ISSN 0021-8944, Journal of Applied Mechanics and Technical Physics, 2025, Vol. 66, No. 1, pp. 80–88. © Pleiades Publishing, Ltd., 2025.
Russian Text © The Author(s), 2025, published in Prikladnaya Mekhanika i Tekhnicheskaya Fizika, 2025, Vol. 66, No. 1, pp. 93–103.

Theory of Thermophoresis of a Large Evaporating Droplet in a Viscous Nonisothermal Binary Gaseous Medium

N. V. Malai[a,*], **P. V. Sokhan'**[a,**], **and Yu. I. Shostak**[a,***]
[a] *Belgorod National Research University, Belgorod, Russia*
**e-mail: malay@bsuedu.ru*
***e-mail: sokhanp95@gmail.com*
****e-mail: juliashostak@mail.ru*

Received April 4, 2024; revised May 5, 2024; accepted June 3, 2024

Abstract—Thermophoretic motion of a spherical evaporating droplet in a viscous binary gas medium at arbitrary relative temperature differences in its vicinity is theoretically described in a quasistationary approximation at low Reynolds and Peclet numbers. A system of gas-dynamic equations is solved, including a velocity-linearized system of Navier–Stokes equations, as well as heat and mass transfer equations. The properties of a gaseous medium are described with account for the power-law dependence of the transfer (viscosity, diffusion, and thermal conductivity) and density coefficients on temperature. The resulting numerical estimates suggest that the dependences of the thermophoretic force and the velocity of the droplet on the average temperature of its surface are nonlinear.

Keywords: gas thermophoresis, thermophoresis of evaporating droplet, linearized Navier–Stokes equation, binary mixture

DOI: 10.1134/S0021894425010092

INTRODUCTION

In gaseous media with nonuniform temperature distribution, there is an ordered motion of airborne particles due to the action of molecular interaction forces. The motion of airborne particles in a gaseous medium because of the presence of a temperature gradient therein is known as thermophoresis [1–3]. If the key mechanism of thermophoresis is thermal sliding, then it is a fair statement that the action of a thermophoretic force causes the particles to move from a higher-temperature region to a lower-temperature region.

Thermophoresis causes great interest due to the fact that this phenomenon has a significant effect on the process of particle sedimentation in heat and mass exchangers and can be used (1) in fine cleaning of small volumes of gases, collecting aerosol samples and applying special coatings of a given thickness, as well as (2) in the design of experimental devices in which it is necessary to ensure and regulate the directed motion of aerosol particles.

Moreover, the particles involved in thermophoresis may have a variety of characteristics: their surface can have various shapes (spherical, cylindrical, spheroidal, etc.); their form can be solid and liquid; their composition can be heterogeneous; also, these particles can possess anisotropy of thermophysical properties and have a phase transition on their surface.

Despite the fact that thermophoresis was discovered at the beginning of the 20th century [1], interest in this phenomenon is constantly growing and new fields of its application are emerging [4–8].

The description of the thermophoresis process is a complex mathematical problem. This is due to the fact that this particle motion in gases is described by a nonlinear system of gas-dynamic equations. In addition, the particle motion is affected by processes occurring both on the surface (during direct interaction of the molecules of the gaseous medium with the particle surface) and in the volume due to heterogeneity of the hydrodynamic, temperature, and concentration fields.

In previously published works, the theory of thermophoresis of evaporating droplets was mainly based on small relative temperature differences in their vicinity (see, e.g., [2–4]). If T_{iS} denotes the average surface temperature of an evaporating droplet and T_∞ denotes the temperature at a long distance from it, then the following inequality is obtained: $(T_{iS} - T_\infty)/T_\infty \ll 1$. When this condition is fulfilled, the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion) can be considered as constant, which significantly simplifies the solution to the system of gas-dynamic equations describing thermophoresis.

In the present paper, the theory of thermophoresis of an evaporating droplet is based on arbitrary relative temperature differences in its vicinity. There are a few works that consider the case $(T_{iS} - T_\infty)/T_\infty \geq 1$. In this case, it is necessary to account for the dependence of the transfer coefficients on temperature; here the gaseous medium is called nonisothermal (gravity-induced motion of heated large solid particles [9], as well as photo-, thermo-, and diffusiophoresis of heated large solid particles [10–12]). As shown in [9–12], heating the surface of solid particles has a significant effect on their motion.

1. PROBLEM STATEMENT. KEY EQUATIONS AND BOUNDARY CONDITOINS

Let a constant small temperature gradient ∇T that is directed along the polar axis $z = r\cos\theta$ be maintained in an unlimited stationary binary gas mixture using external sources.

Thermophoresis is described in a spherical coordinate system (r, θ, φ) associated with the center of mass of the evaporating droplet. With this choice of the position of the origin, the droplet can be considered as motionless, while the center of gravity of the external environment moves relative to the particle with velocity U_∞ in the direction opposite to the direction of its actual motion with the velocity $U_{\text{th}} = -U_\infty$ (U_{th} is the thermophoresis velocity). The distributions of velocities, pressures, relative concentrations, and temperatures are axially symmetric relative to the Oz axis.

It is assumed that the average distance between the droplets is significantly greater than the droplet diameter. Due to this, the mutual diffusion-hydrodynamic and thermal effect of the droplets can be neglected. The particle is considered to be heated, i.e., the condition $(T_{iS} - T_\infty)/T_\infty \geq 1$ is satisfied. The reason that the particle surface becomes heated up, for example, can be the presence of internal heat sources with density q_i. Heating leads to the fact that the average temperature of the droplet surface differs significantly from the temperature of the gaseous medium at a long distance from it. With this problem formulation, the type of internal heat sources is not given, which allows one to use the available results for a wider range of problems.

When describing the properties of a binary gas mixture, it is necessary to account for the dependence of viscosity μ_e, diffusion D_{12}, density ρ_e, and the thermal conductivities λ_e and λ_i on temperature. In this work, we use the power dependences [13, 14]

$$\mu_e(t_e) = \mu_\infty t_e^{\beta}, \quad \lambda_e(t_e) = \lambda_\infty t_e^{\alpha}, \quad D_{12}(t_e) = D_\infty t_e^{1+\omega}, \quad \lambda_i(t_i) = \lambda_{i0} t_i^{\gamma}, \quad \rho_e = \rho_\infty / t_e,$$

where $t_e = T_e/T_\infty$; $t_i = T_i/T_\infty$; $\mu_\infty = \mu_e(T_\infty)$; $\rho_\infty = \rho_e(T_\infty)$; $\lambda_\infty = \lambda_e(T_\infty)$; $D_\infty = D_{12}(T_\infty)$; $\lambda_{i0} = \lambda_i(T_\infty)$; $0.5 \leq \alpha \leq 1.0$, $0.5 \leq \beta \leq 1.0$; $0.5 \leq \omega \leq 1.0$, $-1 \leq \gamma \leq +1$; subscripts e and i correspond to a binary gas mixture and a particle; S denotes the values of physical quantities at the average temperature of the particle surface; "∞" denotes physical quantities characterizing the gas environment far from the droplet.

A binary gas mixture consists of two components, whose relative concentrations are denoted by C_1 and C_2. In this case, $C_1 = n_1/n_e$, $C_2 = n_2/n_e$, and $n_e = n_1 + n_2$ is the total number of molecules in a unit volume of the mixture; $\rho_1 = n_1 m_1$, $\rho_2 = n_2 m_2$, $\rho_e = \rho_1 + \rho_2$, m_1, n_1 and m_2, n_2 is the mass and numerical concentration of molecules of the first and second mixture components. The composition of the first component with concentration C_1 is the same as that of the substance of the liquid droplet, and the second component with concentration C_2 is considered to be the main one (carrier).

It is assumed that $C_1 \ll C_2$, i.e., diffusion evaporation is considered, in which the main influence on the process of heat and mass transfer in the vicinity of the droplet is determined by molecular diffusion. The circulation of the substance inside the droplet and its evaporation (reactive effect) are taken into account. The droplet radius is assumed to be constant (the variation time of the droplet radius is much greater than the relaxation time of diffusion and thermal inhomogeneities near it) [15]. As the droplet moves, it remains spherical in shape, i.e., the surface tension forces are much greater than the viscous resistance force. Due to the short time of thermal and diffusion relaxation, the process of heat and mass transfer in the particle–gas system is quasistationary and free convection can be neglected (the Grashof number is small). The problem is solved by the hydrodynamic method, i.e., the equations of hydrodynamics and heat and mass transfer with the corresponding boundary conditions are solved.

Within the framework of the assumptions formulated above, we solve a system of gas-dynamic equations [16]

$$\begin{gathered}\frac{\partial}{\partial x_k}P_e = \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial U_k^e}{\partial x_j}+\frac{\partial U_j^e}{\partial x_k}-\frac{2}{3}\delta_k^j\frac{\partial U_n^e}{\partial x_n}\right)\right], \quad \operatorname{div}(\rho_e U_e) = 0, \quad P_e = n_e k_{\rm B} T_e;\\ \mu_i \Delta U_i = \nabla P_i, \quad \operatorname{div} U_i = 0, \quad \operatorname{div}(\lambda_e \nabla T_e) = 0, \quad \operatorname{div}(\lambda_i \nabla T_i) = -q_i, \quad \operatorname{div}\left(\frac{n_e^2 m_1 m_2}{\rho_e} D_{12}\nabla C_1\right) = 0,\end{gathered} \tag{1}$$

describing the distribution of the mass velocity fields U_e and pressure fields P_e in a binary gas mixture and inside an evaporating droplet, as well as the temperature fields T_e and T_i and the relative concentration of the first component C_1.

At infinity ($y \to \infty$) and at $y \to 0$ (the finiteness of the physical quantities characterizing the droplet), the following boundary conditions are valid:

$$\begin{gathered} y \to \infty: \quad U_r^e = U_\infty \cos\theta, \quad U_\theta^e = -U_\infty \sin\theta, \quad P_e = P_\infty, \quad T_e = T_\infty + |\nabla T| r\cos\theta,\\ C_1 = C_{1\infty}, \quad y = r/R;\\ y \to 0: \quad T_i \neq \infty, \quad P_i \neq \infty, \quad |U_i| \neq \infty. \end{gathered} \tag{2}$$

The droplet is considered to be large, so the boundary conditions do not account for the linear corrections of the Knudsen number (temperature jump, isothermal slip, etc.) [3], the thermal diffusion, and the Dufour effect (second-order smallness effects). On the droplet surface ($y = 1$), the following conditions are satisfied:

$$n_2 U_r^e + D_{12}\frac{n_e^2 m_1}{\rho_e R}\frac{\partial C_1}{\partial y} = 0, \quad n_1 U_r^e - D_{12}\frac{n_e^2 m_2}{\rho_e R}\frac{\partial C_1}{\partial y} = n_{1i} U_r^i; \tag{3}$$

$$U_\theta^e - U_\theta^i = K_{TS}^{(0)}\frac{\nu_e}{R T_e}\frac{\partial T_e}{\partial \theta} + K_{DS}^{(0)}\frac{D_{12}}{R}\frac{\partial C_1}{\partial \theta}, \quad T_e = T_i; \tag{4}$$

$$-\lambda_e\frac{\partial T_e}{\partial y} + \lambda_i\frac{\partial T_i}{\partial y} = L\frac{n_e^2 m_1 m_2}{\rho_e}D_{12}\frac{\partial C_1}{\partial y} - \sigma_0\sigma_1 R(T_i^4 - T_\infty^4); \tag{5}$$

$$\mu_e\left(\frac{\partial U_\theta^e}{\partial y} + \frac{1}{y}\frac{\partial U_r^e}{\partial \theta} - \frac{U_\theta^e}{y}\right) + \frac{\partial \sigma}{\partial T_i}\frac{\partial T_i}{\partial \theta} = \mu_i\left(\frac{\partial U_\theta^i}{\partial y} + \frac{1}{y}\frac{\partial U_r^i}{\partial \theta} - \frac{U_\theta^i}{y}\right). \tag{6}$$

Equations (3) account for the impermeability of the radial mass flow for the second component of the binary gas mixture and the continuity of the radial flow for the first component. Equation (4) accounts for the difference in the tangential components of the velocities of the internal and external environments, equal to the sum of the thermal and diffusion slip velocities, as well as the equality of the temperatures. Equation (5) accounts for the continuity of the radial heat flows, including the heat spent on the phase transition of the droplet substance into the first component of the binary gas mixture and on radiation. Equation (6) shows the continuity of the tangential components of the viscous stress tensor and the dependence of the surface tension coefficient on temperature. In system (3)–(6), $n_1 U_r^e$, $n_2 U_r^e$ and $D_{12}(n_e^2 m_2/(\rho_e R))\partial C_1/\partial y$, $D_{12}(n_e^2 m_1/(\rho_e R))\partial C_1/\partial y$ are the radial convective and diffusion flows of the first and second components, respectively; x_k are the Cartesian coordinates; U_k^e are the mass velocity components; $K_{TS}^{(0)}$ and $K_{DS}^{(0)}$ denote the thermal and diffusion slip coefficients, which are determined from the solution of the Boltzmann equation for the Knudsen layer (in the general case, their form depends on the molecular interaction model, the masses of the molecules, and the average surface temperature of the particle [17, 18]); σ is the surface tension coefficient; σ_0 is the Stefan–Boltzmann constant; σ_1 is the integral emissivity of the particle; L is the specific phase transition heat; $U_\infty = |U_\infty|$ is the free-stream velocity; n_{1i} denotes the number of molecules per unit volume of a droplet; ν_e is the kinematic viscosity; $k_{\rm B}$ is the Boltzmann constant; U_r, U_θ denotes the radial and tangential mass velocity components in spherical coordinates.

Relative concentration of molecules of the first component (undergoing phase transition) of the external mixture $C_1 = C_{1S}(T_{iS})$ near the droplet surface in linear approximation with respect to the temperature perturbation $\delta T_i(y,\theta)$ satisfies the condition $C_1 = C_{1S}^{(H)}(T_{iS}) + C_{1S}^{*}(T_{iS})\delta T_i$. Here the value of $\delta T_i(y,\theta)$ is determined from the boundary conditions on the droplet surface; $C_{1S}^{(H)}(T_{iS}) = n_{1S}^{(H)}/n_e$; $n_{1S}^{(H)}$ denotes the maximum concentration of molecules of the first component, which depends on the average temperature of the droplet surface T_{iS}; $C_{1S}^{*} = (1/n_e)\partial n_{1S}^{(H)}/\partial T_i$ is derived from the maximum concentration.

The key parameters in the problem are the material constants μ_∞ and λ_∞, as well as quantities R, T_∞, $|\nabla T|$, and U_∞ retained during the particle motion. From these parameters, one can construct a dimensionless combination characterizing the temperature difference on the particle and playing the role of a small parameter of the problem: $\varepsilon = R|\nabla T|/T_\infty \ll 1$ [2, 3]. The calculation of the force and velocity of thermophoresis is to be limited to the first order of smallness.

For $\varepsilon \ll 1$, the incoming flow has only a perturbing effect, so the solution to the equations of gas dynamics is sought for in the form of an expansion in a small parameter.

2. FIELDS OF VELOCITIES, PRESSURES, TEMPERATURES, AND RELATIVE CONCENTRATION OF THE FIRST COMPONENT

For most gases, the thermal conductivity of a droplet is much greater than the thermal conductivity of gas. This means that the equation of dynamic viscosity can be constructed without the dependence on angle θ in the particle–gaseous medium system; also, it can be assumed that the mixture viscosity depends only on temperature $t_{e0}(y)$, i.e., $\mu_e(t_e(y,\theta)) \approx \mu_e(t_{e0}(y))$. In this case, $t_e(y,\theta) = t_{e0}(y) + \delta t_e(y,\theta)$, where $\delta t_e(y,\theta)$, $t_{e0}(y)$ are determined from the solution to the thermal problem.

With this assumption, the hydrodynamic part of the problem can be considered separately from the thermal and diffusion parts and the relationship between these parts can be established using boundary conditions. The method applied here is taken from [9–12], in which it was shown that the velocity-linearized Navier–Stokes equations (1) were reduced to a third-order inhomogeneous linear differential equation with an isolated singular point. The solution to the resulting differential equation was obtained in the form of generalized power series, and the existence theorem for the resulting solution is proven too [19].

Thus, for the mass velocity components U_e and the pressure components P_e, the following expressions are obtained that satisfy the boundary conditions (2):

$$U_r^e(y,\theta) = U_\infty \cos\theta G(y), \quad G(y) = A_1G_1(y) + A_2G_2(y) + G_3(y),$$

$$U_\theta^e(y,\theta) = -U_\infty \sin\theta g(y), \quad g(y) = A_1G_4(y) + A_2G_5(y) + G_6(y),$$

$$P_e(y,\theta) = P_\infty + \frac{\mu_\infty U_\infty}{R} t_{e0}^{\beta}\left\{\frac{y^2}{2}G''' + y\left(3 + \frac{\beta-1}{2}yf\right)G'' + \left(2 - y^2 f' - \frac{\beta}{2}y^2 f^2 + (\beta-2)yf\right)G'\right.$$
$$\left. + \left[\frac{f}{3} - \frac{y^2 f''}{2} - yf'\left(2 + \frac{y\beta f}{2}\right)\right]G\right\}.$$

Here

$$G_1(y) = \frac{1}{y^3}\sum_{n=0}^{\infty} C_n^{(1)} l^n, \quad G_3(y) = \sum_{n=0}^{\infty} C_n^{(3)} l^n + \omega_3 \ln(y) G_1(y),$$

$$f(y) = \frac{1}{t_{e0}(y)}\frac{dt_{e0}(y)}{dy}, \quad G_2(y) = \frac{1}{y}\sum_{n=0}^{\infty} C_n^{(2)} l^n + \omega_2 \ln(y) G_1(y), \quad l(y) = \frac{\Gamma_0}{y + \Gamma_0},$$

$$g(y) = G(y) + \frac{y}{2}(G'(y) - f(y)G(y)), \quad G_k(y) = \left(1 + \frac{l}{2(1+\alpha)}\right)G_{k-3}(y) + \frac{1}{2}yG'_{k-3}(y),$$
$$k = 4, 5, 6,$$

the primes next to f and G denote the derivatives of the corresponding functions; coefficients $C_n^{(1)}$, $C_n^{(2)}$, and $C_n^{(3)}$ are determined using the recurrence formulas from [19].

The general solution to system (1), describing the velocity and pressure fields inside an evaporating droplet, has the form [16]

$$U_r^i(y,\theta) = U_\infty \cos\theta(A_3 + A_4 y^2), \quad U_\theta^i(y,\theta) = -U_\infty \sin\theta(A_3 + 2A_4 y^2),$$

$$P_i(y,\theta) = P_0 + 10(\mu_i/R)U_\infty \cos\theta A_4 y^2.$$

The integration constants A_1, A_2, A_3, A_4, and Γ_0 are determined using the boundary conditions of the problem.

The heat conduction equations outside and inside the droplet are solved by the variable separation method, and the solution to the diffusion equation is given in the form of generalized power series [20].

Thus, the general solutions to the heat and mass transfer equations that satisfy the boundary conditions (6) have the form

$$t_e(y,\theta) = t_{e0}(y) + \varepsilon t_{e1}(y,\theta), \quad t_i(y,\theta) = t_{i0}(y) + \varepsilon t_{i1}(y,\theta),$$

$$C_1(y,\theta) = C_{10}(y) + \varepsilon C_{11}(y,\theta),$$

where

$$t_{e1}(y,\theta) = \frac{\cos\theta}{t_{e0}^{\alpha}}\left(y + \frac{\Gamma_1}{y^2}\right), \quad C_{10}(y) = C_{1\infty} + M_0(t_{e0}^{(1+\alpha-\omega)} - 1),$$

$$t_{i0}(y) = \left(B_0 + \frac{H_0}{y} - \frac{1}{y}\int_y^1 \psi_0 dy + \int_y^1 \frac{\psi_0}{y} dy\right)^{1/(1+\gamma)}, \quad H_0 = \frac{(1+\gamma)R^2}{3\lambda_{i0}T_\infty} J_0,$$

$$C_{11}(y,\theta) = \cos\theta M_1 \Phi_1(y), \quad \psi_0(y) = -\frac{R^2(1+\gamma)}{2\lambda_{i0}T_\infty} y^2 \int_{-1}^{+1} q_1 dx, \quad t_{e0}(y) = \left(1 + \frac{\Gamma_0}{y}\right)^{1/(1+\alpha)},$$

$$t_{i1}(y,\theta) = \frac{\cos\theta}{t_{i0}^{\gamma}}\left[B_1 y + \frac{H_1}{y^2} + \frac{1}{3}\left(y\int_1^y \frac{\psi_1}{y^2} dy - \frac{1}{y^2}\int_1^y \psi_1 y dy\right)\right], \quad H_1 = \frac{RJ_1}{3\lambda_{i0}T_\infty},$$

$$J_0 = \frac{1}{V}\int_V q_i dV, \quad J_1 = \frac{1}{V}\int_V q_i z dV,$$

$$\psi_1(y) = -\frac{3R^2}{2\lambda_{i0}T_\infty} y^2 \int_{-1}^{+1} q_1 x dx, \quad V = \frac{4}{3}\pi R^3, \quad x = \cos\theta, \quad z = r\cos\theta,$$

$$\Phi_1(y) = \frac{1}{y^2}\sum_{n=0}^{\infty} \Delta_n^{(1)} l^n, \quad \Delta_0^{(1)} = 1, \quad dV = r^2 \sin\theta dr d\theta d\varphi,$$

$\int_V q_i z dV$ is the dipole moment of the heat source density [21, 22]; integration is carried out over the entire volume of the evaporating droplet.

For $n \geq 1$, we determine $\Delta_n^{(1)}$ using the recurrence formulas

$$\Delta_n^{(1)} = \frac{1}{n(n+3)}\left[(n+1)\left(2(n-1) - \frac{\omega}{1+\alpha}\right)\Delta_{n-1}^{(1)} - (n-2)\left(n - 1 - \frac{\omega}{1+\alpha}\right)\Delta_{n-2}^{(1)}\right],$$

for $n < 0$, $\Delta_n^{(1)} = 0$.

The average value of the droplet surface temperature T_{iS} is obtained by solving the system

$$t_{iS} = t_{eS}, \quad \Gamma_0 = t_{eS}^{1+\alpha} - 1, \quad M_0 = \frac{C_{1S}^{(H)} - C_{1\infty}}{t_{eS}^{(1+\alpha-\omega)} - 1},$$

$$\frac{l^{(S)}}{1+\alpha} t_{eS} = \frac{R^2}{3\lambda_{eS}T_\infty} J_0 - L\frac{n_\infty^2 m_1 m_2 D_{12}^{(S)}}{\rho_\infty T_\infty \lambda_{eS}} \frac{1+\alpha-\omega}{1+\alpha} M_0 l^{(S)} t_{eS}^{\alpha-\omega} - \sigma_0\sigma_1 \frac{RT_\infty^3}{\lambda_{eS}}(t_{eS}^4 - 1).$$

Here $T_{iS} = T_\infty t_{iS}$; $T_{eS} = T_\infty t_{eS}$; $t_{iS} = t_{i0}\big|_{y=1}$; $t_{eS} = t_{e0}\big|_{y=1}$; $\lambda_{eS} = \lambda_\infty t_{eS}^{\alpha}$; $\lambda_{iS} = \lambda_{i0} t_{iS}^{\gamma}$; $n_\infty = n_e(T_\infty)$; $\rho_\infty = \rho_e(T_\infty)$; $D_{12}^{(S)} = D_\infty t_{eS}^{1+\omega}$; $l^{(S)} = l\big|_{y=1}$.

3. THERMOPHORETIC FORCE AND VELOCITY. RESULTS AND DISCUSSION

The resulting force acting on the particle is determined by integrating the stress tensor over the surface [16]:

$$F_z = \int\limits_{(S)} (-P_e \cos\theta + \sigma_{rr} \cos\theta - \sigma_{r\theta} \sin\theta) r^2 \sin\theta \, d\theta \, d\varphi \Big|_{r=R}. \qquad (7)$$

Here σ_{rr} and $\sigma_{r\theta}$ are the stress tensor components [16].

After the resulting expressions are substituted into Eq. (7) and integrated, the resulting force becomes the sum of the viscous resistance force of the medium F_μ and the thermophoretic force F_{th}:

$$F_\mu = 6\pi R \mu_\infty f_\mu U_\infty n_z, \qquad F_{\text{th}} = -6\pi R \mu_\infty f_{\text{th}} |\nabla T| n_z \qquad (8)$$

(n_z is the unit vector in the direction of the Oz axis).

The values of f_μ and f_{th} can be calculated using the following expressions:

$$f_\mu = \frac{2 N_2(1) + N_3(1)\mu_{eS}/(3\mu_{iS})}{3 N_1(1) + N_4(1)\mu_{eS}/(3\mu_{iS})}, \qquad N_1(1) = G_1(1)G_2'(1) - G_2(1)G_1'(1),$$

$$N_3(1) = G_3(1)G_1''(1) - G_1(1)G_3''(1) + \left(2 + \frac{l^{(S)}}{1+\alpha}\right)(G_3(1)G_1'(1) - G_1(1)G_3'(1)),$$

$$N_4(1) = G_2(1)G_1''(1) - G_1(1)G_2''(1) + \left(2 + \frac{l^{(S)}}{1+\alpha}\right)(G_2(1)G_1'(1) - G_1(1)G_2'(1)),$$

$$N_2(1) = G_1(1)G_3'(1) - G_3(1)G_1'(1), \qquad a_0 = 1 + 4\sigma_0\sigma_1 \frac{R T_\infty^3 t_{iS}^3}{\lambda_{iS}}, \qquad \nu_{eS} = \nu_e(T_{iS}),$$

$$f_{\text{th}} = 4\frac{\lambda_{eS}}{\lambda_{iS}} \frac{G_1(1)}{t_{eS}^{\alpha}\delta T_\infty (N_1(1) + N_4(1)\mu_{eS}/(3\mu_{iS}))} \left\{ K_{TS}^{(0)} \frac{\nu_{eS}}{t_{eS}} + K_{DS}^{(0)} D_{12}^{(S)} C_{1S}^{*} T_\infty + \frac{R}{3\mu_{iS}} \frac{\partial\sigma}{\partial t_i} + \frac{D_{12}^{(S)} n_\infty^2}{n_{2\infty}} \right.$$
$$\left. \times \left[C_{1S}^{*} T_\infty \frac{\Phi_1'(1)}{\Phi_1(1)} \left(a_1 - \frac{1}{n_{1i} t_{eS}}\right) - \frac{1+\alpha-\omega}{1+\alpha} M_0 l^{(S)} t_{eS}^{1+\alpha-\omega} \left((1+\omega)a_1 - \frac{\omega}{n_{1i} t_{eS}}\right)\right]\right\},$$

$$\delta = a_0 + 2\frac{\lambda_{eS}}{\lambda_{iS}} - L \frac{n_\infty^2 m_1 m_2}{\rho_\infty T_\infty \lambda_{iS} t_{eS}} D_{12}^{(S)} \left(C_{1S}^{*} T_\infty \frac{\Phi_1'(1)}{\Phi_1(1)} - \omega \frac{1+\alpha-\omega}{1+\alpha} l^{(S)} M_0 t_{eS}^{\alpha-\omega} \right),$$

$$a_1 = \frac{m_1 G_4(1)}{\rho_\infty G_1(1)} \left(1 - \frac{\mu_{eS}}{3\mu_{iS}} \frac{G_4'(1) + G_1(1) - G_4(1)}{G_4(1)} \right), \qquad \mu_{eS} = \mu_e(T_{iS}), \qquad \mu_{iS} = \mu_i(T_{iS}).$$

The total force is equated to zero: the droplet uniformly moves in a straight line as the force of viscous resistance of the medium is balanced by the photophoretic force. Thus, the following expression is obtained for the photophoretic velocity of a large spherical evaporating droplet

$$U_{\text{th}} = -\frac{f_{\text{th}}}{f_\mu} |\nabla T| n_z. \qquad (9)$$

This paper developes the general theory of thermophoresis of large evaporating droplets at arbitrary relative temperature differences in their vicinity.

Coefficient f_{th} included in the expressions for the thermophoresis force and velocity is the sum of four terms. The first term, which is proportional to the thermal slip coefficient $K_{TS}^{(0)}$, corresponds to the motion of the evaporating droplet in the direction of a temperature drop in the environment, i.e., from the higher-temperature region to the lower-temperature region. The fourth term (describing the reactive component of the momentum acting on the droplet) is due to the phase transition and circulation of the substance inside the droplet (internal flow). These terms are included in the expression for f_{th} with different signs. The droplet can move in the direction of both temperature growth and temperature drop. The third term is due to the presence of variable interfacial tension on the droplet surface. For most liquids, the surface tension decreases with an increase in the temperature ($\partial\sigma/\partial t_i < 0$), so the third term contributes to the force and velocity in the direction of temperature growth in the environment of the droplet. In the presence of the second term, which describes the diffusion slip proportional to $K_{DS}^{(0)}$, the droplet can move in the direction of both temperature growth and drop, depending on the mass of the binary gas mixture components. If the molecular mass of the external mixture component undergoing the phase transition on the droplet surface is $m_1 < m_2$, then $K_{DS}^{(0)} > 0$. Otherwise, $K_{DS}^{(0)} < 0$. The above-considered terms show that changes are observed not only in the thermophoresis force and velocity, but also in their direction. Moreover, the expression for f_{th} includes functions G_1, G_2, G_3, and Φ_1, as well as their first derivatives, which depend on the average temperature of the droplet surface and on indices α, β, ω, and γ. The average temperature of the droplet surface depends significantly on the heat source density q_i, which, in the case of absorption of electromagnetic radiation, can be estimated, for example, using the formula

$$q_i = \frac{4\pi n_k a_k}{n_s \lambda_0} I_0 B_k,$$

where $m_k = n_k + ia_k$ is the complex refractive index of the droplet; n_s is the refractive index of the medium; λ_0 and I_0 denote the wavelength and intensity of the incident radiation; B_k is the coordinate function calculated using the Mie theory [21]. The specific numerical estimates suggest that the presence of heat sources also affects the thermophoresis force and velocity.

Equations (8) and (9) for the resistance force, the thermophoresis force, and the thermophoresis velocity can also be used for small relative temperature differences. In this case, the average temperature of the particle surface differs slightly from the temperature of the ambient gas far from it; as $\Gamma_0 \to 0$ we have $G_1 = 1$, $G_1' = -3$, $G_1'' = 12$, $G_2 = 1$, $G_2' = -1$, $G_2'' = 2$, $G_3 = 1$, $G_2' = 0$, $G_3'' = 0$, $N_1 = 2$, $N_2 = 3$, $N_3 = 6$, $N_4 = 6$, $G_4 = -1/2$, $G_4' = 3/2$, $\Phi_1 = 1$, and $\Phi_1' = -2$.

In this case, Eqs. (8) and (9) transform into the corresponding expressions [1–3]. In particular, if the boundary conditions of continuity of radial mass and heat flows are replaced with the corresponding boundary conditions [4] and the corresponding transformations are performed, then Eq. (9) coincides with the formula obtained in [4]. With fulfillment of the conditions

$$L \to 0, \quad \frac{\mu_{eS}}{\mu_{iS}} \to 0, \quad D_{12}^{(S)} \to 0, \quad \frac{\partial\sigma}{\partial T_i} \to 0, \quad \frac{\partial\sigma}{\partial t_i} \to 0, \quad C_{1S}^{(H)}(T_{iS}) \to 0$$

and with the assumption that the value of the thermal slip coefficient is equal to the Maxwell value, the Epstein formula is obtained [1].

The effect of heating the surface of the evaporating droplet on the thermophoresis force at arbitrary temperature differences is described. Figures 1 and 2 demonstrate how the average particle surface temperature T_{iS} affects functions $f_{\text{th}}^* = f_{\text{th}}/f_{\text{th}S}$ and $f_{\text{th}m}^* = f_{\text{th}m}/f_{\text{th}mS}$. The calculation of $f_{\text{th}m}^*$ is based on the equations of thermophoresis of a large evaporating droplet at small temperature differences [2, 3]. However, the molecular transfer coefficients are calculated using the average droplet surface temperature. This is necessary to determine the possibility of using equations for describing thermophoresis at small relative temperature differences in the case where one not only accounts for surface heating, but also for the dependence of the transfer coefficients on temperature. Figures 1 and 2 suggest that there is a significant error in the results of the calculations based on the equations of the thermophoresis force and velocity obtained at small temperature differences. The values of the coefficients are taken from [13, 14, 23].

CONCLUSIONS

This paper presents the equations of the thermophoresis force and velocity. These equations describe the motion of large evaporating droplets in a temperature gradient field and account for the power-law dependence of the molecular transfer coefficients (viscosity, thermal conductivity, and diffusion), as well

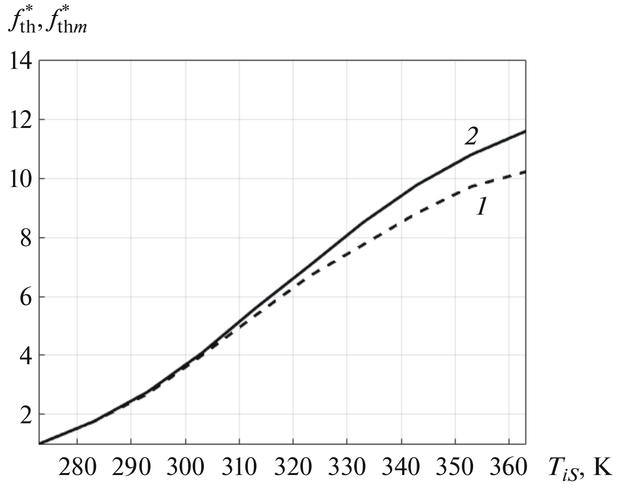

Fig. 1. Dependences of functions f_{th}^* (*1*) and $f_{\text{th}m}^*$ (*2*) on the average temperature T_{iS} of the surface of the airborne water droplet having the radius $R = 30\times10^{-6}$ m at $T_\infty = 273$ K, $P_\infty = 10^5$ Pa, $C_{1\infty} = 0.001$, $\alpha = 0.765$, $\beta = 0.693$, $\omega = 0.652$, $K_{TS}^{(0)} = 1.161$, and $K_{DS}^{(0)} = 0.3$.

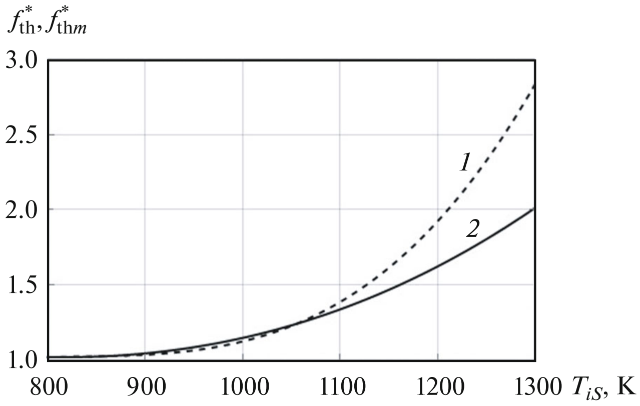

Fig. 2. Dependences of functions f_{th}^* (*1*) and $f_{\text{th}m}^*$ (*2*) on the average temperature T_{iS} of the surface of the airborne lithium droplet having the radius $R = 30\times10^{-6}$ m at $T_\infty = 288$ K, $P_\infty = 10^5$ Pa, $C_{1\infty} = 0.001$, $\alpha = 0.765$, $\beta = 0.693$, $\omega = 0.652$, $K_{TS}^{(0)} = 1.161$, and $K_{DS}^{(0)} = 0.3$.

as the density of the gaseous medium on temperature at arbitrary relative temperature differences. The numerical estimates show that the dependences of the thermophoresis force and velocity on the average temperature of the particle surface are nonlinear.

FUNDING

This work was supported by ongoing institutional funding. No additional grants to carry out or direct this particular research were obtained.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

REFERENCES

1. Epstein, P.S., Zur theorie des radiometers, *Z. Phys.*, 1929, vol. 54, nos. 7/8, pp. 537–539.

2. Yalamov, Yu.I., Gaidukov, M.N., and Shchukin, E.R., Theory of Thermophoresis and Diffusiophoresis of Moderately Large and Large Droplets of Airborne Particles, *Izhenerno-Fizicheskii Zhurnal*, 1975, no. 2, pp. 505–507.
3. Galoyan, V.S. and Yalamov, Yu.I., *Dinamika kapel' v neodnorodnykh vyazkikh sredakh* (Droplet Dynamics in Inhomogeneous Viscous Media, Erevan: Luis, 1985).
4. D'yakonov, S.N., Efremov, E.V., Morozov, A.A., The Effect of the Evaporation Coefficient on the Thermophoresis of a Volatile Single-Component Drop in a Binary Gas Mixture, *Zhurnal Tekhnicheskoi Fiziki*, 2002, vol. 72, no. 3, pp. 11–16 [*Tech. Phys.*, 2002, vol. 47, no. 3, pp. 306–312; DOI: 10.1134/1.1463119].
5. Selim, A., Hossain, M.A., and Rees D.A.S., The Effect of Surface Mass Transfer on Mixed Convection Flow Past a Heated Vertical Flat Permeable Plate With Thermophoresis, *Intern. J. Thermal Sci.*, 2003, vol. 42, pp. 973–982;].
https://doi.org/10.1016/S1290-0729(03)00075-9
6. Reineck, P., Wienken, C.J., and Braun, D., Thermophoresis of Single Stranded DNA, *Electrophoresis*, 2010, vol. 31, no. 2, pp. 279–286;
https://doi.org/10.1002/elps.200900505
7. Bubenchikov, M.A., Potekaev, A.I., and Bubenchikov, A.M., Thermophoresis of Ultrafine and Nanosized Particles, *Izvestiya Vuzov. Fizika*, 2013, vol. 56, no. 7, pp. 59–61 [*Russ. Phys. J.*, vol. 56, no. 7, pp. 785–790; DOI: 10.1007/s11182-013-0100-y].
8. Jerabek-Willemsen, M., André, T., Wanner, A., et al., MicroScale Thermophoresis: Interaction Analysis and Beyond, *J. Molec. Structure*, 2014, vol. 107, pp. 101–113;
https://doi.org/10.1016/j.molstruc.2014.03.009
9. Malai, N.V., Shchukin, E.R., Stukalov, A.A., et al., Gravity-Induced Motion of a Uniformly Heated Solid Particle in a Gaseous Medium, *Prikl. Mekh. Tekh. Fiz.*, 2008, vol. 49, no. 1, pp. 74–80 [*J. Appl. Mech. Tech. Phys.*, 2008, vol. 49, no. 1, pp. 58–63; DOI: 10.1007/s10808-008-0008-4].
10. Malai, N.V., Limanskaya, A.V., Shchukin, E.R., et al., Photophoresis of Heated Large Spherical Aerosol Particles, *Zhurnal Tekhnicheskoi Fiziki*, 2012, vol. 82, no. 10, pp. 42–50 [*Tech. Phys.*, 2012, vol. 57, no. 10, pp. 1364–1371; DOI: 10.1134/S1063784212100131].
11. Malai, N.V., Limanskaya, A.V., and Shchukin, E.R., Thermophoretic Motion of Large Heated Aerosol Spherical Particles, *Prikl. Mekh. Tekh. Fiz.*, 2016, vol. 57, no. 2, pp. 164–171 [*J. Appl. Mech. Tech. Phy.*, 2016, vol. 57, no. 2, pp. 337–343; DOI: 10.1134/S0021894416020176].
12. Malai, N.V., Shchukin, E.R., Shulimanova, Z.L., and Efimtseva, D.N., Diffusiophoresis and Photophoresis of Heated Large Nonvolatile Aerosol Spherical Particles, *Prikl. Mekh. Tekh. Fiz.*, 2020, vol. 61, no. 3, 129–139 [*J. Appl. Mech. Tech. Phy.*, vol. 61, no. 3, pp. 426–435; DOI: 10.1134/S0021894420030141].
13. Bretsznajder, St., *Wlasnosci gazow i cieczy*, 1962, Wydawn. Naukowo-Techiczne.
14. Vargaftik, N.B., *Handbook of Physical Properties of Liquids and Gases: Pure Substances and Mixtures*, 1976, Hemisphere Publ.
15. Kuz'min, M.K., Generalized Equations for the Variation Rate of Radii of Large Aerosol Droplets During Evaporation and Condensation, *Vestnik Moskovskogo Gosudarstvennogo Oblastnogo Universiteta. Seriya Fizika—Matematika*, 2018, no. 4, pp. 155–156.
16. Landau, L.D. and Lifshitz, E.M., *A Course in Theoretical Physics*, Vol. 6: *Fluid Mechanics*, 1959, Pergamon Press.
17. Poddoskin, A.B., Yushkanov, A.A., and Yalamov, Yu.I., Theory of Thermophoresis of Moderately Large Aerosol Particles, *Zhurnal Tekhnicheskoi Fiziki*, 1982, vol. 52, no. 11, pp. 2253–2261.
18. Yushkanov, A.A., Savkov, S.A., and Yalamov, Yu.I., Dependence of Slip Coefficients on the Intermolecular Interaction Model, *Izhenerno-Fizichskii Zhurnal*, 1986, vol. 51, no. 4, pp. 686–687
19. Malai, N.V., Limanskaya, A.V., and Shchukin, E.R., 2015. Solution of a Boundary Value Problem for the Navier–Stokes Equations Linearized With Respect to Velocity: Nonisothermal Flow of a Gaseous Medium Past a Uniformly Heated Sphere, *Diff. Equat.*, 2015, vol. 51, no. 10, pp. 1319–1329;
https://doi.org/10.1134/S0012266115100079
20. Kamke, E., *Differentialgleichungen, Losungsmethoden und Losungen*, 1956. Akademische Verlagsgesellschaft Geest & Portig,
21. Bohren, C.F. and Huffman, D.R., *Absorption and Scattering of Light by Small Particles*, 1986, John Wiley & Sons, Inc.
22. Beresnev, S.A. and Kochneva, L.B., Asymmetry Factor of Radiation Absorption and Photophoresis of Aerosols, *Optika Atmosfery i Okeana*, 2003, vol. 16, no. 2, pp. 134–141.
23. Smithells, K.J., *Metals: Reference Book*, 1976, Butterworth and Co. (Publishers) Ltd.