt{\xi^2+\varepsilon}}{\varepsilon}\right)\right),\quad(38)$$

where $R_{\text{INT}}^{(s)(1)}$ and $R_{\text{INT}}^{(s)(2)}$ interference spectral functions describing, respectively, the interference of PXR and DTR from the amorphous layer and the interference of PXR and DTR from the front boundary of the layer from the layered structure.

The obtained expressions (27), (32), and (36) describing the spectral-angular densities of DTR and PXR and their interference are the main result of this chapter. These expressions take into account all possible interference effects, as well as effects associated with the asymmetry of reflection (parameter ε). These expressions can be used to analyze the spectral-angular characteristics of the radiation of a relativistic electron in the three-layer structure under consideration depending on the parameters of the layers included in the target and the energy of the emitting electrons.

NUMERICAL CALCULATIONS

Using the expressions obtained in this work for the spectral-angular densities of PXR and DTR and their interference, we will carry out numerical calculations for an example. Let us assume that a relativistic electron with the Lorentz factor $\gamma = 500$ intersects the structure of amorphous layer-vacuum-periodic layered medium. The layered structure consists of periodically arranged layers of silicon Si, carbon C, and tungsten W with thicknesses of respectively $l_1 = l_2 = l_3 = T/3$, where $T = 2$ nm. Thickness of the layered structure is $L = 2$ μm. Let us set the angle between the axis of the beam of relativistic electrons and the reflective layers (Bragg angle) $\theta_B = 2.25°$, while the Bragg frequency is $\omega_B = 8$ keV. In this case, we will consider an electron moving along the beam axis, then we will assume $\psi_\perp = \psi_\parallel = 0$. For the radiation frequency under consideration, the real part of the dielectric susceptibility of silicon, carbon, and tungsten have the values $\chi_{\text{Si}}' = -1.53\times10^{-5}$, $\chi_{\text{C}}' = -2.25\times10^{-5}$, and $\chi_{\text{W}}' = -9.6\times10^{-5}$. Let us choose tungsten W as the thickness of the first layer $a = 2.1$ μm and air layer thickness is $b = 2.7$ μm. These thicknesses are calculated in such a way that, at the Bragg frequency under consideration, conditions (33) and (34) of the constructive interference of transition radiation from the first and second boundaries of the first tungsten layer and the interference of TR from the first layer and TR from the front boundary of the third layer are satisfied. We will choose the angle between the target surface and the reflective layers $\delta = \pi/6$. The asymmetry parameter in this case is $\varepsilon = 1.15$. We will perform calculations for σ-polarized waves ($s = 1$).

Figure 2 shows curves constructed using formulas (27)–(31) describing the contribution of TR from the first amorphous layer $F_1^{(1)}$ and the contribution of the TR generated at the front boundary of the third layer of a periodic layered medium $F_2^{(1)}$, as well as their

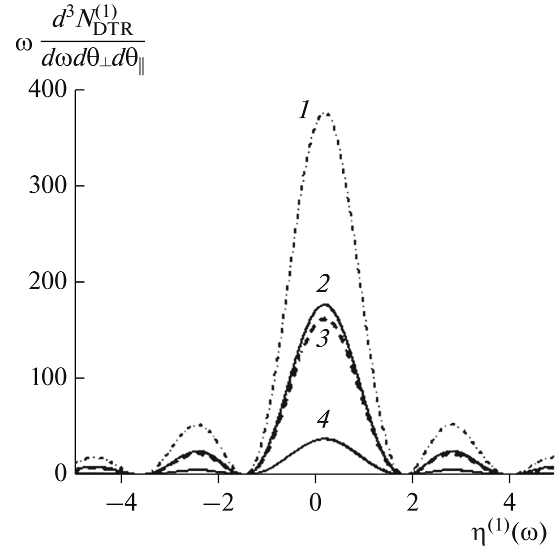

Fig. 2. Spectral-angular density of DTR at a fixed observation angle under conditions of constructive interference (33) and (34). Viewing angle: $\theta_\perp = 2$ mrad and $\theta_\parallel = 0$. Designations: (*1*) $F^{(1)}$; (*2*) $F_1^{(1)}$; (*3*) $F_{\text{INT}}^{(1)}$; and (*4*) $F_2^{(1)}$.

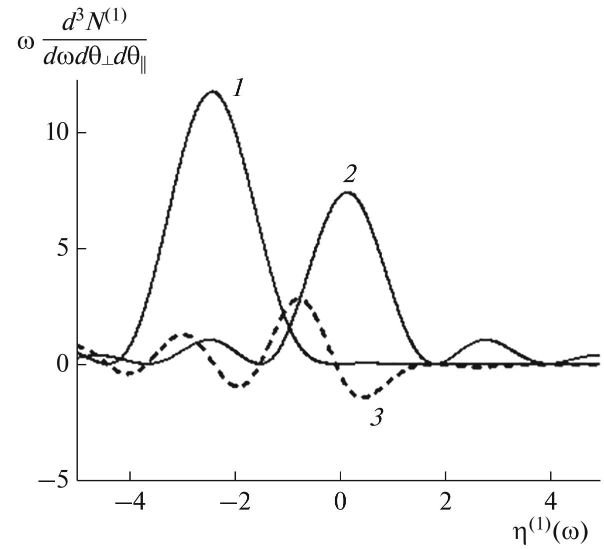

Fig. 3. Spectral-angular densities of PXR and DTR and their interference. Viewing angle: $\theta_\perp = 7$ mrad and $\theta_\parallel = 0$. Designations: (*1*) $\omega\frac{d^3N^{(1)}_{\rm PXR}}{d\omega d\theta_\perp d\theta_\parallel}$; (*2*) $\omega\frac{d^3N^{(1)}_{\rm DTR}}{d\omega d\theta_\perp d\theta_\parallel}$; and (*3*) $\omega\frac{d^3N^{(1)}_{\rm INT}}{d\omega d\theta_\perp d\theta_\parallel}$.

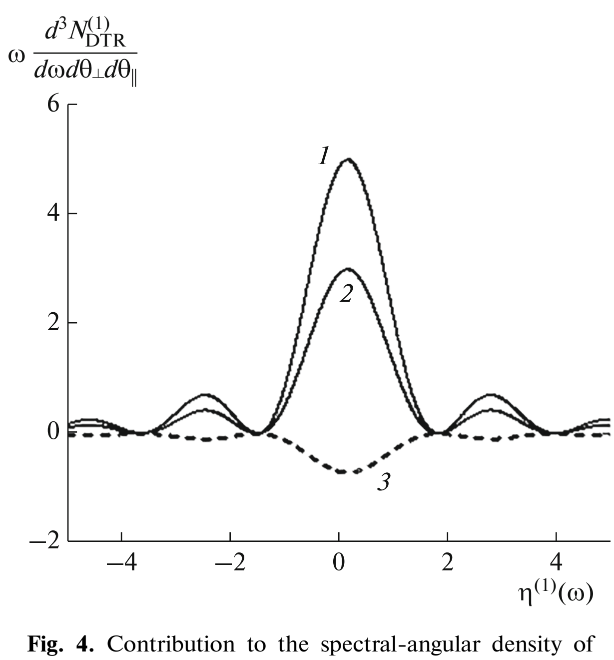

Fig. 4. Contribution to the spectral-angular density of DTR transition radiation from the first layer $F_1^{(s)}$ and the front boundary of the third layer $F_2^{(s)}$ and their interference $F_{\rm INT}^{(s)}$. Designations: (*1*) $F_1^{(1)}$; (*2*) $F_2^{(1)}$; and (*3*) $F_{\rm INT}^{(1)}$.

interference $F_{\rm INT}^{(1)}$, to the total spectral-angular density of DTR. The curves are plotted at a fixed viewing angle. $\theta_\perp = 2$ mrad and $\theta_\parallel = 0$ corresponding to the maximum angular density of DTR. From Fig. 2 it follows that in the conditions under consideration the main contribution to the total DTR is given by the transition radiation from the first layer $F_1^{(1)}$ and interference $F_{\rm INT}^{(1)}$. It should be noted that if there were only one third layer of the periodic layered medium, then the spectral-angular density of DTR would be curve $F_2^{(1)}$. Thus, the structure under consideration can increase the spectral-angular density of DTR many times, which will make it possible to obtain intense beams of X-ray radiation even at low energies of relativistic electrons.

We will also provide an example of calculations of the spectral-angular density of PXR, DTR and their interference. Figure 3 shows curves constructed using formulas (27), (32), and (36), describing the spectral-angular densities of PXR and DTR and their interference at a fixed observation angle $\theta_\perp = 7$ mrad and $\theta_\parallel = 0$, which corresponds to the maximum angular density of PXR. Figure 4 shows the contributions of transition radiations and their interference for the transition radiation curve shown in Fig. 3.

CONCLUSIONS

The paper develops a theory of coherent X-ray radiation of relativistic electrons in a composite structure consisting of two amorphous layers and a layer of a periodic layered medium. Within the framework of the two-wave approximation of the dynamic theory of X-ray diffraction in a periodic layered medium, expressions are obtained that describe the amplitude of the Fourier transform of the electric field of radiation, which is presented as the sum of PXR and DTR amplitudes. Next, we consider the case where the second layer is a vacuum. Expressions are obtained that describe the spectral-angular densities of diffracted transition radiation and parametric X-ray radiation and their interference.

It is shown that the contribution to the total DTR is formed from transition radiation generated in the amorphous layer and at the front boundary of the periodic layered structure, which under certain conditions can interfere both constructively and destructively. The transition radiation is then reflected in the layered medium in the direction of Bragg scattering, forming DTR. Thus, it is possible to increase or decrease the contribution of DTR to the total coherent X-ray radiation due to constructive or destructive interference of transition radiation.

The possibility of a significant increase in the spectral-angular density of DTR due to the total contribution of transition radiation waves from the amorphous layer and the front boundary of the periodic layered structure and their constructive interference is shown.

The possibility of the existence of interference between PXR and DTR in the considered three-layer structure is demonstrated.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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